Proposal of Combination of NOMA and Beamforming Methods for Downlink Multi-users systems

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***Abstract*—In this paper, a downlink multi-users (MUs) system, where the MUs are clustered into several clusters based on the located regions, is taken into consideration. A combinition of beamforming and no-orthogonal multiple access (NOMA) methods is proposed and the base station (BS) communicates instantaneously with all users in multiple clusters by the proposed method. The close-form expressions of outage probability and ergodic capacity are derived to evaluate our proposed method. The derived analytical results are verified by Monte Carlo simulations and the proposed method is compared with conventional orthogonal multiple access (OMA) method. The results indicate that our proposed method can improve the performance of downlink MUs systems more than the OMA method.**

***Index Terms*—Non orthogonal multiple access, Successive interference cancellation, close-form expression, outage probability, ergodic capacity.**

I. INTRODUCTION

The non-orthogonal multiple access (NOMA) is considered as a promising multiple access technique for the fifth generation (5G) mobile networks due to its superior spectral efficiency [1]–[3]. The main technique of NOMA is to use the power domain for multiple access which adopts the superposition code at the transmitter and successive interference cancellation (SIC) method to detect the signals at the receiver [4]. While decoding the received signal, the SIC operates at each user to separate superimposed symbols and removes the inter-user interference. In the downlink NOMA scenario, less transmission power is allocated to users having better channel conditions, whereas more transmission power is allocated to users having worse channel conditions [5], [6]. The purpose of this strategy is to achieve a trade-off between system throughput and fairness of users [7]. Moreover, NOMA systems allocate powers based on priority of users, users that have higher priority are allocated more power whereas users of lower priority are allocated less power [8], [9].

In addition, compared with conventional multiple access systems, such as TDMA, CDMA, FDMA and so on, the NOMA systems offer better fairness of users, even for users with weak CSI.

The previous works which are focusing on analyzing performance of NOMA in different scenarios are summarized as follows. In [10], the authors investigated a NOMA downlink system, where all users locate randomly, and derived the close-form expressions of outage probability as well as ergodic capacity. In [11], the authors investigated both downlink and uplink of NOMA systems. The dynamic power allocation with undertaking QoS for different users was proposed. Based on the obtained results, the authors concluded that the proposed dynamic power allocation can provide more whippiness to fairness performance of all users. Outage performance and ergodic capacity were investigated in [12]. In this work, the authors analyzed a NOMA system under condition of fairness of the users quality, leading to the necessarity of allocation of power following the channel gains. In [13], the authors discussed on NOMA scheme aspect of cognitive radio, in which the power allocation coefficient is chosen to satisfy the predefined requirements of user QoS. The authors in [14], [15] and [16] proposed the NOMA multi-users (MUs) multipe input multiple output (MIMO) systems and the results shown that the performance is improved in the case that MUs is gathered into a cluster.

Throughout the reviewed works, the bandwidth efficiency of NOMA system is improved, and then the

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throughput is enhanced. These characteristics lead to the fact that NOMA technology is suitable for MUs. Furthermore, the NOMA technology is providing an ultrahigh connectivity for many billions of electronic devices in the IoT by using the non-orthogonal properties. Especially, the design for NOMA systems has a low-complexity when comparing with other multiple access methods such as: MUs shared access (MUSA), pattern-division multiple access (PDMA) and sparse-code multiple access (SCMA) [2].

The above mentioned works just investigated the performance of the MUs NOMA system and evaluate these problems by simulation. To the best of our knowledge, there is a research that takes multi-beam multiple input single output (MISO) NOMA into consideration [17]. However, the author assumed that each cluster has only two users, namely near user and far user. We are going to extend this work with more users in the same beam. Consequently, in this paper, we propose a combination of NOMA and beamforming methods for downlink MUs systems, in which every user has only one antenna, and several nearby users become a cluster, the base station (BS) is equipped with multiple antennas and transmits signals to clusters by beamforming method. The contributions of our paper are summarized as follows.

- We focus on the downlink MUs system with multiple antennas at the BS and single antenna at the user site. Several users are clustered and becomes a cluster based on their location. To mitigate the cros-wise interference from the other clusters, the beamforming is applied at the BS site. Moreover, the NOMA method is combined with the beamforming method to cancel the interferences from the other users in the same cluster.
- The system performance is analyzed, and close-form expressions of outage probability and ergodic capacity for each user at the arbitrary cluster are derived in order to evaluate the proposed system model. We compare the simulation result with calculation result to confirm the close-form expressions.
- We compare the performance of the proposed method and the conventional OMA system to indicate the proposed method is better.

The rest of the paper is organized as follows. Section II represents the MUs NOMA system model and channel model. The performance analysis is given in Section III. Numerical results are depicted in Section IV. Finally, the Section V concludes the work.

II. SYSTEM MODEL

In this model, we consider a combination of NOMA and beamforming for downlink MUs-MISO systems, where the BS is equipped M antennas to serve the M clusters, while each user has a single antenna because of limited size. Let the number of users in every cluster is the same and equals N. To simply compose clusters, the users are clustered by spatial direction methods such as Global Positioning System (GPS) technique or user location tracking algorithms and so on.Note that the users in each cluster have different propagation distances, however we assume the channel state information (CSI) is perfect. Without loss of generality, the CSI is obtained at the BS in feedback uplink transmission slot.

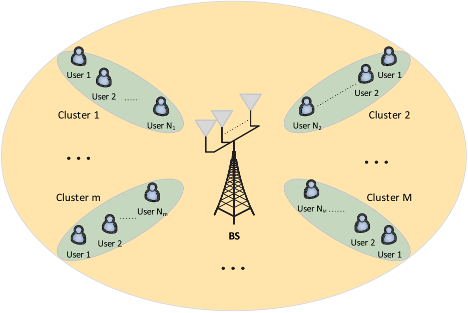

Fig. 1. System Model.

To keep the trade-off between complexity of implementation and performance of systems, we use zero-force beamforming (ZFBF) at the BS. We design a weight $\mathbf{w}_m$ for the mth cluster to mitigate the interference from the other clusters.

The channel matrix between the BS and nth user in mth cluster (hereafter called (m,n)th user) is denoted by $\mathbf{h}_{m,n}$, where $\mathbf{h}_{m,n} = [h_{m,1}, h_{m,2}, \cdots, h_{m,M}] \in \mathbb{C}^{1\times M}$ with $m \in \{1,\cdots,M\}$ and $n \in \{1,\cdots,N\}$. The channel coefficients are denoted by $\mathrm{h}_{m,n} \sim \mathcal{CN}(0,\Omega_{m,n})$, where $\mathbb{E}\{|\mathrm{h}_{m,i}|^2\} = \Omega_{m,i}$ with $i \in \{1,\cdots,N\}$ are variances of the channel gains, where $\mathbb{E}\{\cdot\}$ denotes the average operator. In this model, we assume that all channels are quasi-static independent and identically distributed (i.i.d.) and keeps constants in a time slot, however they changes independently over time slots meaning block fading.

Every antenna at the BS transmits the superposition code, which includes N signals of the mth cluster, it can be described by $\mathbf{x}_{\mathrm{S},m} = [x_{m,1},\cdots,x_{m,N}]^T$, where $x_{m,n}$ is the signal of (m,n)th user, $\mathbb{E}\{|\mathbf{x}_{\mathrm{S},m}|^2\} = P_{\mathrm{S}}$ and $[\cdots]^T$ denotes the transpose matrix. The signals are multiplied beamforming vector, $\mathbf{w}_m$, together at the output antenna based on the principle of zero-forcing (ZF) method. Hence the signals which are intended to the mth cluster, are given by

$$\mathbf{x}_{\mathrm{S},m} = \mathbf{w}_m \sum_{n=1}^{N} \sqrt{a_{m,n} P_{\mathrm{S}}} x_{m,n}, \tag{1}$$

where $a_{m,n}$ denotes the power allocation coefficient for the (m, n)th user and $\sum_{n=1}^{N} a_{m,n} = 1$. In order to maximize the channel gain while cancelling the inter-cluster interference, $\mathbf{w}_m$ can be represented as the projection of $\mathbf{h}_m$ in the null space of the interference channels related to the mth cluster. Mathematically, $\mathbf{w}_m$ is given as

$$\mathbf{w}_m = \frac{\mathbf{B}\mathbf{h}_m}{\|\mathbf{B}\mathbf{h}_m\|}, \tag{2}$$

where $\mathbf{B} = \mathbf{I}_N - \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$ and the structure of matrix of the system, $\mathbf{H}_m = [\mathbf{h}_{m,n}]^{M \times N}$, at the BS is given in detail as follows.

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{m-1}, \mathbf{h}_{m+1}, \cdots, \mathbf{h}_M]^T, \tag{3}$$

Hence, we have

$$\mathbf{h}_m^H \mathbf{w}_j = 0, \forall m \neq j. \tag{4}$$

In this paper, we assume that the users determine the decoding signal order based on only the channel gains. According to assumption of the large-scale fading channel model, the distances from BS to the users are sorted as $d_{m,1} > d_{m,2}, \cdots, > d_{m,N}$. Therefor without loss of generality, we assume the channel gains of the mth cluster are sorted as follow:

$$|\mathbf{w}_m\mathbf{h}_{m,1}|^2 \leq |\mathbf{w}_m\mathbf{h}_{m,2}|^2 \leq, ..., \leq |\mathbf{w}_m\mathbf{h}_{m,N}|^2. \tag{5}$$

Hence, following the principle of NOMA, the power allocation coefficient for each user is ordered as follows:

$$a_{m,1} \geq a_{m,2} \geq, ..., \geq a_{m,N}. \tag{6}$$

The structure of output signal after SIC process at the (m, n)th user with perfect beamforming is given by

$$\begin{aligned} y_{m,n} &= \mathbf{h}_{m,n}\mathbf{w}_m \sum_{n=1}^{N} \sqrt{a_{m,n} P_{\mathrm{S}}} x_{m,n} + \mathbf{n}_{m,n} \\ &= \underbrace{\mathbf{h}_{m,n}\mathbf{w}_m \sqrt{a_{m,n} P_{\mathrm{S}}} x_{m,n}}_{\text{desired signal of } (m,n)\text{th user}} \\ &\quad + \underbrace{\mathbf{h}_{m,n}\mathbf{w}_m \sum_{i=n+1}^{N} \sqrt{a_{m,i} P_{\mathrm{S}}} x_{m,i}}_{\text{interference of other user}} \\ &\quad + \underbrace{\mathbf{h}_{m,n}\mathbf{w}_m \sum_{k=1}^{n-1} \sqrt{a_{m,k} P_{\mathrm{S}}} x_{m,k}}_{\text{interference of imperfect SIC}} + \mathbf{n}_{m,n}, \end{aligned} \tag{7}$$

where $\mathbf{n}_{m,n} = [n_{m,1}, \cdots, n_{m,N}] \in \mathbb{C}^{1 \times N}$ with $n_{m,n} \sim \mathcal{CN}(0, \sigma_{m,n}^2)$ is an i.i.d additive white Gaussian noise (AWGN) at the (m, n)th user. In case of perfect SIC, the term $\mathbf{h}_{m,n}\mathbf{w}_m \sum_{k=1}^{n-1} \sqrt{a_{m,k} P_{\mathrm{S}}} x_{m,k}$ equals to zero.

Because the ZFBF is applied, it can cancel the inter-cluster interference, however the intra-cluster interference meaning the interference between users in the same cluster is still existing. Hence the successive interference cancellation (SIC) technique should be used to remove the intra-cluster interference. Note that, in this paper we assume the perfect SIC at all users.

At receiver site, each user operates the SIC to remove the interference from the other users which have worse channel gain, i.e., the interference from $(m, n + 1 \cdots N)$th users is removed by (m, n)th user. The signal interference noise ratio (SINR) of (m, n)th user is denoted by $\gamma_{m,n}$, and from (7), the SINR is given as

$$\gamma_{m,n} = \frac{P_{\mathrm{S}} a_{m,n} |\mathbf{h}_{m,n}\mathbf{w}_m|^2}{\sum_{i=n+1}^{N} P_{\mathrm{S}} a_{m,i} |\mathbf{h}_{m,n}\mathbf{w}_m|^2 + \sigma_{m,n}^2}. \tag{8}$$

The (m, N)th user operate the SIC to cancel all interference signals of the other users, thus the SNR of (m, N)th user is described as follows.

$$\gamma_{m,N} = \frac{P_{\mathrm{S}} a_{m,N} |\mathbf{h}_{m,N}\mathbf{w}_m|^2}{\sigma_{m,N}^2}. \tag{9}$$

Since the nomalized $\mathbf{w}_m$ is designed independently of $\mathbf{h}_{m,n}$, the $|\mathbf{h}_{m,i}\mathbf{w}_m|^2$, $i \in \{1, \cdots, N\}$, is Chi-square distributed with 2 degree of freedom [18]. If a variable is random and has Chi-square distribution with 2 degree of freedom, it becomes a Rayleigh distribution [19, pp. 16]. Therefore, $|\mathbf{h}_{m,n}\mathbf{w}_m|^2$ is the exponential random variables. Our detail system model is following the system model described in [20].

III. Performance Analysis

A. Outage probability of (m, n)th user

In this section, we derive the outage probability of (m, n)th user in our proposed system. In order to keep fairness for all users, the required data rate from the BS to each user is assumed to be the same and denoted by r. Hence, the outage event happens if the instantaneous end-to-end capacity, $\log_2(1 + \gamma_{m,n})$, is smaller than r. The mathematical outage probability can be given as

$$\mathrm{OP}_{m,n} = \Pr\Big(\gamma_{m,n} \leq \gamma_{\mathrm{th}}\Big). \tag{10}$$

where $\gamma_{\mathrm{th}} = 2^r - 1$ is the outage threshold.

Replacing (8) into (10), we have

$$\mathrm{OP}_{m,n} = \Pr\left(\frac{P_{\mathrm{S}} a_{m,n} |\mathbf{h}_{m,n}\mathbf{w}_m|^2}{\sum_{i=n+1}^{N} P_{\mathrm{S}} a_{m,i} |\mathbf{h}_{m,n}\mathbf{w}_m|^2 + \sigma_{m,n}^2} \leq \gamma_{\mathrm{th}}\right). \tag{11}$$

After some manipulations, the mathematical outage probability expression is given as

$$\mathrm{OP}_{m,n} = \Pr\left(\underset{n:\,1\div N}{X_{m,n}} \leq \frac{\gamma_{\mathrm{th}}}{P_{\mathrm{S}}\left(a_{m,n} - \gamma_{\mathrm{th}} b_{m,n}\right)} = \theta_i \right), \tag{12}$$

where

$$\begin{aligned} b_{m,n} &= \sum\nolimits_{i=n+1}^{N} a_{m,i} \\ X_{m,n} &= |\mathbf{w}_m \mathbf{h}_{m,n}|^2. \end{aligned}$$

It is clear that in (12) the outage always occurs if $a_{m,n} \leq \gamma_{\mathrm{th}} b_{m,n}$. Hence we need to allocate more power for users who have small channel gain.

Based on the ordered variable $X_{m,n}$ in term of PDF is given by [21, eq, (6.58)].

$$\begin{aligned} f_{X_{m,n}}(x) =& \frac{N!}{(N-n)!(n-1)!} \sum_{j=0}^{N-n} (-1)^j \binom{N-n}{j} \\ & \times f_{X_k}(x_k) \Big[F_{X_k}(x_k) \Big]^{n+j-1}, \end{aligned} \tag{13}$$

where

$$f_{X_k}(x_k) = \frac{1}{\Omega_k} \exp\Big(-\frac{x_k}{\Omega_k} \Big), \tag{14}$$

$$F_{X_k}(x_k) = 1 - \exp\Big(-\frac{x_k}{\Omega_k} \Big). \tag{15}$$

Replace (14) and (15) into (13), and then use binomial expansion, we obtain the PDF of SINR of the (m,n)th user. Based on the defined of outage probability $\mathrm{OP} = \int_0^{\theta^*} f_{X_{m,n}}(x)dx$ and after some manipulations we have the close-form outage probability expression of the (m,n)th user as

$$\begin{aligned} \mathrm{OP}_{m,n} =& \frac{N!}{(N-n)!(n-1)!} \sum_{k=0}^{N-n} \frac{(-1)^k}{n+k} \binom{N-n}{k} \\ & \times \left[1 - \exp\Big(-\frac{\theta^*}{P_{\mathrm{S}} \Omega_{m,k}} \Big) \right]^{n+k}, \end{aligned} \tag{16}$$

where $k \leq n \leq N$, $\theta^* = \max\limits_{i=1:N} \{ \theta_i = \frac{\gamma_{\mathrm{th}}}{P_{\mathrm{S}}(a_{m,n} - \gamma_{\mathrm{th}} b_{m,n})} \}$.

IV. NUMERICAL RESULTS

The aim of this section is to provide numerical results for evaluating the performance of proposed combination of beamforming and NOMA for downlink MUs systems, and validating the derived analytical results. In most of published works, the number of users in one superposition signal is up to three, there is not the case of larger number of users. Hence, in this work we consider three users per cluster, where the BS generates perfect beam for each cluster.

All of the channels between the BS and each user are assumed to be Rayleigh distribution. Without loss of generality, we assume the first user is the farthest one, and the third user is the nearest one from the BS. Thus, we can choose average channel gain as $\Omega_{m,1} = 1$, $\Omega_{m,2} = 2$, $\Omega_{m,3} = 3$, and then the power allocation coefficients are sorted as $a_1 > a_2 > a_3$. In the generality of cases, the power allocation coefficient is given by $a_n = \frac{N-n+1}{\psi}$, where $\psi = \frac{N(N+1)}{2}$ to ensure that $\sum_{n=1}^{N} \sqrt{a_i} = 1$.

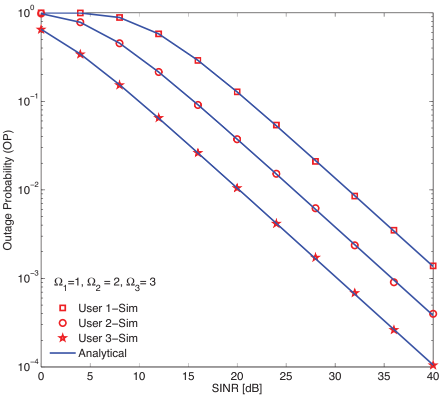

Fig. 2. Outage probability of every user in the mth cluster of single cellular with BFZF.

In Fig. 2, we present the outage probability of each user versus SNR in dB. In this scenario, the transmit power is allocated based on the channel gain, i.e., $a_1 = 0.7$, $a_2 = 0.2$ and $a_3 = 0.1$. As shown in this figure, the theoretical analysis matches the simulation results. And the performance of the third user is the best among the users, although its allocated power is the lowest. This reason is the third user is the closest to the BS meaning its channel gain is the highest (The decay of magnitude of signal is according to a square of distance (of the multi-path fading) [22, pp.33]).

When BPSK, QPSK and 8-PSK modulations are respectively applied, the number of bits per symbol is changed as $r = 1, r = 2$ and $r = 3$. Fig. 3 depicts the outage probability of the user with different number of bits per symbol, while the other parameters are the same as above. The increase of modulation order leads the performance of system to decrease, this is explained that, the transmit power of all cases is fixed, consequently the power of every bit is decreased when the modulation order increases, $\gamma_b = \gamma_s / \log_2(\mathcal{M})$, where $\mathcal{M}$ and γ_s denote the modulation order and the power of sysmbol, respectively. Finally, the Monte Carlo simulations guarantee the correctness of the analytical results.

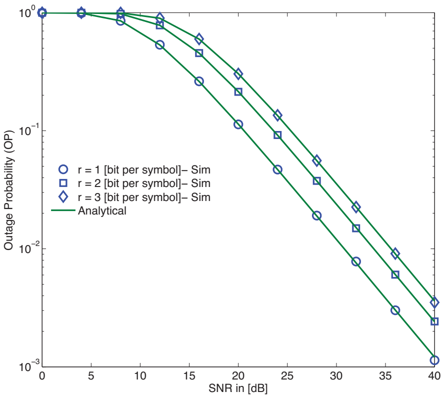

Fig. 3. Outage probability of the second user versus average SINR with different number of bits per symbol.

V. CONCLUSION

A downlink MUs system with ZFBF was proposed and the theoretical closed-form expressions of its outage probability was derived and verified by the Monte-Carlo simulations. The proposed system was compared to OMA systems and the calculation result indicated that the proposed system outperforms OMA systems.

The BS with multi-antennas was considered and the beamforming method was applied to cancel the inter-cluster interference. The beamforming method was chosen with the purpose of keeping a trade-off between the complexity and system performance. On the other hand, the NOMA with SIC technology also was applied to cancel the intra-cluster interference. The proposed combination of beamforming and NOMA technologies outperforms the conventional OMA sytem. However, both beamforming and NOMA technologies were assumed to be implemented perfectly, hence the inter-cluster and intra-cluster interferences were clearly cancelled. We will consider the imperfect CSI due to channel estimation error, and then the effect of intra-cluster and inter-cluster interferences will be discussed in our future works.

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