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Features of the Spectral Characteristics of Narrow-Band Optical Filters with Oblique Incidence of the Radiation Beam

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Abstract—The performance of narrow-band optical filters on the basis of frustrated total internal reflection was analyzed taking into account the Gaussian distribution of the angle of radiation incidence. The spectral characteristics of these optical filters are modelled with and without divergence of the incident beam. It is shown that, when measuring the spectral characteristics of optical filters, the divergence of the incident radiation beam should not be greater than a certain limit value that does not exceed several angular minutes.

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Narrow-band optical filters, the operation of which is based on the phenomenon of frustrated total internal reflection (FTIR), were created relatively recently. This type of optical filter has several advantages compared to optical filters operating on the basis of interference with a normal incidence of radiation. These include, in particular, the minimum number of interference layers that make up the optical filters, wide suppression zones, and the possibility of using them as polarizers. The narrowband FTIR optical filter considered in [1] consists of three alternating dielectric layers made of materials with low and high refractive indices located between media with the same refractive indices. The layers are formed on a substrate made in the form of a triangular prism and made of an optical material with a high refractive index, one of the acute angles of which is greater than the limiting angle of total internal reflection at the prism–layer interface. The layer adjoining the substrate has a low refractive index, the middle layer has the same refractive index as that of the substrate, and the third layer with a low refractive index borders on a prism similar to a substrate prism. In [2], the layers adjoining the prisms are replaced by air gaps. Another option of narrow-band optical filter based on the FTIR effect considered in [3] differs in that the number of used interference layers increases to 21.

In the calculations (ideal condition), the angle of incidence of radiation on the coating from which narrow-band FTIR optical filters are formed must be equal to some constant value α_0. When measuring the spectral characteristics of these filters in spectrophotometers, the results are often much worse than theoretical calculations. Perhaps the main cause of this problem is the divergence of the incident radiation beam used in spectrophotometers, since narrow-band optical filters based on FTIR effect operate when the radiation is incident at an angle to which their spectral characteristics are very sensitive. Therefore, it is easy to show that the incident beams used should be strictly parallel. In practice, well-collimated light beams can satisfy this requirement. Laser beams are not strictly parallel [4–6]; therefore, coincidence of the calculated and experimental curves is unlikely. In this paper, we consider the condition that determines the allowable divergence of the incident beam when measuring the spectral characteristics of narrowband filters based on the FTIR effect. The real divergence of laser beams is several fractions of milliradians, i.e., several angular minutes [7–9].

To improve the accuracy of the calculation of the characteristics of these filters, it is necessary to take into account the divergence of the incident beam. In this case, the radiation beams are incident on the coating of the optical filters not strictly at angle α_0, but in a certain interval of angles in the limit of their divergence, which may lead to a deviation of the experimental spectral characteristics of the optical filters from the calculated ones. Suppose that the incident beams diverge in the limit of a small angle $\Delta\theta$ and the angle of incidence of radiation on the coating deviates by a small value $\Delta\alpha_0$ (Fig. 1). In the general case, the radiation is incident on the filters from the air, the refractive index of which is equal to unity. From Fig. 1, it is easy to see that $\Delta\alpha_0 = \Delta\theta'$, where $\Delta\theta'$ is the angle of refraction of the radiation in the first prism of the filters corresponding to the deviation of the incident beam at angle $\Delta\theta$. This angle can be determined from Snell's law:

$$\sin\Delta\theta' = \frac{1}{n_0}\sin\Delta\theta, \quad (1)$$

where n_0 is the refractive index of the material from which the prisms of optical filters are made. Since the divergence of the incident beam is small, the following approximations are valid: $\sin\Delta\theta' = \Delta\theta'$ and $\sin\Delta\theta = \Delta\theta$. Therefore,

$$\Delta\alpha_0 = \Delta\theta' = \frac{\Delta\theta}{n_0}. \quad (2)$$

The maximum deviation of the incident beam is $\pm\Delta\theta$. According to (2), the deviation of the angle of incidence of light on the filter coating between the prisms is $\pm\Delta\theta/n_0$. Consequently, the light rays from the beam are incident on the filter coating at different angles in the range of $[\alpha_0 - \Delta\theta/n_0; \alpha_0 + \Delta\theta/n_0]$. The direction of incidence of light rays can be considered as a random variable, which allows the use of the Gaussian distribution (or the normal distribution) to describe them [10, 11]. This distribution is characterized by a probability density function. For the angle of incidence of light rays on the coating, according to [12], the probability density function is expressed as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\alpha_0)^2}{2\sigma^2}\right), \quad (3)$$

where σ is the mean square deviation of the angle of incidence of light, x is the angle of incidence of radiation on the coating, and the expected value is angle of incidence of light α_0. To ensure an incidence on this coating at least 99.8% of the light rays at an angle in the range $[\alpha_0 - \Delta\theta/n_0; \alpha_0 + \Delta\theta/n_0]$ the mean square deviation of the angle of radiation incidence should satisfy the equality $3\sigma = \Delta\theta/n_0$. From here, we obtain $\sigma = \Delta\theta/3n_0$. The probability of some arbitrary deviation of the angle of incidence of the radiation fall into the interval a–b, where $a < b$, is equal to

$$\rho = \int_a^b f(x)dx. \quad (4)$$

The energy transmittance of the considered filters can be determined from the following expression:

$$T(x,\lambda) = \frac{(1-R)^2}{1 - 2R\cos 2\varphi_0 + R^2}, \quad (5)$$

where λ is the wavelength of the incident radiation, φ_0 is the phase thickness of the middle layer, and R is the reflection coefficient of the layers adjacent to the prisms of the optical filters. The phase thickness of the middle layer is determined by the expression

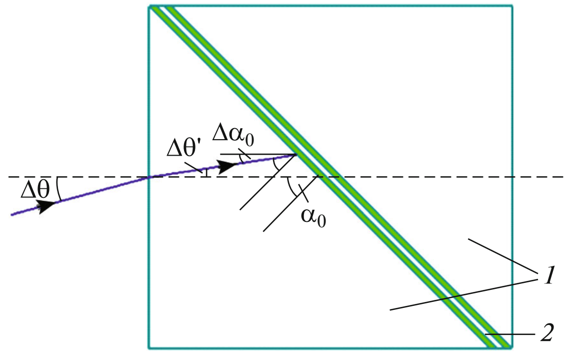

Fig. 1. Illustration of the deviation of the angle of incidence of radiation on the optical filter. (*1*) Prisms included in the filter, (*2*) the coating between prisms.

$$\varphi_0(x) = \frac{2\pi n_0 d_0}{\lambda}\cos x, \quad (6)$$

where d_0 is the thickness of the middle layer. The reflection coefficient of layers with a low refractive index is described by the following expression:

$$R(x,\lambda) = \frac{(n_0^2 n^{-1}\sin\varphi - n\sin\varphi)^2}{4n_0^2\cos^2\varphi + (n_0^2 n^{-1}\sin\varphi + n\sin\varphi)^2}, \quad (7)$$

where n is the refractive index of the layers adjacent to the prisms and φ is the phase thickness of these layers, which is defined as

$$\varphi(x) = \frac{2\pi d}{\lambda}\sqrt{n^2 - n_0^2\sin^2 x}, \quad (8)$$

where d is the thickness of the layers adjacent to the prisms.

Taking into account the Gaussian distribution of incident radiation, the expression for determining the energy transmittance of this filter can be rewritten as

$$T(\lambda) = \int_{\alpha_0-\Delta\theta/n_0}^{\alpha_0+\Delta\theta/n_0} T(x,\lambda)f(x)dx. \quad (9)$$

Expression (9) makes it possible to determine the energy transmittance of the narrow-band FTIR optical filters taking into account the divergence of the incident radiation beam in a certain angular limit $\pm\Delta\theta$. Therefore, it allows one to determine limiting value $\Delta\theta$ at which the maximum transmission of these optical filters is still observed.

Taking into account the data presented in [1], one possible design variant of narrow-band FTIR optical filters was made from the following materials: two identical prisms made of Russian K8 glass, layers with a low refractive index of MgF_2, and a middle layer of SiO_2. Norland 61 optical glue was used to connect the prisms. The appearance of this optical filter is shown in Fig. 2.

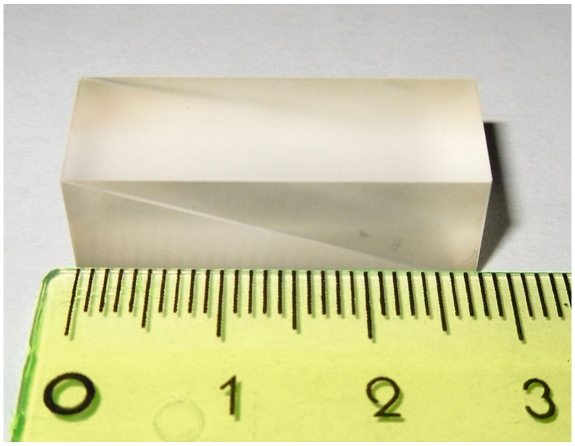

Fig. 2. The appearance of an FTIR optical filter with dimensions of 8 × 8 × 28 mm.

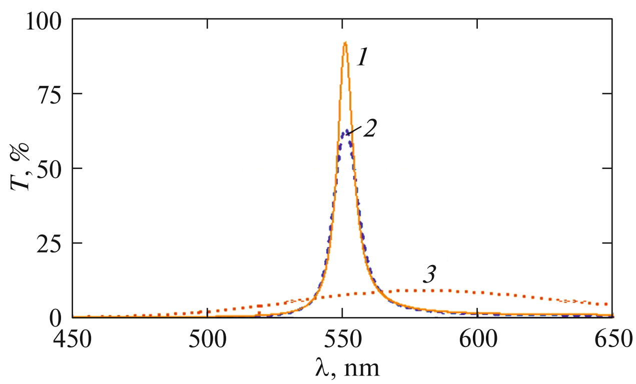

Fig. 3. Spectral characteristics of an FTIR optical filter for the divergence of the incident radiation beam: $\Delta\theta$ = (*1*) 0′, (*2*) 5′, and (*3*) 90′.

When measuring the spectral characteristics of this filter in spectrophotometers, for example, in a PE-5400UF spectrophotometer, the transmission maximum is not detected (curve *3* in Fig. 3). This circumstance can be explained by the fact that the radiation beam in the indicated spectrophotometer is not a parallel beam, with its divergence angle being 1.5°.

Analysis of Fig. 3 shows that the divergence of the incident beam strongly distorts the spectral characteristics of the narrowband optical filter, operation of which is based on the FTIR phenomenon in the interference layers. To observe a transmittance of at least 60% at the maximum in the transparency zone, this optical filter should operate with a diverging incident beam of radiation of no more than 5 angular minutes.

Thus, for narrow-band optical filters operating with oblique incidence of radiation, a small divergence of the incident beam can significantly distort their spectral characteristics. The results obtained allow us to establish the tolerance for the divergence of the incident radiation beam used in the operation or measurement of the spectral characteristics of these optical filters. For the considered design of the narrowband filter based on the FTIR phenomenon in the interference layers, the value of the limiting divergence of the incident radiation beam is 5 angular minutes.

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