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BUILDING QUASI-TIME-OPTIMAL CONTROL LAWS FOR BALL AND BEAM SYSTEM

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***Abstract*—In this paper, a methodology for rapid impact control of Ball and Beam system is proposed and its performance is compared with the LQR method. Ball and Beam system is a large nonlinear system parameters are difficult to estimate accurately and easily in which affect by noise. In this work, the model of the system is transformed into a Jordanian model by using the differential transformation so that the system is in the form of a quasi-time-optimal equation. As a result, synthesis of control laws does not need to linearize the system, this is the superiority of this method. In addition, applying new control laws ensure that the system not only obtains optimal time corresponding to the desired output value but also stabilizes with varying parameters and noise interference. The simulation results illustrate the effectiveness of the proposed method.**

***Index Terms*—Ball and Beam system, quasi-time-optimal control, controllable Jordan form, parameter uncertainty**

I. Introduction

Ball and Beam system is a typical system used in study of modern and intelligent control theory. Unstable dynamics of the system is similar to the problems of study in aerospace industry. Ball and Beam system consists of a bar rotated in the vertical plane by the torque of the actuator, and a ball moves freely along the rod axis.

Among recent studies, [1] presented an approach based on Lyapunov method combined with calibration process when starting to show positive results. Moreover, quality of the designing controller is not really impressive. Some studies using intelligent control methods such as [2-4] presented successful results when designing controller is using Neural Networks and Type-2 Fuzzy Control.

This study focuses on an issue related to quasi-time-optimal control. Controllers are synthesized as nonlinear state feedback based on the Jordan formula. When combining efficient controllers in rapidity of action, it is not necessary to rely on the linearized equation system as the current contribution.

The rest of the paper is organized as follows. Part II is for physical modeling of Ball and Beam system. Part III is an overview of the effective control method in rapid impact. Applying the method to synthesize the rapid impact controllers for Ball and Beam system is in Part IV. There, the output of the feedback controller is built and is sufficient for the local stable property of the system. In Part V, the simulation results were presented and compared with the LQR method. Finally, in Part VI, the main contributions of the paper are summarized and future research is indicated.

II. DYNAMICAL MODEL OF THE BALL AND BEAM SYSTEM

A. Experimental Setup

We consider a well-known Ball and Beam system, which is schematically depicted in Fig. 1.

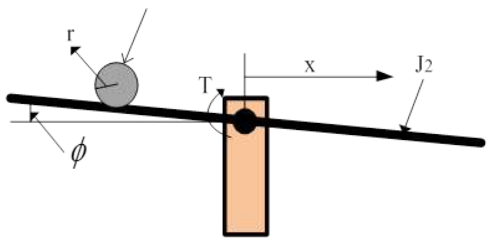

Fig. 1. Ball and Beam system.

The system consists of a ball which can roll on a beam without slipping. The beam rotates around its pivot point where a torque T can be applied. Position (x) of the ball is measured with respect to the pivot point and the beam angle (ϕ) is defined relative to the horizontal plane. The values of the mass (m) and radius (r) of the ball, its moment of inertia (J_1) as well as the moment of inertia (J_2) of the beam are given in Table I together with the gravity constant (g) and are the same as those used in [1,6]. They will be used for the simulations in Section V.

TABLE I
THE PARAMETER OF BALL AND BEAM

Symbol	Description	Value	Unit
M	ball mass	0.05	kg
r	ball radius	0.01	m
J_1	ball moment of inertia	2.10^{-6}	$kg.m^2$
J_2	beam moment of inertia	0.02	$kg.m^2$
g	gravity	9.81	$m^2/_{s^2}$

A DC motor is used to generate the torque (T) on the beam. The ball position at time (t) $x(t)$ is measured with an ultrasonic sound distance sensor. The rotation angle is measured using an incremental encoder. Due to this, we do not measure the absolute rotation angle $\phi(t)$, but rather the relative angular difference with respect to the beginning of the measurement at $t = 0, i.e.\phi(t) - \phi_0$, where ϕ_0 is the angular offset.

B. Equations of Motion

The Lagrange's function of the system as follows [1,9]:

$$L = \frac{1}{2}J_2\dot{\phi}^2 + \frac{1}{2}J_1(\frac{\dot{x}}{r} + \dot{\phi})^2 + \frac{1}{2}m(\dot{x}^2 + \dot{x}^2\dot{\phi}^2) - mgx\sin(\phi) \quad (1)$$

L is defined as the difference between the kinetic energy and the potential energy employed, yielding the following equations of motion:

$$(J_1 + J_2 + mx^2)\ddot{\phi} + 2mx\dot{x}\dot{\phi} + mgx\cos(\phi) = T \quad (2)$$

$$(\frac{J_1}{r^2} + m)\ddot{x} - mx\dot{\phi}^2 + mg\sin(\phi) = 0 \quad (3)$$

C. State-Space Representation

Introducing the state vector $x = (x_1, x_2, x_3, x_4)^T = (x, \dot{x}, \varphi, \dot{\varphi})^T$ and the input $u = T \in R$, we obtain the space-state representation $\dot{x} = f(x, u)$, $x(0) = x_0$ with:

$$f(x,u) = \begin{pmatrix} x_2 \\ \frac{mx_1x_4^2 - mg\sin(x_3)}{A} \\ x_4 \\ \frac{u - 2mx_1x_2x_4 - mgx_1\cos(x_3)}{B + mx_1^2} \end{pmatrix} \quad (4)$$

where $A = {J_1}/{r^2} + m$ and $B = J_1 + J_2$ are employed for notational convenience.

III. APPROACH TO QUASI-TIME-OPTIMAL CONTROL

With this approach, it is possible to solve the synthesis of control law for a large nonlinear object class, in which the control law provides many advantages for the system, such as quasi-time-optimal control, asymptotic stability and sustainable.

The concept of quasi-time-optimal control is shown below, suppose the system model in state-space is taken by Jordan in the form:

$$\begin{cases} \dot{x}_i = f_i\left(x_1, x_2, \ldots x_{i+1}\right),\ i = \overline{1, n-1}; \\ \dot{x}_n = f_n\left(x_1, x_2, \ldots x_n\right) + u. \end{cases} \quad (5)$$

where $f_i(\circ)$ is an analytic function, which means that the derivative exists according to all variables $x_1, x_2, \ldots x_{i+1}$, $\forall i < n \rightarrow {\partial f_i}/{\partial x_{i+1}} \neq 0$, and $u(t)$ is the control signal.

The synthetic method of rapid-impact nearly-optimal control laws for the system (5) by using the differential transformations puts system (5) on the virtual system of equations of the form (6):

$$\begin{cases} \dot{y}_i = \nu_i \frac{y_i}{\sqrt{{y_i}^2 + {\varepsilon_i}^2}} + g_i\left(y_1, y_2, ..., y_{i-1}, y_{i+1}, ...y_{i+1}, \varepsilon_i\right) \\ \dot{y}_n = \frac{y_n}{\sqrt{{y_n}^2 + {\varepsilon_n}^2}} \end{cases} \quad (6)$$

where $i = \overline{1, n-1}$, $h(y, \varepsilon) = \frac{y}{\sqrt{y^2 + \varepsilon^2}}$, u_n is quasi-time-optimal function, $g_i(\circ)$ is a continuous function, $\varepsilon_i; i = \overline{1, n}$ is the parameter of quasi-time-optimal control.

When controlling needs to choose some outputs such that the purpose of quasi-time-optimal control for the output is achieved, we use the quasi-time-optimal control equations rank k which has the following form:

$$\begin{cases} \dot{y}_1 = -v_1^m h(y_1, \varepsilon_1) + y_2;\ \ldots\ ; \dot{y}_k = v_k^m h(y_k, \varepsilon_k) + y_{k+1} \\ \dot{y}_{k+1} = -\frac{y_{k+1}}{\varepsilon_{k+1}} + y_{k+2};\ \ldots\ ; \dot{y}_{n-1} = -\frac{y_{n-1}}{\varepsilon_{n-1}} + y_n \\ \dot{y}_n = -\frac{y_n}{\varepsilon_n} \end{cases} \quad (7)$$

It is easy to see that the system of equations (7) is asymptotically stable according to Liapunov with the Liapunov's function $V = \frac{1}{2}(y_1 + y_2 + ... + y_n)^2$, here $\varepsilon_i \leq 0.5$; $i = \overline{1, n}$; and $\nu_i > \frac{2}{h(\max(y_i), \varepsilon_i)}$; $i = \overline{1, k}$. From this solving the equation with $y_1 = \phi(x_1)$, we get the quasi-time-optimal control control laws of the desired variable.

IV. SYNTHESIS OF CONTROL LAWS FOR BALL AND BEAM SYSTEM

A. LQR Optimal Control Law for Ball and Beam system

First, the nonlinear system (4) will be put on a linear form: $\dot{X} = AX + Bu$ at point (0,0,0,0).

where $A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_2}{\partial u} & \frac{\partial f_3}{\partial u} & \frac{\partial f_4}{\partial u} \end{bmatrix}$

The matrix K of the optimal control value: $u(t) = -KX(t)$ is obtained when quality norm J reaching the minimum value: $J = \int_0^\infty (X^TQX + u^TRu)dt$ where Q is a positive determinant (or semi-positive), R is the positive determinant matrix. If the optimal K matrix is defined by the Riccati equation of the form: $K = R^{-1}B^TP$, the matrix P must satisfy the equation: $PA + A^TP + Q - PBR^{-1}B^TP = 0$.

The control law, u(t), with the parameters mentioned above is achieved according to the LQR method with matrix values Q and R:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; R = 100;$$

$$u(t) = -0.9911x_1(t) - 0.3822x_2(t) + 0.9522x_3(t) + 0.2193x_4(t)$$

B. The quasi-time-optimal control law

Considering the model (4), if the component $mx_1x_4^2$ is the angular velocity of the bar, which has a small value, we change the form of Jordan by removing components nonlinear $mx_1x_4^2$. Then, we obtain the state equation of the Ball and Beam system as follows:

$$f(x,u) = \begin{pmatrix} x_2 \\ \frac{-mg\sin(x_3)}{A} \\ x_4 \\ \frac{u-2mx_1x_2x_4-mgx_1\cos(x_3)}{B+mx_1^2} \end{pmatrix} \quad (8)$$

The system (8) is Jordan-controlled and still nonlinear [3]. The virtual system is selected in the form (9) when the Ball position is guaranteed to be placed in the preset-position with the quasi-time optimal control. The quasi-time-optimal control law following u distance is found when resolving (9) with $y_1 = x_1$. The formulas of u are:

$$\begin{cases} \dot{y}_1 = -v\frac{y_1-y_{sp}}{\sqrt{(y_1-y_{sp})^2+{\varepsilon_1}^2}} + y_2 \\ \dot{y}_2 = -\frac{y_2}{\varepsilon_2} + y_3 \\ \dot{y}_3 = -\frac{y_3}{\varepsilon_3} + y_4 \\ \dot{y}_4 = -\frac{y_4}{\varepsilon_4} \end{cases} \quad (9)$$

V. SIMULATION AND EVALUATION OF THE RESULT OF RAPID-IMPACT NEARLY-OPTIMAL CONTROL

Following the proposed method, the control law is synthesized to provide stable control for Ball and Beam system. The results of system simulation with initial system values are as follows: $x_{sp} = 0$, $x(0) = 0.5$, $\dot{x}(0) = 0$, $\phi(0) = 0.4$, $\dot{\phi}(0) = 0$. The parameters of quasi-time-optimal control: $v = 1$, $\varepsilon_1 = 0.2$, $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.3$. Figure 2 shows the response to the position of the ball with the two methods mentioned above. It is evident that the position response of system with quasi-time-optimal control law has a better stability results. Particularly, the oscillation and overshoot do not occur as in the response produced by LQR method. Figure 3 shows the angle response of Beam under the quasi-time-optimal control method , which is faster and more stable than the counterpart.

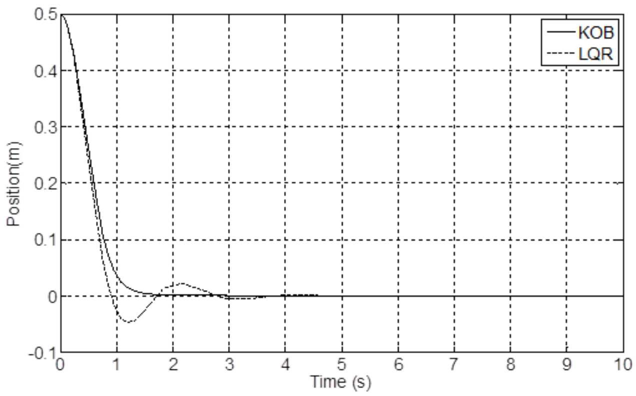

Fig. 2. The position response of Ball.

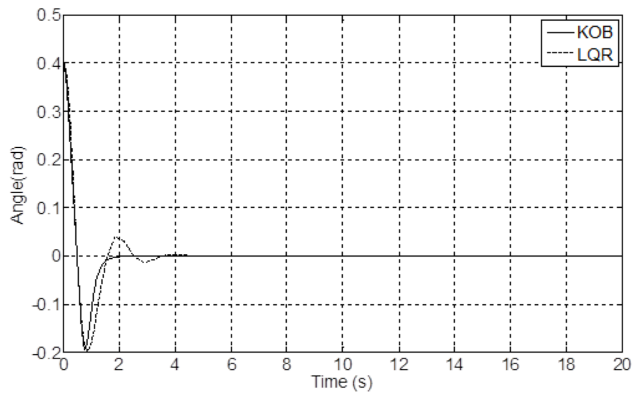

Fig. 3. The angle response of Beam.

CONCLUSION

The results obtained when synthesizing the control law for the Ball and Beam system have demonstrated the rapid optimization for high-complex nonlinear systems. Compared with the traditional LQR method, the position and angle responses produced by using the proposed composite controller are better. Future studies will add adaptive control laws when parameters of control systems are varying or the systems are not the form of Jordan.

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