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Parametric Spectral Analysis of Noisy Signals with Unimodal Spectrum

V. G. Andrejev[1*], Ngoc L. Tran[1], and Tien P. Nguyen[2]
[1]*Ryazan State Radio Engineering University, Ryazan, Russia*
[2]*Le Quy Don Technical University, Hanoi, Vietnam*
**e-mail: andrejev.v.g@rsreu.ru*

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Abstract—The article proposes a method for the coefficients reconstruction of a discrete autocorrelation function of random signals with a unimodal power spectral density to construct their parametric models. The method is based on finding the optimal values of the relative width ΔFT and the weighting factor $\alpha \in [0; 1]$ of the spectral mode, which characterizes the share of the Gaussian and resonant components in the spectrum envelope. The proposed method makes it possible to reduce by 1.5–4 times the discrepancy between the control and estimated spectra in comparison with the known approaches to parametric spectral analysis. An increase in the adequacy of spectral estimation makes it possible to reduce the length M of the analyzed time sample by a factor of 2.3–4 times, while maintaining the accuracy of the spectral analysis achieved by other known parametric methods. Winnings are achieved by using a priori information about the spectral properties of the analyzed process.

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INTRODUCTION

Reducing the influence of noise that complicates the spectral estimation of quasi-stationary signals in various application areas is an actual problem in various areas of radio engineering [1–3]. The solution of this problem is characteristic of parametric spectral analysis based on the recovered coefficients of the autocorrelation sequence [4, 5].

Such a restoration is possible in those problems where the power spectral density (PSD) of the original signals that are unaffected by noise is unimodal with partially known parameters of the spectral mode. Such tasks include the processing and estimation of parameters of electromagnetic waves reflections from extended objects (clouds, fog, flocks of birds and insects, dust clouds, the underlying surface), as well as technical and medical diagnostics [4, 6–8].

It is possible to compensate the effect of additive white Gaussian noise on the evaluation of the autocorrelation properties of the experimental signal by finding the optimal values of the relative width ΔFT (F is the frequency, T is the time period between samples) of the spectral mode and the weight coefficient α that characterizes the Gaussian fraction and the fraction of $(1 - \alpha)$ resonant components in the unimodal envelope of the spectrum [1, 9, 10].

Inaccurate estimates of the autocorrelation coefficients R_j lead to a significant deterioration in the quality of the parametric spectral evaluation [11–13]. It is possible to construct more adequate estimates of the PSD based on the recovered autocorrelation coefficients $\tilde{R}_j$, for example, using autoregressive approaches [1, 6].

Below the gains in the quality of spectral estimation are shown that are obtained by using this method as compared to the results obtained with known methods of reducing the impact of noise on a useful signal. It is expedient to carry out the comparison according to the formal criterion that can be represented by the standard deviation of the obtained spectral estimates of **s** from the control spectrum **c** [14, 15]. The control spectrum **c** is the spectral estimate based on non-noisy data.

Usually an autoregressive (AR) model [16–18] with a parametric approach is used for the spectral estimation of narrowband signals ($\Delta FT < 0.3$), especially when there are significant limitations on the duration of observations M ($M < 1000$). Since the work considers sufficiently narrow-band processes with a

small relative width ΔFT of the spectrum ($\Delta FT < 0.3$), it is preferable to use the AR model of optimized order **p** as the parametric method of spectral estimation [1, 19].

Representing the correlation matrix **R** as the sum of the analyzed process useful $\breve{\mathbf{R}}$ correlated and uncorrelated interfering $P_{\text{n}}\mathbf{I}$ components respectively:

$$\mathbf{R} = \breve{\mathbf{R}} + P_{\text{n}}\mathbf{I}, \tag{1}$$

where $\breve{\mathbf{R}}$ is the normalized to dispersion $[(p + 1)\times(p + 1)]$-dimensional autocorrelation matrix of the useful signal, **I** is the $[(p + 1)\times(p + 1)]$-dimensional unit matrix, P_{n} is the relative noise power.

The matrix **R** is Hermitian and Toeplitz [1, 20] for the considered quasistationary interfering process. Therefore, the structure of the matrix **R** has a band form:

$$\mathbf{R} = \begin{bmatrix} 1+P_{\text{n}} & \breve{R}_1^* & \cdots & \breve{R}_j^* & \cdots & \breve{R}_p^* \\ \breve{R}_1 & 1+P_{\text{n}} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \breve{R}_j^* \\ \breve{R}_j & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \breve{R}_1^* \\ \breve{R}_p & \cdots & \breve{R}_j & \cdots & \breve{R}_1 & 1+P_{\text{n}} \end{bmatrix}, \tag{2}$$

where $\breve{R}_j$ are the jth order autocorrelation coefficients normalized to the dispersion of the correlated (useful) component; $j = 0, 1, \ldots, p$; "*" is the sign of complex conjugation.

It is necessary to consider the class of processes that are unimodal in the spectrum and stationary in the observation interval while having a mixed (Gaussian and resonant) shape of the envelope of the spectral power density. If the spectrum mode has the mixed form [6, 12], the modules of the coefficients $\breve{R}_j$ of the autocorrelation of such processes are reduced to the form:

$$\breve{\mathbf{R}} = \alpha\breve{\mathbf{R}}_{\text{G}} + (1-\alpha)\breve{\mathbf{R}}_{\text{r}}, \tag{3}$$

where $\breve{\mathbf{R}}_{\text{G}}$ is the $[(p + 1)\times(p + 1)]$-dimensional dispersion-normalized autocorrelation matrix for the Gaussian envelope of the spectrum, whose elements are

$$R_{\text{G}_{j,k}} = \exp[-(\pi\Delta FT|j-k|)^2 / 2.8], \tag{4}$$

$\breve{\mathbf{R}}_{\text{r}}$ is the $[(p + 1)\times(p + 1)]$-dimensional autocorrelation matrix for the resonant envelope with a unit dispersion of a random process that is unimodal across the spectrum, the elements of which are

$$R_{\text{r}_{j,k}} = \exp(-\pi\Delta FT|j-k|), \tag{5}$$

α, $(1 - \alpha)$ are the proportions of the Gaussian and resonant components in the envelope of the unimodal energy spectrum, respectively, $j, k = 0, 1, \ldots, p$.

To compensate for the destructive effect of noise on the accuracy of the autocorrelation coefficients R_j estimation, it is necessary to find the optimal values of α and ΔFT characterizing the mode of the spectrum. The optimization is performed by the criterion of the minimum of the square of the length E of the $(p + 1)$-dimensional vector $\boldsymbol{\varepsilon}$ of the discrepancy:

$$E = \boldsymbol{\varepsilon}^{\text{H}}\boldsymbol{\varepsilon},$$

where

$$\boldsymbol{\varepsilon}=\mathbf{R}\mathbf{i}-\alpha(\breve{\mathbf{R}}_{\mathrm{G}}+P_{\mathrm{n}}\mathbf{I})\mathbf{i}-(1-\alpha)(\breve{\mathbf{R}}_{\mathrm{r}}+P_{\mathrm{n}}\mathbf{I})\mathbf{i}, \tag{6}$$

where $\mathbf{i}$ = [1, 0, …, 0] is the (p + 1)-dimensional leftmost column vector of the identity matrix $\mathbf{I}$; "H" is a symbol of complex conjugation and transposition.

While using (4) and (5) when normalizing to the dispersion of the useful signal of the matrix $\breve{\mathbf{R}}+P_{\mathrm{n}}\mathbf{I}$, the function (6) at the position of the spectral mode of the useful signal at zero relative frequency takes the form:

$$E(\alpha,\Delta FT)=\sum_{k=1}^{p}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}^2\rightarrow\min_{\substack{\alpha\in[0,1],\\ \Delta FT\in[0.01,\,0.3]}}, \tag{7}$$

where R_k are the elements of the matrix $\mathbf{R}$ of the observed random process.

Restrictions on the relative width of the ΔFT spectrum are due to the assumption of it being narrowband, which is typical of many practical applications [4, 5].

To determine the optimal values of α and ΔFT, which characterize the spectral mode, we use the gradient descent method. Taking the derivative with respect to α of the minimized expression (7):

$$\frac{\mathrm{d}E}{\mathrm{d}\alpha}=\sum_{k=1}^{p}\left\{\frac{\mathrm{d}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}^2}{\mathrm{d}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}}\times\frac{\mathrm{d}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}}{\mathrm{d}\alpha}\right\} \tag{8}$$

$$=2\sum_{k=1}^{p}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}\left\{\frac{-\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}+\frac{\exp(-\pi\Delta FTk)}{1+P_{\mathrm{n}}}\right\}.$$

Taking the derivative with respect to ΔFT of the objective function (7):

$$\frac{\mathrm{d}E}{\mathrm{d}\Delta FT}=\sum_{k=1}^{p}\left\{\frac{\mathrm{d}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}^2}{\mathrm{d}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}}\times\frac{\mathrm{d}\left\{R_k-\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\mathrm{n}}}-\frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\mathrm{n}}}\right\}}{\mathrm{d}\Delta FT}\right\}$$

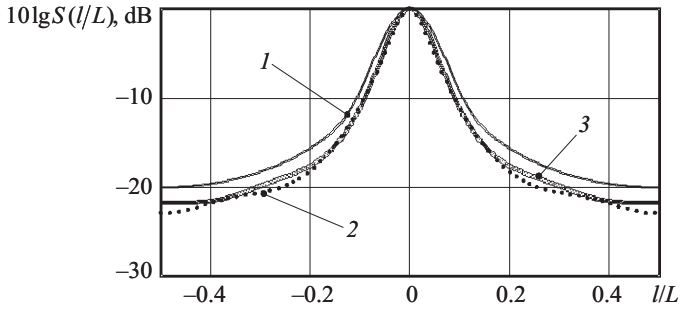

Fig. 1.

$$= 2\sum_{k=1}^{p}\left\{R_k - \frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\text{n}}} - \frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\text{n}}}\right\}$$

$$\times\left\{\frac{-\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\text{n}}}\frac{\text{d}[-(\pi\Delta FTk)^2/2.8]}{\text{d}\Delta FT} - \frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\text{n}}}\frac{\text{d}[-(\pi\Delta FTk)]}{\text{d}\Delta FT}\right\}$$

$$= 2\sum_{k=1}^{p}\left\{R_k - \frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]}{1+P_{\text{n}}} - \frac{(1-\alpha)\exp(-\pi\Delta FTk)]}{1+P_{\text{n}}}\right\}$$

$$\times\left\{\frac{\alpha\exp[-(\pi\Delta FTk)^2/2.8]\cdot 2\pi^2k^2\Delta FT}{2.8(1+P_{\text{n}})} + \frac{(1-\alpha)\exp(-\pi\Delta FTk)\pi k}{1+P_{\text{n}}}\right\}. \quad (9)$$

The use of the optimal values of α and ΔFT allows to reconstruct the autocorrelation coefficients $\breve{R}_j$ of the useful component of the analyzed random process. The effectiveness of such a restoration and its positive effect on the accuracy of spectral estimates of **s** process, which is unimodal in the spectrum, with a partially known form of the power spectral density is confirmed below.

Fig. 1 shows the results of spectral estimation using parametric methods in conditions of weak noise. The ratio P_{n} of the powers of the correlated signal under study and the additive white noise is $P_{\text{n}} = 10^{-6}$. The relative width ΔFT of the spectral mode is assumed to be 10% of the entire observed frequency spectrum ($\Delta FT = 0.1$). Such value of ΔFT corresponds to the typical radio reflections from clouds [21, 22] for a centimeter pulsed radar system with a low repetition rate of probing pulses [14, 16].

Fig. 1 has the following legend: $S(l/L)$ is the PSD that is normalized to its maximum value, given in relative units; l/L is the relative frequency that is numerically equal to the ratio of the number l of the current spectral sample to the total number L of spectral samples ($L = 1024$); solid thin curve *1* is the control spectrum (PSD that is obtained from theoretical data using the AR model of the thirtieth order); dotted curve *2* is the PSD that is obtained using the well-known AR model; solid bold curve *3* is the spectrum estimation using the proposed approach. Curves *2*, *3* were obtained with the same order $p = 5$ of the compared methods of parametric spectral analysis.

Fig. 1 allows to conclude that the quality of spectrum estimation by the proposed approach is better than the well-known parametric method of autoregression of the same order p.

To make the effectiveness assessment of the compared methods of spectral analysis more objective, vector $\boldsymbol{\varepsilon}_{\text{s}}$ of the discrepancy is formed between the L-dimensional vectors of the spectral power densities:

$$\boldsymbol{\varepsilon}_{\text{s}} = \mathbf{c} - \mathbf{s}, \quad (10)$$

where **c** is the L-dimensional vector PSD of the control model that uses the energy spectrum obtained using a high-order AR model ($p = 30$) with unlimited sampling $M \to \infty$ (theoretically calculated autocorrelation

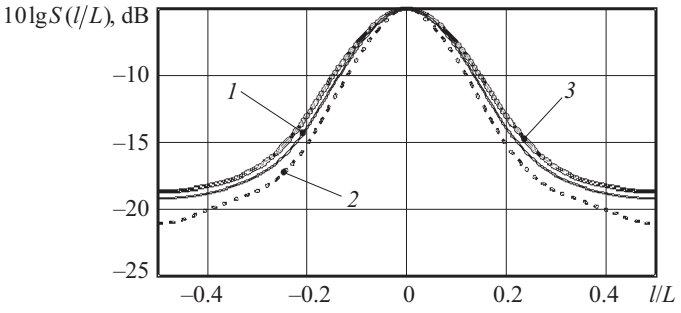

Fig. 2.

Table 1

P_n	Method	
	AP	Proposed
10^{-2}	1.572×10^{-3}	4.446×10^{-4}
10^{-3}	9.712×10^{-4}	3.827×10^{-4}
10^{-4}	5.737×10^{-4}	3.341×10^{-4}
10^{-5}	4.287×10^{-4}	2.856×10^{-4}

coefficients R_j are taken); **s** is the L-dimensional PSD vector, obtained by methods that are compared to the control model.

The value E_S of the square of the length of the L-dimensional column vector $\boldsymbol{\varepsilon}_S$ of the discrepancy, which is normalized to the number L of spectral samples, is taken as the criterion of adequacy:

$$E_S = \boldsymbol{\varepsilon}_S^H \boldsymbol{\varepsilon}_S / L . \quad (11)$$

The adequacy comparison of the spectral estimation using various approaches allows to conclude that the normalized square of the length E_S of the vector $\boldsymbol{\varepsilon}_S$ of the discrepancy, calculated from expressions (10), (11), for the proposed approach ($E_S = 2.46\times10^{-4}$) has approximately 1.5 times smaller value than the well-known parametric AR-method ($E_S = 3.672\times10^{-4}$) with the length M of the sample, which was used to estimate the autocorrelation matrix **R**, $M = 1000$, $p = 5$, $\alpha = 0.5$, $\Delta FT = 0.1$, $P_n = 10^{-6}$.

The influence of additive white Gaussian noise on the adequacy of the spectral estimation of the correlated process described above is analyzed in Table 1.

When setting up the dependence of E_S value on the relative power P_n of the uncorrelated noise given in Table 1, the following parameters are taken: $L = 1024$, $\alpha = 0.5$, $\Delta FT = 0.1$, $p = 5$, and $M = 1000$.

Analysis of the data presented in Table 1 showed that the normalized squares of the lengths E_S of the discrepancy vectors $\boldsymbol{\varepsilon}_S$ for spectral estimation by the known autoregressive method in a wide range of relative power changes $P_n = [10^{-5}; 10^{-2}]$ noise have large values and, consequently, low adequacy compared with the proposed approach. So, for example, at $P_n = 10^{-2}$, the gains by criterion (11) for the proposed approach to spectrum evaluation reach 4 times in comparison with the known AR-approach.

Fig. 2 shows the results of spectral estimation with a twofold increase in the relative width ΔFT of the spectrum ($\Delta FT = 0.2$) and $P_n = 10^{-6}$.

The legend of Fig. 2 is similar to Fig. 1: continuous thin curve *1* is the control spectrum; dotted curve *2* is the PSD, obtained using the well-known AR model; solid bold curve *3* is the spectrum estimation using the proposed method.

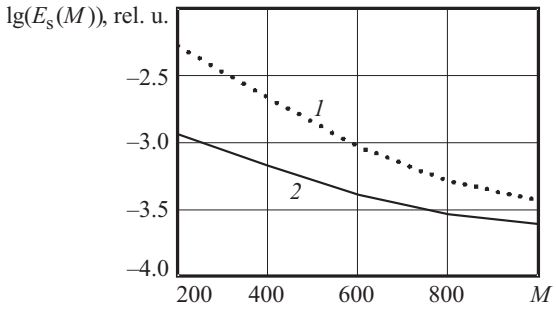

Fig. 3.

Table 2

p	Method	
	AP	Proposed
3	1.072×10^{-3}	8.796×10^{-4}
5	3.672×10^{-4}	2.46×10^{-4}
7	2.118×10^{-4}	1.023×10^{-4}
9	1.901×10^{-4}	7.702×10^{-5}
11	8.185×10^{-5}	2.31×10^{-5}

The analysis of Fig. 2 indicates that the advantages of the proposed approach are preserved with an increase in the relative width of the spectrum ΔFT of the process under study. The quality of spectrum estimation by the proposed method is better than that of the well-known AR-method with similar values $L = 1024$, $\alpha = 0.5$, $p = 5$, $M = 1000$ and the relative noise power $P_n = 10^{-6}$.

The objective assessment of the effectiveness of describing processes with wider bandwidth ($\Delta FT = 0.2$) requires to compare the normalized squares of the lengths of the E_s vectors of the discrepancies between the estimated and control spectra. Computer experiments have shown that in this case the square of the length E_s of the discrepancy vector for the proposed model ($E_s = 1.149\times10^{-3}$) is 4 times smaller than for the method of the AR-model ($E_s = 4.911\times10^{-3}$) of the same order ($p = 5$).

It is necessary to analyze the influence of the order p of models on the adequacy of spectral estimation (Table 2) with values similar to the first example: $M = 1000$, $\alpha = 0.5$, $L = 1024$, $P_n = 10^{-6}$, $\Delta FT = 0.1$ (Fig. 1).

The analysis of Table 2 shows that the adequacies of the proposed approach and the well-known AR-method increase with increasing order p of models, while increasing the number of autocorrelation coefficients R_j, $\widetilde{R}_j$ used for spectral estimation, $j = 0, 1, \ldots, p$.

The next step is to study how the number of samples (length) M affects the adequacy of spectral estimation using the proposed approach with similar previous values $p = 5$, $L = 1024$, $\Delta FT = 0.1$ and relative noise power $P_n = 10^{-2}$ (Fig. 3).

Figure 3 shows the dependences of the normalized squares of the lengths E_s of the vectors $\boldsymbol{\varepsilon}_s$ of the discrepancy between the compared methods of spectral analysis and the control spectrum. The dotted line *1* reflects the dependence $E_s(M)$ for the well-known AR approach; solid line *2* reflects the proposed method.

The dependences in Fig. 3 show that the accuracy E_s of spectral estimation with the same length M is higher for the proposed method than for the well-known parametric AR-approach. While $M = 200$, the accuracy of the proposed method by criterion (11) is more than 4 times higher compared to the well-known AR model, which makes it possible to significantly (up to several times) reduce the number M of time

samples used for spectral estimation, while maintaining the specified model adequacy. Fig. 3 shows that it is possible to reduce by 2.3–4 times the length M compared to the known AR method for $p = 5$ for a given spectral estimation accuracy $\lg E_s < -2.9$.

CONCLUSIONS

Thus, the authors have proposed and investigated a method for recovering the coefficients R_j of a discrete autocorrelation sequence of random signals that are unimodal by spectrum with a partially known form of their spectral mode. The method is based on finding the optimal values of the relative width ΔFT and the weighting factor $\alpha \in [0; 1]$ of the spectral mode, which characterizes the fraction α of the Gaussian and the fraction $(1-\alpha)$ of the resonant components in the envelope of the unimodal spectrum. Finding the values of ΔFT and α makes it possible to restore the autocorrelation coefficients R_j of the random process, distorted by noise, to improve the quality of its spectral estimation.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

ADDITIONAL INFORMATION

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