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Background Calibration of Multiple Channel Mismatches in Time-Interleaved ADCs

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***Abstract*—Time-Interleaved Analog-to-Digital Converter (TIADC) is a promising approach to meet the requirement of high speed wireless communication systems. However, mismatches between the channels of TIADCs generate unwanted distortions at the output spectrum. This paper demonstrates a fully digital background calibration technique to identify and compensate three all deviations which are offset, gain and timing mismatches for TIADCs. In which, offset mismatch is corrected by computing the average of the output samples of each channel. The gain and timing mismatches are compensated by using LMS algorithm and the modulation matrix $m[n]$. The simulation results show that the performance of the ADC is improved by 41.32 dB for SNDR and 64.6 dB for SFDR.**

***Index Terms*—channel mismatches, TIADC, output spectrum**

I. Introduction

Wide-band applications require high speed and low power Analog-to-Digital Converters (ADCs). To fulfill this requirement, TIADC architecture is an efficient solution to increase the overall system sample rate, while keeping high accuracy [1]. A TIADC is a single converter having higher sampling rate [2], [3], because TIADC consists of M channel ADCs, these ADCs take the sample at the same rate but interleaved time, as in Fig. 1.

Regrettably, the performance of TIADC is sensitive to channel mismatches, i.e., offset, gain and timing mismatches [1] as in Fig. 2. These mismatches affect the amplitude and frequency of the input signal, generate unwanted distortions in the output spectrum and thus decrease the signal-to-noise and distortion ratio (SNDR) and the spurious-free dynamic range (SFDR) [4]. Currently, gain mismatch and timing mismatch have been corrected [5]–[11]. Papers [5]–[7] analyzed two-channel TIADC systems, [8]–[11] calibrated only timing mismatch, [11] considered four-channel system and corrected timing and gain mismatches but not offset mismatch. In this paper, we propose a method for correcting three deviations: offset, gain and timing mismatches to improve the performance of TIADC M channels.

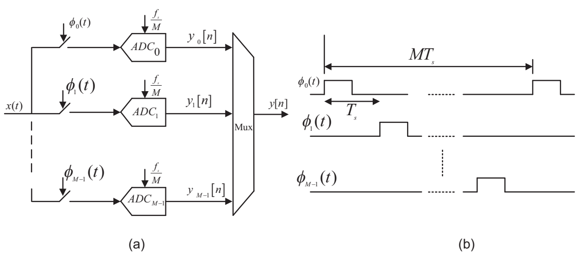

Fig. 1. Structure and Timing diagram of TIADC with M channels.

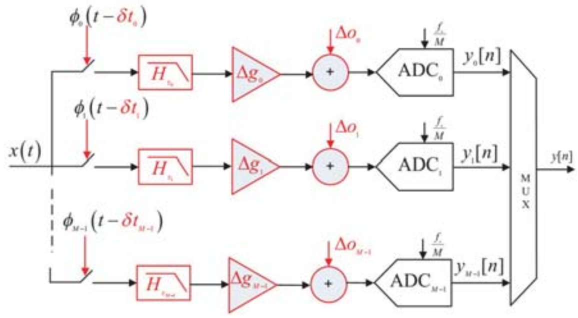

Fig. 2. The channel mismatches in TIADC.

The remainder of this paper is organized as follows. In Section II, the TIADC model with offset, gain and timing mismatches is reviewed. Section III demonstrates a fully digital background calibration technique for channanel mismatches. The simulation results are given in Section IV. Finally, conclusion is included in Section V.

II. System Model

An M-channel TIADC with each sub-ADC being characterized by the offsets o_m, the gains g_m and the relative timing deviation $r_m T_s$ for $m = 0, 1, ..., M-1$, as shown in Fig. 3. The offset mismatch arises because of the addition of a different offset in each sub-ADC, the gain mismatch is caused by the difference in

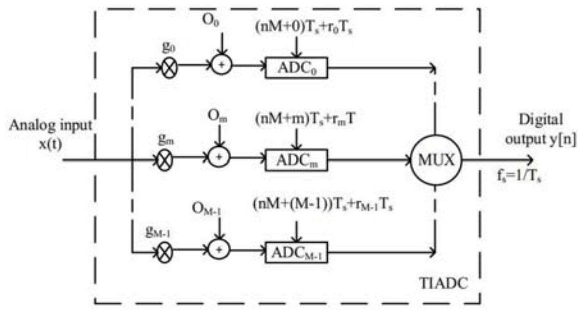

Fig. 3. An M-channel TIADC with channel mismatches.

gain of the channels while timing mismatch is due to imprecision of the sampling pulse between channels that leads to uneven sampling.

The input signal is bandlimited $X\left(j\Omega\right) = 0$, with $|\Omega| \geq B$ and $B \leq \frac{\pi}{T_s}$, TIADC here has M-channel, the output can be written as

$$Y\left(e^{j\omega}\right) = \sum_{k=o}^{M-1} \left(\alpha_k \left(e^{j\left(\omega - k\frac{2*\pi}{M}\right)}\right) X\left(e^{j\left(\omega - k\frac{2*\pi}{M}\right)}\right)\right) \quad (1)$$

$$+ \frac{1}{M} \sum_{m=0}^{M-1} o_m e^{-jk\frac{2*\pi}{M}}$$

where

$$a_k\left(e^{j\omega}\right) = \frac{1}{M} \sum_{m=0}^{M-1} g_m e^{r_m H_d\left(e^{j\omega}\right)} e^{-jk\frac{2\pi}{M}m} \quad (2)$$

and

$$H_d(e^{j\omega}) = j\omega, \qquad \omega \in [-\pi; \pi] \quad (3)$$

is the frequency response of an ideal discrete-time differentiator [12]. If the TIADC is ideal, it means TIADC has no mismatches, we have $o_m = 0$, $g_m = 1$ and $r_m = 0$. Because time offsets r_m are small compared with T_s, we can apply the Taylor expansion for component $e^{r_m H_d\left(e^{j\omega}\right)}$, and ignore the high-order component, we get

$$e^{r_m H_d\left(e^{j\omega}\right)} \approx 1 + r_m H_d\left(e^{j\omega}\right) \quad (4)$$

Getting (4) for (1) and taking the inverse discrete-time Fourier transform of (1), the output $y[n]$ can be written as

$$y\left[n\right] = G_0 x\left[n\right] + e\left[n\right] \quad (5)$$

where $e\left[n\right]$ is the error which is given due to offset, gain and timing mismatches.

III. PROPOSED METHOD

Error $e\left[n\right]$ in (5) can be written as a vector

$$e\left[n\right] = c_g^T x_g\left[n\right] + c_r^T x_r\left[n\right] + o_m \quad (6)$$

where o_m, c_g and c_r are the offset mismatch coefficients vector, gain mismatch coefficients vector and timing mismatch coefficients vector, respectively. The proposed technique performs offset mismatch correction before gain and timing mismatches correction.

A. *Offset calibration*

Assume that $\hat{o}_i$ is the estimate of the offset o_i of the i^{th} channel ADC, quantization values are small and can be neglected. To calibrate the offset mismatch, the offset $\hat{o}_i$ of the individual channel ADC is estimated. Assume that the input signal is Wide-Sense-Stationary (WSS), expected value of the input is approximately zero, i.e.

$$\frac{1}{N} \sum_{k=0}^{N-1} \left(1 + g_i\right) x\left(\left(kM + i\right)T_s + r_i\right) \approx 0 \quad (7)$$

By averaging the output of each sub-ADC over N samples, the offset estimate of the i^{th} sub-ADC is given as

$$\begin{aligned} \hat{o}_i =& \frac{1}{N} \sum_{k=0}^{N-1} y_i\left[k\right] \quad (8) \\ =& \frac{1}{N} \sum_{k=0}^{N-1} \left(\left(1 + g_i\right) x\left(\left(kM + i\right)T_s + r_i\right) + o_i\right) \\ =& \underbrace{\frac{1}{N} \sum_{k=0}^{N-1} \left(1 + g_i\right) x\left(\left(kM + i\right)T_s + r_i\right)}_{\approx 0} + o_i \approx o_i \end{aligned}$$

Here if N is small, the estimated value $\hat{o}_i$ does not cling to o_i , so N must be large enough in order to $\hat{o}_i$ reach o_i, resulting in the output being eliminated offset mismatch, as shown in Figure 4. The value N is large enough, depending on the type of input signal, for example signal in IV requires $N \geq 2^6$.

Once estimated offset is known, offset mismatch induces error which is the second term of (1) can be subtracted from the sub-ADC output to generate the correct signal as shown in Fig. 4. Consequently, the output of TIADC consist of gain and timing mismatches which will be calibrated, the detail is shown in the follow part.

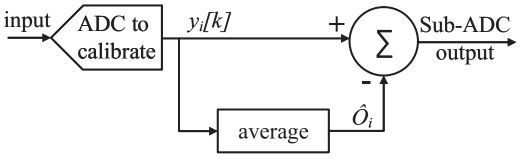

Fig. 4. Offset mismatch calibration for each sub-ADC.

B. Analysis signal only consisting gain and timing mismatches

The signal without offset mismatch is written as

$$Y\left(e^{j\omega}\right)=\sum_{k=o}^{M-1}\left(\alpha_k\left(e^{j\left(\omega-k\frac{2*\pi}{M}\right)}\right)X\left(e^{j\left(\omega-k\frac{2*\pi}{M}\right)}\right)\right) \tag{9}$$

Fig. 5 shows the facilitated model of discrete-time system of an M-channel TIADC.

And (6) becomes

$$e[n]=c_g^T x_g[n]+c_r^T x_r[n] \tag{10}$$

where c_g and c_r are written as (11) and (12)

$$c_g=\left(\Re\{G_1\},\Im\{G_1\},...,\Re\{G_1\},\Im\{G_1\},..., \Re\left\{G_{\frac{M}{2}-1}\right\},\Im\left\{G_{\frac{M}{2}-1}\right\},G_{\frac{M}{2}}\right)^T \tag{11}$$

$$c_r=\left(\Re\{R_1\},\Im\{R_1\},...,\Re\{R_1\},\Im\{R_1\},..., \Re\left\{R_{\frac{M}{2}-1}\right\},\Im\left\{R_{\frac{M}{2}-1}\right\},R_{\frac{M}{2}}\right)^T \tag{12}$$

where $\Re(x)$ and $\Im(x)$ are the real part and the imaginary part of x, respectively.

$$G_k=\frac{1}{M}\sum_{m=0}^{M-1}g_m e^{-jk\frac{2*\pi}{M}m} \tag{13}$$

$$R_k=\frac{1}{M}\sum_{m=0}^{M-1}g_m r_m e^{-jk\frac{2*\pi}{M}m} \tag{14}$$

whereas $x_g[n]$ and $x_r[n]$ are the modulated signal vector and the modulated and differentiated signal vector, respectively, as (15) and (16)

$$x_g[n]=m[n]\,x[n] \tag{15}$$

$$x_r[n]=m[n]\,x[n]*h_d[n] \tag{16}$$

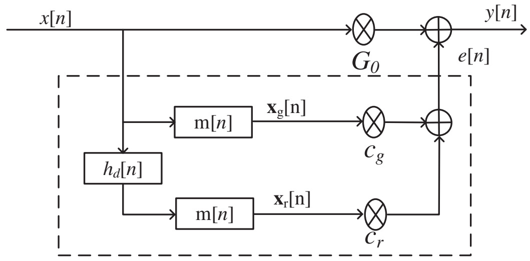

Fig. 5. The simplified model of discrete-time system.

with

$$\begin{aligned}\mathrm{m[n]}=&\Big(2\cos\Big(1\frac{2\pi}{M}n\Big),-2\sin\Big(1\frac{2\pi}{M}n\Big),...,\\ &2\cos\Big(k\frac{2\pi}{M}n\Big),-2\sin\Big(k\frac{2\pi}{M}n\Big),...,\\ &2\cos\Big(\Big(\frac{M}{2}-1\Big)\frac{2\pi}{M}n\Big),-2\sin\Big(\Big(\frac{M}{2}-1\Big)\frac{2\pi}{M}n\Big),\\ &(-1)^n\Big)^T\end{aligned} \tag{17}$$

and $h_d(n)$ is the impulse response of the ideal discrete-time differentiator.

C. Structure of gain and timing mismatches calibration

In this part, the blind estimation structure using the LMS algorithm [13] is shown in Fig. 6. As discussed above, by estimating the gain and timing offset coefficients vector, c_g and c_r, respectively, the error vector $e[n]$ is estimated. This error is then subtracted so that the input signal is restored according to the following

$$\hat{x}[n]=y[n]-\hat{e}[n]=G_0x[n]+e[n]-\hat{e}[n] \tag{18}$$

In the blind correction method, the input signal $x(n)$ is unknown, the output signal $y(n)$ of TIADC is used instead of $x(n)$ in (10), (15) and (16) to estimate the error signal.

Therefore, we have

$$\hat{e}[n]=\hat{c}_g[n]\,y_g[n]+\hat{c}_r[n]\,y_r[n] \tag{19}$$

where

$$\begin{aligned}y_g[n]&=m[n]\,y[n]\\ y_r[n]&=m[n]\,y[n]*h_d[n]\end{aligned} \tag{20}$$

The LMS algorithm minimizes the average squared error function $E\left\{\varepsilon^2(n,\omega)\right\}$ where E represents the mean, μ is the adaptive step factor and $\varepsilon[n]$ is the error after the signal passes through a high pass filter as Fig. 6.

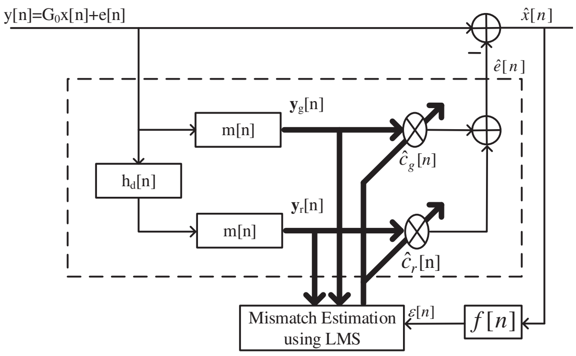

Fig. 6. Gain and timing mismatches calibration structure.

The procedure to implement the LMS algorithm

1. Initialize: $n = 0, \hat{c}_g[0], \hat{c}_r[0], \mu$
2. For $n = 0$ to L do
3. $\hat{e}[n] = \hat{c}_g^T[n] y_g[n] + \hat{c}_r^T[n] y_r[n]$
4. $\hat{x}[n] = y[n] - \hat{e}[n]$
5. $\varepsilon[n] = \hat{x}[n] * f[n]$
6. $\hat{c}_g[n] = \hat{c}_g[n-1] + \mu\varepsilon[n] y_g[n]$
   $\hat{c}_r[n] = \hat{c}_r[n-1] + \mu\varepsilon[n] y_r[n]$
7. End for

IV. SIMULATION RESULTS

To show the effectiveness of the proposed method, the simulated results of an four-chanel TIADC with the sampling frequency of 2.7GHz is shown, assuming that channel 0 have no mismatches, is the reference channel, is indicated in Table I. The input signal is bandlimited with a variance $\sigma = 1$ and 2^{20} sample and using LMS algorithm with adaptive step $\mu_g = 2^{-10}$, $\mu_r = 2^{-12}$ and a 33-tap fixed FIR filter. The signal-to-noise ratio is calculated according to equation (21), (22) for $y[n]$ and $\hat{x}[n]$ as [14]

$$SNR_y = 10\log_{10}\left(\frac{\sum_{n=0}^{N-1}|G_0 x[n]|^2}{\sum_{n=0}^{N-1}|G_0 x[n] - y[n]|^2}\right) \tag{21}$$

TABLE I
THE TABLE OF CHANNEL MISMATCH VALUES

Sub ADC	Channel mismatches		
	o_m	g_m	r_m
ADC_0	0.026883	0	0
ADC_1	0.091694	0.0037618	$0.00098039T_s$
ADC_2	-0.11294	0.0082516	$0.00007280T_s$
ADC_3	0.043109	0.0042512	$-0.00090824T_s$

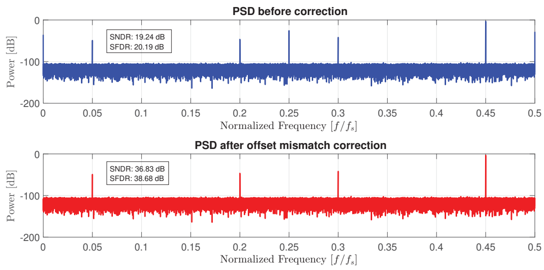

Fig. 7. Output spectrum of four-channel TIADC before and after offset calibration.

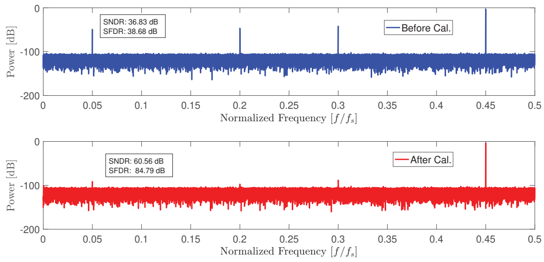

Fig. 8. Output spectrum of four-channel TIADC before and after calibration in gain and timing.

$$SNR_{\hat{x}} = 10\log_{10}\left(\frac{\sum_{n=0}^{N-1}|G_0 x[n]|^2}{\sum_{n=0}^{N-1}|G_0 x[n] - \hat{x}[n]|^2}\right) \tag{22}$$

The simulation results in Fig. 7 and Fig. 8 show that the effect of mismatches in the output spectrum of the TIADC with the proposed correction technique have been eliminated. The SNDR after calibration is 60.56 dB, gives an enhancement of 41.32 dB compared to the unrecompensed output which is 19.24 dB, this factor is only 34.1dB improvement in [14]. The SFDR after calibration is 84.79 dB, shows an increase of 64.6 dB compared to the unrecompensed output which is 20.19 dB. Thus, the performance of TIADC is significantly improved.

The convergence of the estimated offset coefficients o_m, the estimated gain coefficients c_g and the estimated timing coefficients c_r are shown in Fig. 9, Fig. 10 and Fig. 11, respectively. As can be seen, after 2^6 samples, 2^{13} samples and 2^{14} samples, the offset coefficients $\hat{o}_m$, the gain coefficient $\hat{c}_g$ and the timing coefficients $\hat{c}_r$, respectively, have converged.

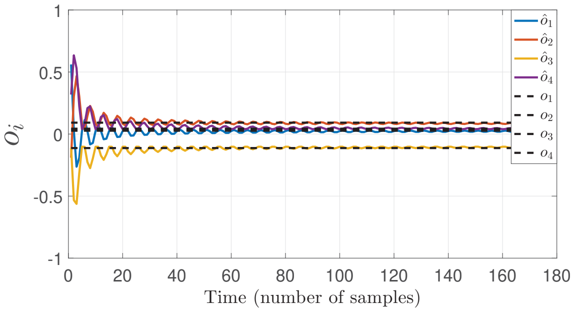

Fig. 9. The convergence of the offset mismatch coefficients $\hat{o}_m$.

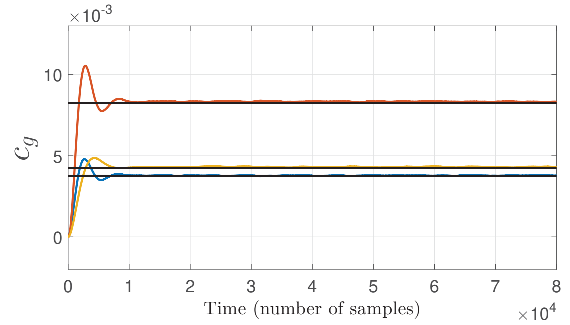

Fig. 10. The convergence of the gain mismatch coefficients $\hat{c}_g$.

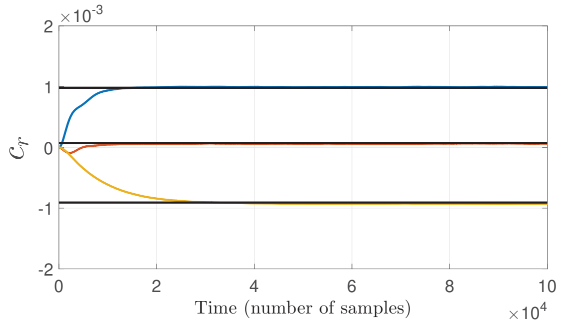

Fig. 11. The convergence of the timing mismatch coefficients $\hat{c}_r$.

V. Conclusion

This paper presented an efficient method to calibrate the offset, gain and timing mismatches in an M-channel TIADC. Compared with the other papers, a system to compensate three all channel mismatches was proposed. The input signal is bandlimited WGN, the results of an 4-channel TIADC that has demonstrated a higher improvement in the SNDR and SFDR were simulated.

Acknowledgment

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References

[1] C. Vogel, "The impact of combined channel mismatch effects in time-interleaved adcs," *IEEE transactions on instrumentation and measurement*, vol. 54, no. 1, pp. 415–427, 2005.

[2] W. C. Black and D. A. Hodges, "Time interleaved converter arrays," *IEEE Journal of Solid-state circuits*, vol. 15, no. 6, pp. 1022–1029, 1980.

[3] W. C. Black, *High speed CMOS A/D conversion techniques*. University of California, Berkeley, 1980.

[4] S. J. Tilden, T. E. Linnenbrink, and P. J. Green, "Overview of ieee-std-1241" standard for terminology and test methods for analog-to-digital converters"," in *Instrumentation and Measurement Technology Conference, 1999. IMTC/99. Proceedings of the 16th IEEE*, vol. 3. IEEE, 1999, pp. 1498–1503.

[5] S. M. Jamal, D. Fu, M. P. Singh, P. J. Hurst, and S. H. Lewis, "Calibration of sample-time error in a two-channel time-interleaved analog-to-digital converter," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 51, no. 1, pp. 130–139, 2004.

[6] M. Seo, M. J. Rodwell, and U. Madhow, "Blind correction of gain and timing mismatches for a two-channel time-interleaved analog-to-digital converter," in *Signals, Systems and Computers, 2005. Conference Record of the Thirty-Ninth Asilomar Conference on*. IEEE, 2005, pp. 1121–1125.

[7] S. Huang and B. C. Levy, "Adaptive blind calibration of timing offset and gain mismatch for two-channel time-interleaved adcs," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 53, no. 6, pp. 1278–1288, 2006.

[8] ——, "Blind calibration of timing offsets for four-channel time-interleaved adcs," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 54, no. 4, pp. 863–876, 2007.

[9] C. Vogel, "A frequency domain method for blind identification of timing mismatches in time-interleaved adcs," in *Norchip Conference, 2006. 24th*. IEEE, 2006, pp. 45–48.

[10] D. Marelli, K. Mahata, and M. Fu, "Linear lms compensation for timing mismatch in time-interleaved adcs," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 56, no. 11, pp. 2476–2486, 2009.

[11] J. Matsuno, T. Yamaji, M. Furuta, and T. Itakura, "All-digital background calibration technique for time-interleaved adc using pseudo aliasing signal," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 5, pp. 1113–1121, 2013.

[12] A. V. Oppenheim, *Discrete-time signal processing*. Pearson Education India, 1999.

[13] B. Farhang-Boroujeny, *Adaptive filters: theory and applications*. John Wiley & Sons, 2013.

[14] S. Saleem and C. Vogel, "On blind identification of gain and timing mismatches in time-interleaved analog-to-digital converters," in *33rd International Conference on Telecommunications and Signal Processing, Baden (Austria)*, 2010, pp. 151–155.