An Efficient Design of Precoding and Equalization to Reduce BER of Multi-path MIMO Channels

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***Abstract*—The block transmission via multi-path MIMO channels has attracted much interest of many researchers because of its wide applications. In this paper, authors propose an approach to decrease the BER of the multi-path MIMO channels based on an effective combination of precoding and equalization. The logical analysis and simulation results demonstrate that the proposal design can take advantage of channel energy, therefore, reduce the BER. In comparison with the conventional design, the proposed design can improve the system performance in some scenarios.**

***Index Terms*—Precoding, Equalization, ISI MIMO systems, redundancy, BER, block transmission**

I. Introduction

In wireless broadband communications, block transmission systems are really suitable for high data rate transmission as it can efficiently combat with inter-block interference (IBI) caused by independent frequency selective fading channels as in [1], [2], [3] and [4]. For many years, since the joint precoding and equalization could improve performance of the block transmission systems, it has attracted much attention of researchers worldwide and applied for MIMO systems as in [5], [6] and [7].

In oder to avoid the inter-symbol interference (ISI), a guard interval in the form of zero padding (ZP) or cyclic prefix (CP) interval is employed as in [1], [6] and [8]. However, it also makes a part of channel energy to be lost during the cancellation of the guard interval, therefore, the spectrum efficiency is reduced, especially for the channels with long impulse response [9]. Consequently, an amount of redundancy is added to improve the system performance or reduce the bit error rate (BER), resulting in a decrease of data transmission rate [10]. The redundancy is generally understood as the length of the guard intervals [11], [12] and defined as the difference between the input symbol blocks and the transmit or receive symbol blocks.

In this paper, we will combine the ideas reported in [8] and [11] and propose a method of joint optimal precoding and equalization for the frequency selective ISI MIMO channels that depends on suitable distribution of redundancy for both transmitter and receiver during transmission. This helps to reduce the loss in the channel energy and produce lower BER than that in previous schemes, therefore, increase the system performance.

The rest of the paper is organized as follows: Section 2 shows the system model, Section 3 illustrates the simulation results and Section 4 gives the conclusion. In this paper, notations are used as following: boldface font is used for vector and matrix; The set of complex numbers is denoted by symbol $\mathbb{C}$; $(\cdot)^H$ and $(\cdot)^T$ is the Hermitian transpose and transpose operation, respectively.

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II. System Model

This paper considers the system model of block transmission based on the linear precoding and equalization for the ISI MIMO channels illustrated in Fig.1. The MIMO channel with T transmit and R receive antennas is assumed to be stationary, frequency selective and have finite impulse response (FIR) of order of L. The channel impulse response (CIR) is given in matrices $\mathbf{H}[0], \mathbf{H}[1], ..., \mathbf{H}[L]$ where $\mathbf{H}[l] \in \mathbb{C}^{R\times T}, (l = 0, ..., L)$.

Fig.1.a shows the system model of precoding and equalization for the ISI MIMO channels. One can see that the input symbol stream $s[n]$ is converted into symbol vectors $s[i]$ with the size of $N \times 1$ by the serial-to-parallel converter. After that, the symbol vector $s[i]$ is transfered to the precoder in order to generate symbol vectors $x[i]$ with the size of $PT \times 1$. And then, the symbol vectors $x[i]$ are divided into P vectors with the size of $T \times 1$ and passed through the ISI MIMO channel.

At the channel output, the shaded serial-to-parallel converter makes symbol vectors $y[i]$ of the size $PR\times 1$ from P received symbol vectors each of which has the size of $R\times 1$. Because of existing of noise, the received symbol vector $y[i]$ consists of two parts, an information symbol vector $r[i]$ and a noise samples vector $v[i]$. In the receiver side, the symbol vectors $y[i]$ are transfered to the equalizer to make symbol vectors $\hat{\mathbf{s}}[i]$ with the size of $N \times 1$. Consequently, the output symbol stream $\hat{s}[n]$ is obtained by the parallel-to-serial converter from the symbol vectors $\hat{\mathbf{s}}[i]$.

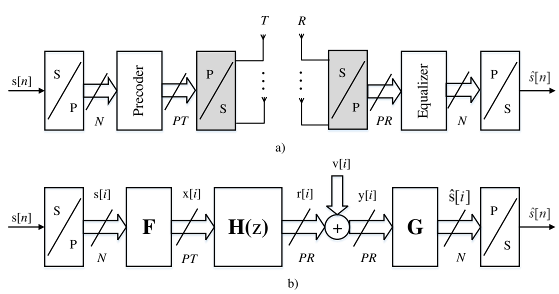

Figure 1. The System model of Precoding and equalization for ISI MIMO channels. a) System block diagram; b) Equivalent system model with channel matrix

With the input symbol stream $\mathbf{s}[n]$ and the sampled version of received signal $\mathbf{y}[n]$, the symbol vectors in Fig. 1.b is defined as

$$\begin{aligned}
\mathbf{s}[i] &= [s[iN], s[iN+1], \ldots, s[iN+N-1]]^T \\
\mathbf{x}[i] &= [x[iPT], x[iPT+1], \ldots, x[iPT+PT-1]]^T \\
\mathbf{y}[i] &= [y[iPR], x[iPR+1], \ldots, y[iPR+PR-1]]^T \\
\hat{\mathbf{s}}[i] &= [\hat{s}[iN], \hat{s}[iN+1], \ldots, \hat{s}[iN+N-1]]^T \\
\mathbf{v}[i] &= [v[iPR], x[iPR+1], \ldots, v[iPR+PR-1]]^T
\end{aligned}$$

where $\mathbf{v}[i]$ is the vector of noise samples of length PR and is assumed to be the additive white Gaussian noise (AWGN).

In [8], when $P \geq L$, the symbol blocks $\hat{\mathbf{s}}[i]$ are given by

$$\hat{\mathbf{s}}[i] = \mathbf{G}\mathbf{H}_0\mathbf{F}\mathbf{s}[i] + \mathbf{G}\mathbf{H}_1\mathbf{F}\mathbf{s}[i-1] + \mathbf{G}\mathbf{v}[i] \tag{1}$$

where $\mathbf{F} \in \mathbb{C}^{PT\times N}$ is the precoder, $\mathbf{G} \in \mathbb{C}^{N\times PR}$ is the equalizer. $\mathbf{H}_0$ and $\mathbf{H}_1$ are $PR \times PT$ matrices illustrated by the following equations

$$\mathbf{H}_0 = \begin{bmatrix} \mathbf{H}[0] & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \mathbf{H}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{H}[L] & \cdots & \ddots & \cdots & \vdots \\ \vdots & \ddots & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{H}[L] & \cdots & \mathbf{H}[0] \end{bmatrix}, \tag{2}$$

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{H}[L] & \cdots & \mathbf{H}[1] \\ \vdots & \ddots & \mathbf{0} & \ddots & \vdots \\ \mathbf{0} & \cdots & \ddots & \cdots & \mathbf{H}[L] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}. \tag{3}$$

In order to optimize the linear precoder and equalizer in [8], one can be applied for the ISI MIMO channels with the assumption that $PT = M + LT$, $(M \geq N)$, and the IBI can be eliminated either by trailing zero (TZ) approach where the last LT rows of the precoder $\mathbf{F}$ are set to zero or leading zero (LZ) approach where the first LR columns of the equalizer $\mathbf{G}$ are set to zero.

In this paper, we will focus on the LZ case. Since the first LR columns of the equalizer $\mathbf{G}$ are set to zero, the equalizer has following form

$$\mathbf{G} = \begin{bmatrix} \mathbf{0}_{N\times LR} & \mathbf{G}_0 \end{bmatrix}, \tag{4}$$

where the equalizer $\mathbf{G}_0 \in \mathbb{C}^{N\times(P-L)R}$ will be jointly designed with the precoding $\mathbf{F}$ under either zero-forcing (ZF) or minimum mean square error (MMSE) criteria.

When the IBI is eliminated, equation (1) can be written as

$$\hat{\mathbf{s}}[i] = \mathbf{G}_0\mathbf{H}\mathbf{F}_0\mathbf{s}[i] + \mathbf{G}_0\mathbf{v}[i], \tag{5}$$

where $\mathbf{H}$ is the last M rows of $\mathbf{H}_0$ and $\mathbf{F}_0 = \mathbf{F}$.

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}[\mathrm{L}] & \cdots & \mathbf{H}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}[\mathrm{L}] & \cdots & \mathbf{H}[0] \end{bmatrix} \tag{6}$$

with the transmit power to be constrained to p_0, the MMSE algorithm [8] is used to optimize the precoder and equalizer as following

$$\mathbf{F}_0 = \mathbf{V}\mathbf{\Phi}\mathbf{U}^H, \tag{7}$$

$$\mathbf{G}_0 = \mathbf{R}_{ss}\mathbf{F}_0^H\mathbf{H}^H(\mathbf{R}_{vv} + \mathbf{H}\mathbf{F}_0\mathbf{R}_{ss}\mathbf{F}_0^H\mathbf{H}^H)^{-1}, \tag{8}$$

where $\mathbf{R}_{ss}$ is the input symbol covariance matrix and $\mathbf{R}_{vv}$ is the noise covariance matrix. In addition, $\mathbf{U}$ and $\mathbf{V}$ are the unitary matrices that can be determined by the eigenvalue decompositions (EVD) as in [8]

$$\mathbf{R}_{ss} = \mathbf{U}\mathbf{\Delta}\mathbf{U}^H, \tag{9}$$

$$\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^V, \tag{10}$$

the matrices $\mathbf{\Phi}$, $\mathbf{\Delta}$ and $\mathbf{\Lambda}$ are diagonal with nonnegative elements. Moreover, the main diagonal elements of $\mathbf{\Phi}$ are found out by the MMSE as following

$$|\mathit{\Phi}_{jj}|^2 = \frac{p_0 + \sum_{i=1}^{k}\left(\lambda_{ii}^{-1}\right)}{\sum_{i=1}^{k}\left(\lambda_{ii}^{-1/2}\delta_{ii}^{1/2}\right)}\frac{1}{\sqrt{\lambda_{jj}\delta_{jj}}} - \lambda_{jj}^{-1}\delta_{jj}^{-1} \tag{11}$$

with $\lambda_{jj}, \delta_{jj}$ are the jth main diagonal elements of $\mathbf{\Delta}$ and $\mathbf{\Lambda}$, respectively. k is the number of $\mathit{\Phi}_{jj}$ satisfying $|\mathit{\Phi}_{jj}|^2 > 0$.

The use of guard intervals can help to eliminate the ISI in the dispersive channels. However, it also makes a part of channel energy to be lost during the cancellation of guard interval at the receiver side because the first LR rows of $\mathbf{H}_0$ will be discarded by the equalizer. Besides, one can see that transmission over the MIMO channels, the ISI MIMO channel is decomposed into a set of independent flat fading MIMO channels. The SNR on each of this parallel flat fading MIMO channels is independent and the system error performance will be dominated by the performance of subchannels with low SNRs. Thus problem here is how to avoid the loss in the channel energy in whole matrix $\mathbf{H}_0$ and discard some subchannels having too low SNRs as in [13] while the quality is ensured to whole the system.

To overcome this problem, we combine the ideas reported in [8] and [11] and propose a method of joint optimal design of precoding and equalization that can reduce the loss in matrix $\mathbf{H}_0$ or in orther words, reduce the loss in channel energy. Besides, in order to guarantee that some subchannels with too low eigenvalues will be discarded so that they do not affect the system BER. As a result, the proposed design can provide a better performance than the conventional. The proposed design is based on the solutions in [8], but the guard interval now is shared between the transmitter and the receiver. At the transmitter, instead of setting last LT rows of the precoder matrix to zero, we only set last $KT = \left\lfloor \frac{LT}{2} \right\rfloor$ rows to zero and the first $(L-K)R$ columns of $\mathbf{G}$ are also set to zero at the receiver. Thus the precoder and equalizer have the following forms.

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_0 \\ \mathbf{0}_{KT\times N} \end{bmatrix}, \tag{12}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{0}_{N\times(L-K)R} & \mathbf{G}_0 \end{bmatrix}, \tag{13}$$

where $\mathbf{F}_0 \in \mathbb{C}^{(P-K)T\times N}$ and $\mathbf{G}_0 \in \mathbb{C}^{N\times(P-L+K)R}$. The MMSE [5] is used to optimized the precoder and equalizer as following

$$\mathbf{F}_0 = \mathbf{V}\mathbf{\Phi}_{\mathbf{f}}, \tag{14}$$

$$\mathbf{G}_0 = \mathbf{\Phi}_{\mathbf{g}}\mathbf{V}^H\mathbf{H}^H\mathbf{R}_{vv}^{-\mathbf{1}}, \tag{15}$$

where $\mathbf{V}$ is unitary matrices derived from EVD algorithm as following

$$\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H \tag{16}$$

and $\mathbf{\Phi}_{\mathbf{f}}$, $\mathbf{\Phi}_{\mathbf{g}}$ are diagonal matrices with their main diagonal elements are defined by the MMSE and then given by

$$|\phi_{\mathrm{f},jj}|^2 = \frac{p_0 + \sum_{i=1}^{k}\left(\lambda_{ii}^{-1}\right)}{\sum_{i=1}^{k}\left(\lambda_{ii}^{-1/2}\right)}\frac{1}{\sqrt{\lambda_{jj}}} - \lambda_{jj}^{-1}, \tag{17}$$

$$|\phi_{\mathrm{g},jj}|^2 = \left\{ \frac{\sum_{i=1}^{k}\left(\lambda_{ii}^{-1/2}\right)}{p_0 + \sum_{i=1}^{k}\left(\lambda_{ii}^{-1}\right)}\lambda_{jj}^{-1/2} - \left[\frac{\sum_{i=1}^{k}\left(\lambda_{ii}^{-1/2}\right)}{p_0 + \sum_{i=1}^{k}\left(\lambda_{ii}^{-1}\right)}\lambda_{jj}^{-1/2}\right]^2 \lambda_{jj}^{-1} \right\} \lambda_{jj}^{-1}, \tag{18}$$

where λ_{jj} is the main diagonal elements of $\mathbf{\Lambda}$. k are the number of $\phi_{\mathrm{f},jj}$ and $\phi_{\mathrm{g},jj}$ satisfying $|\phi_{\mathrm{f},jj}|^2 > 0$ and $|\phi_{\mathrm{g},jj}|^2 > 0$, respectively.

With optimal linear precoder and equalizer in (14) and (15), the IBI is completely eliminated, equation (1) can be written as

$$\hat{\mathbf{s}}[i] = \mathbf{G}_0\hat{\mathbf{H}}\mathbf{F}_0\mathbf{s}[i] + \mathbf{G}_0\mathbf{v}'[i], \tag{19}$$

where $\hat{\mathbf{H}} \in \mathbb{C}^{(P-L+K)R\times(P-K)T}$ is illustrated as (20)

$$\hat{\mathbf{H}} = \begin{bmatrix} \mathbf{H}[L-K] & \cdots & \mathbf{H}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & \ddots & \ddots & \vdots \\ \mathbf{H}[L] & & & & \ddots & \mathbf{0} \\ \mathbf{0} & \ddots & & & & \mathbf{H}[0] \\ \vdots & \ddots & \ddots & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{H}[L] & \cdots & \mathbf{H}[K] \end{bmatrix} \tag{20}$$

and $\mathbf{v}'[i]$ is the block of noise samples of length $(P-L+K)R$.

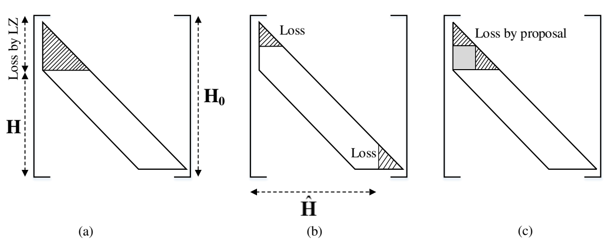

Figure 2. Comparison of the loss in $\mathbf{H}_0$ in two approaches

In order to demonstrate improvement of the proposed design, Fig. 2 compares the loss in $\mathbf{H}_0$ of proposed method and the LZ or TZ method in [8]. From equation (6), one can see that in the LZ case the first LR rows of $\mathbf{H}_0$ are discarded by the equalizer, thus the elements in the shaded triangular in Fig. 2.a will be lost, resulting in a reduction in channel energy. In TZ case, we can easily verify that the system bears exactly the same loss with the LZ case. When the guard interval is shared between transmitter and receiver, the first $(L-K)R$ rows of $\mathbf{H}_0$ are discarded by the equalizer and the last KT columns of $\mathbf{H}_0$ are discarded by the precoder. The loss in $\mathbf{H}_0$ now can be described by the two shaded triangulars as illustrated in Fig. 2.b in which the triangular at the top-left corner and the triangular at bottom-right corner correspond to the loss caused by the equalizer corresponds and the loss caused by the precoder, respectively. We can shift the triangular at the bottom-right corner and compare the loss in two cases as illustrated in Fig. 2c. It is clear that the loss of $\mathbf{H}_0$ in the proposed approach is smaller than the loss in LZ or TZ cases. The elements of $\mathbf{H}_0$ lying in the dotted rectangular area are retained in $\hat{\mathbf{H}}$ and will contribute for the SNR increase because the eigenvalue decomposition usually concentrates in channel energy in some of the largest eigenvalue.

The precoder and equalizer matrices $\mathbf{F}_0$ and $\mathbf{G}_0$ can be then derived under same optimal criteria as in [5] and are calculated as in (14), (15) with matrix $\mathbf{H}$ is replaced by $\hat{\mathbf{H}}$ and $\mathbf{R}_{vv}$ is resized accordingly.

III. Simulation Results

In the simulation scenario, we consider a ISI MIMO channel with two transmit and two receive antennas. Moreover, the order of the CIR is generated from the Saleh-Valenzuela indoor channel model as in [14].

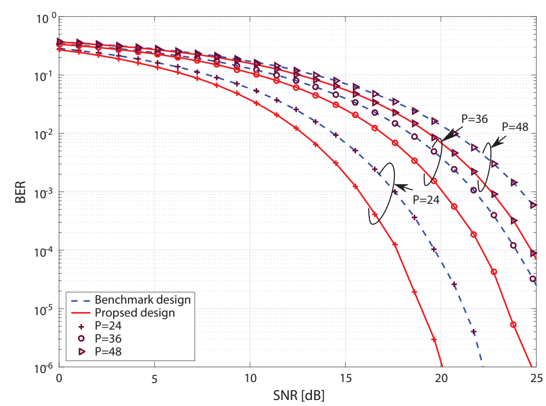

Figure 3. BER performance of the proposed and benchmark designs (P = 24, 36 and 48)

First of all, we will compare the BER performance of the two schemes with differences in the transmit block size P = 24, 36 and 48 while the FIR order L = 11 and QPSK modulation are unchanged as in Fig. 3. It can be seen that at the BER of 4.10^{-3}, the gain in SNR is approximately of 2 dB and is higher at lower BER. Furthermore, when the transmit block size P decreases, the BER rises in the both methods

Second, we will compare the BER performance of the two schemes with differences in the FIR order L = 8, 11 and 14 while the transmit block size P = 24 and QPSK modulation are unchanged as shown in Fig. 4. The proposed method also states its higher performance when the FIR oder is changed. At the BER = 10^{-5}, the achieved gains are approximately of 3.1, 2.5 and 2.4 dB corresponding to the FIR order L = 14, 11 and 8. Moreover, when the FIR oder decreases, the BER decreases in the both designs.

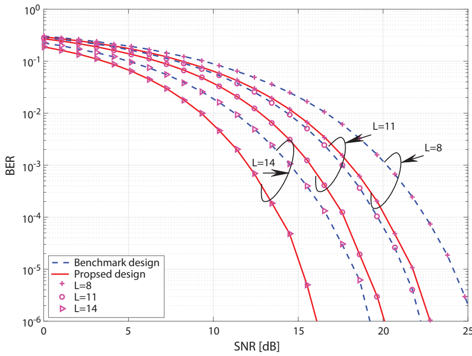

Figure 4. BER performance of the proposed and benchmark designs (L = 8, 11 and 14)

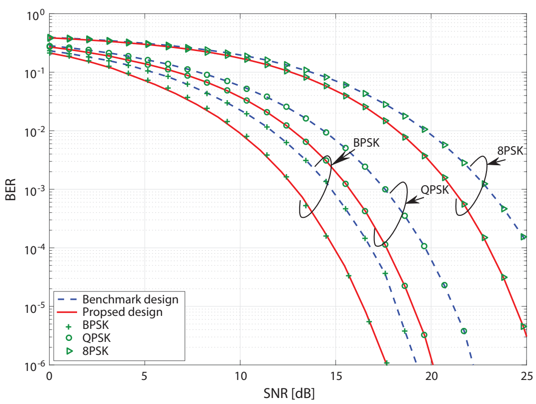

Figure 5. BER performance of the proposed and benchmark designs (BPSK, QPSK and 8PSK)

Third, the BER performance of the two schemes is evaluated with different modulation orders while the transmit block size P = 24 and FIR order L = 11 are unchanged as in the Fig. 5. The system performance is enhanced remarkably by employing the proposed approach with different modulation orders. Moreover, when the modulation order increases (BPSK, QPSK and 8PSK), the gap between the corresponding BER curves becomes closer.

Final, thanks to sharing redundancy during the transmission, the system performance is enhanced significantly. The reason for the improvement in BER performance is not only the improvement in channel gain, but also due to the elimination of subchannels with too low SNR.

IV. Conclusion

In this paper, a combining design of precoding and equalization is proposed. The simulation results show that the proposed design not only takes advantage of the channel energy, but it also discards very low SNR subchannels. As a result, the proposed design can reduce the BER or improve the system performance significantly. In the future, we will optimize some parameters of the proposed system and analyze the change in system complexity when the proposed design is applied.

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