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2018 10TH INTERNATIONAL CONFERENCE ON KNOWLEDGE AND SYSTEMS ENGINEERING

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November 1-3, 2018
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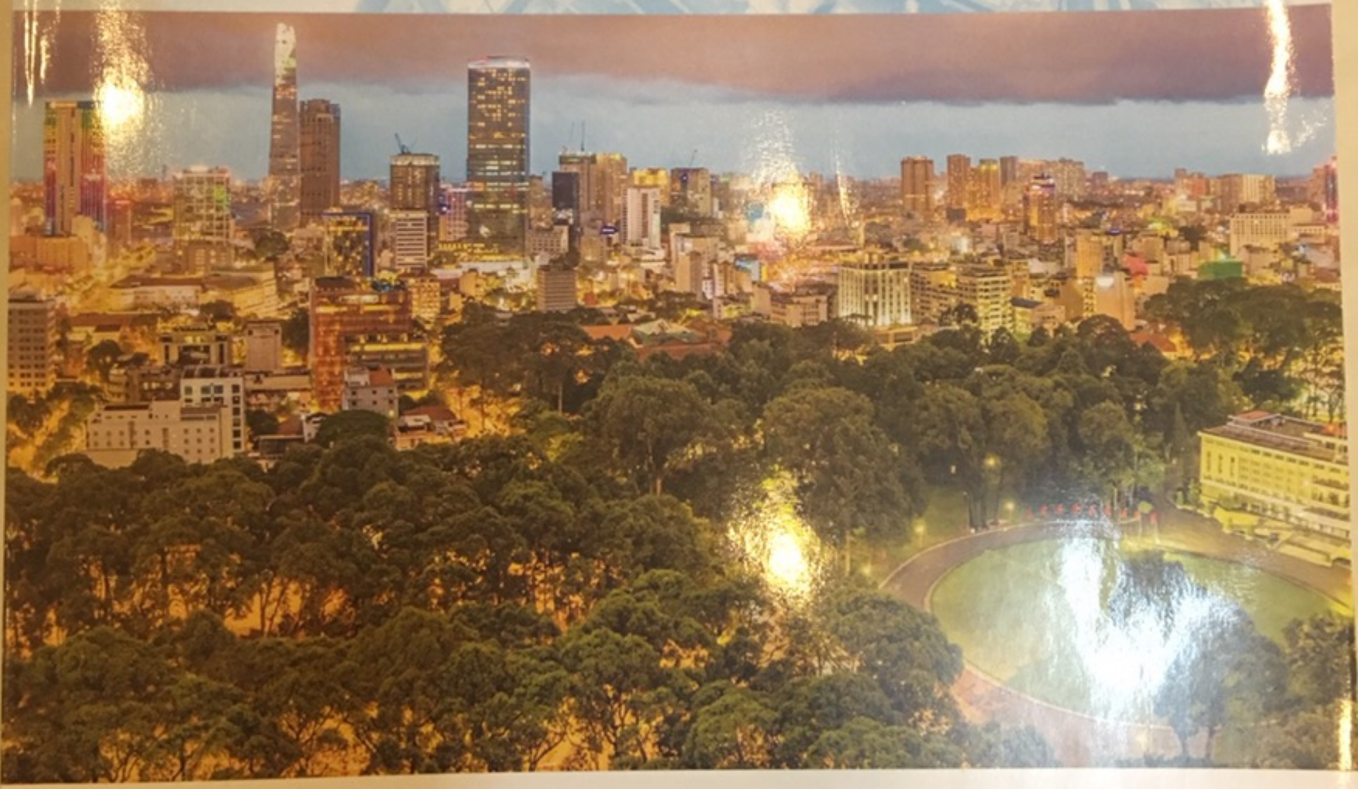

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An improved QR decomposition for color image watermarking

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***Abstract*—Beside many other algorithms such as Singular Value Decomposition (SVD), Discreet Cosine Transform (DCT) and Discrete Wavelet Transform (DWT), QR decomposition is known as an effective method for embedding and extracting watermarked image. In QR decomposition, Gram-Schmidt and Householder [10] are the most popular two algorithms. In this paper, QR factorization is executed by exploiting orthogonality of Q matrix and triangularity of R matrix to find out elements of these matrixes. The algorithm of Sun [12] is used for both embedding and extracting. For embedding watermark, Q and R will be calculated separately where computing R is implemented by solving a set of linear equations and calculating Q is base on Gram-Schmidt algorithm [10] via knowing R. In addition, diagonal elements of R matrix are inspected to ensure their validity except the first diagonal one. For extracting watermark, only the first element $R(1,1)$ of R matrix needs to be computed which is performed by an operation. Experimental results show that the proposed scheme not only has better quality of watermarked image, but also overcomes problems of robustness and computation complexity.**

Keywords*—Improved QR decomposition, color image watermarking, SVD, DCT.

I. INTRODUCTION

A. Background

For solving the problem of copyright protection, digital watermarking technique has been considered as a powerful method. The main goal of image watermarking is to hide some important information into a host image.

There are many image watermarking schemes which based on singular value decomposition (SVD) in [1], [4], [7, [8], [12] and [17] or QR decomposition in [10], [11], [16]. Also, some authors combined SVD with some other methods such as DCT, DWT [3], [5], [13], [15] or associated QR decomposition with SVD or DCT in [2] and [6] to create new watermarking algorithms. For watermarking schemes based on SVD, method of Sun [12] is referred to by many researchers, which embedding and extracting watermark are executed on the first element $D(1,1)$ of triangular matrix D. However, SVD decomposition has a disadvantage that is big time complexity. Hence, in some cases, QR decomposition should be used instead of. From this idea, a combination between QR decomposition and the algorithm of Sun [12] is proposed which uses Gram-Schmidt algorithm [10] for QR factorization and scheme of Sun for embedding as well as extracting watermark. To be similar to SVD decomposition, the first element $R(1,1)$ of upper triangular matrix R is used to embed and extract information.

As experimental results, although this proposed method increases speed of calculating, it gives out worse invisibility and robustness of watermarked image than the one based on SVD decomposition. A reason for this is because Gram-Schmidt algorithm calculates Q and R column by column, so it does not inspect diagonal elements of R matrix if these values are zero or negative.

To convenient for expressing, method of Sun based on SVD is abbreviated to SunSVD and the method based on QR decomposition and Gram-Schmidt algorithm is abbreviated to SunQR in the rest of this paper.

To overcome drawbacks of SunQR, a new watermarking scheme based on QR decomposition is proposed in this paper where R matrix is computed at first and diagonal elements of R are checked and modified if they are zero or negative. Calculating elements of R is performed by solving a set of linear equations; after that, Q is computed based on R. In addition, to improve extracting time, only the first elements $R(1,1)$ of R is need to get out instead of finding out whole elements of Q and R as case of SunQR.

B. Roadmap

The rest of this paper is organized as follows. Section 2 describes the QR decomposition theory and its special features. Section 2.3 introduces the details of the watermark embedding procedure and describes our watermark extraction procedure. Section 3 gives the experimental results. Finally, Section 4 concludes this presentation.

II. PROPOSED METHOD

A. QR decomposition

The QR decomposition, which is a type of matrix decomposition, is defined in [10] as follows:

If A $\in R^{n\times n}$ has linearly independent columns then it can be factored as,

$$A = QR, \tag{1}$$

where Q is an orthogonal matrix ($Q^TQ = QQ^T = I$) and R is an upper triangular matrix with nonzero diagonal elements.

For example, suppose one matrix A of size 4×4 as follows,

$$A = \begin{bmatrix} 42 & 42 & 43 & 42 \\ 41 & 42 & 42 & 41 \\ 39 & 39 & 37 & 38 \\ 37 & 36 & 36 & 38 \end{bmatrix} \tag{2}$$

Using Eq. (1), its upper triangular matrix R and orthogonal matrix Q can be obtained as follows,

$$Q = \begin{bmatrix} 0.5277 & 0.2771 & 0.5992 & -0.5348 \\ 0.5151 & 0.2705 & 0.1365 & 0.8017 \\ 0.4900 & 0.2573 & -0.7889 & -0.2671 \\ 0.4649 & -0.8854 & 0.0000 & 0.0003 \end{bmatrix} \tag{3}$$

$$R = \begin{bmatrix} 79.5927 & 79.1278 & 78.6755 & 79.5676 \\ 0 & 0.8854 & 0.6480 & -1.1425 \\ 0 & 0 & 2.1769 & 0.7889 \\ 0 & 0 & 0 & 0.2673 \end{bmatrix} \tag{4}$$

B. The special feature in Q and R matrix

By multiplying two sides of the equation with A^T (is a transposition of A), (1) in II-A becomes as follows,

$$\begin{aligned} & A^T A = A^T QR \\ \Rightarrow\ & A^T A = (QR)^T QR \\ \Rightarrow\ & A^T A = R^T Q^T QR = R^T R \\ \Rightarrow\ & M = A^T A = R^T R \end{aligned} \tag{5}$$

Note that, Q is an orthogonal matrix, so that $Q^T Q = I$. I is the unit matrix.

Since R is an upper triangular matrix, R can be computed easily by solving a set of linear equations. Supposedly, the host matrix A has size of 4×4. Therefore, M, A^T, R^T and R are also 4×4 matrixes.

The elements of A are represented as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} \mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_4} \end{bmatrix}, \tag{6}$$

where $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \mathbf{a_4}$ are column vectors of A, respectively.

The transposition matrix of A, called A^T , is calculated as follows:

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \tag{7}$$

Therefore, matrix M is calculated as follows:

$$M = A^T A = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \tag{8}$$

R and R^T are also expressed as follows:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{bmatrix} \tag{9}$$

$$R^T = \begin{bmatrix} r_{11} & 0 & 0 & 0 \\ r_{21} & r_{22} & 0 & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} \tag{10}$$

According to (5), we retrieve the following formula:

$$\begin{aligned} & R^T R = M \\ \Longleftrightarrow & \begin{bmatrix} r_{11} & 0 & 0 & 0 \\ r_{21} & r_{22} & 0 & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} * \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{bmatrix} \\ = & \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \end{aligned} \tag{11}$$

According to Eq. (11), the following of parameters can be derived.

$$\begin{aligned} r_{11} * r_{11} = m_{11} &\Rightarrow r_{11} = \sqrt{m}_{11} \\ r_{11} * r_{12} = m_{12} &\Rightarrow r_{12} = m_{12}/r_{11} \\ r_{11} * r_{13} = m_{13} &\Rightarrow r_{13} = m_{13}/r_{11} \\ r_{11} * r_{14} = m_{14} &\Rightarrow r_{14} = m_{14}/r_{11}, \end{aligned} \tag{12}$$

$$\begin{aligned} & r_{12} * r_{12} + r_{22} * r_{22} = m_{22} \Rightarrow r_{22} = \sqrt{m_{22} - r_{12}^2} \\ & r_{12} * r_{13} + r_{22} * r_{23} = m_{23} \Rightarrow r_{23} = (m_{23} - r_{12} * r_{13})/r_{22} \\ & r_{12} * r_{14} + r_{22} * r_{24} = m_{24} \Rightarrow r_{24} = (m_{24} - r_{12} * r_{14})/r_{22}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} & r_{13} * r_{13} + r_{23} * r_{23} + r_{33} * r_{33} = m_{33} \\ & \qquad \Rightarrow r_{33} = \sqrt{m_{33} - r_{13}^2 - r_{23}^2} \\ & r_{13} * r_{14} + r_{23} * r_{24} + r_{33} * r_{34} = m_{34} \\ & \qquad \Rightarrow r_{34} = (m_{34} - r_{13} * r_{14} - r_{23} * r_{24})/r_{33}, \end{aligned} \tag{14}$$

then

$$\begin{aligned} & r_{14} * r_{14} + r_{24} * r_{24} + r_{34} * r_{34} + r_{44} * r_{44} = m_{44} \\ & \qquad \Rightarrow r_{44} = \sqrt{m_{44} - r_{14}^2 - r_{24}^2 - r_{34}^2}, \end{aligned} \tag{15}$$

Calculating Q is performed base on the Gram-Schmidt algorithm [10].

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix} = \begin{bmatrix} \mathbf{q_1} & \mathbf{q_2} & \mathbf{q_3} & \mathbf{q_4} \end{bmatrix} \tag{16}$$

The Gram-Schmidt algorithm, which was introduced in [10], computes Q column by column. According to that, the first column of Q is as follows:

$\tilde{\mathbf{q_1}} = \mathbf{a_1} = [a_{11} \quad a_{21} \quad a_{31} \quad a_{41}]^{\mathbf{T}}$ and $\mathbf{q_1} = \frac{1}{r_{11}} \tilde{\mathbf{q_1}}$.

$\tilde{\mathbf{q_2}} = \mathbf{a_2} - r_{12} * \mathbf{q_1} = [a_{12} \quad a_{22} \quad a_{32} \quad a_{42}]^T - r_{12} * [q_{11} \quad q_{21} \quad q_{31} \quad q_{41}]^T$ and $\mathbf{q_2} = \frac{1}{r_{22}}\tilde{\mathbf{q_2}}$.

$\tilde{\mathbf{q_3}} = \mathbf{a_3} - r_{13} * \mathbf{q_1} - r_{23} * \mathbf{q_2} = [a_{13} \quad a_{23} \quad a_{33} \quad a_{43}]^T - r_{13} * [q_{11} \quad q_{21} \quad q_{31} \quad q_{41}]^T - r_{23} * [q_{12} \quad q_{22} \quad q_{32} \quad q_{42}]^T$ and $\mathbf{q_3} = \frac{1}{r_{33}}\tilde{\mathbf{q_3}}$.

$\tilde{\mathbf{q_4}} = \mathbf{a_4} - r_{14} * \mathbf{q_1} - r_{24} * \mathbf{q_2} - r_{34} * \mathbf{q_3} = [a_{14} \quad a_{24} \quad a_{34} \quad a_{44}]^T - r_{14} * [q_{11} \quad q_{21} \quad q_{31} \quad q_{41}]^T - r_{24} * [q_{12} \quad q_{22} \quad q_{32} \quad q_{42}]^T - r_{34} * [q_{13} \quad q_{23} \quad q_{33} \quad q_{43}]^T$ and $\mathbf{q_4} = \frac{1}{r_{44}}\tilde{\mathbf{q_4}}$.

In general, if the host matrix A has size of $n \times n$, we have

$\tilde{\mathbf{q_1}} = \mathbf{a_1}$.

$\tilde{\mathbf{q_i}} = \mathbf{a_i} - (r_{1i}\mathbf{q_1} + r_{2i}\mathbf{q_2} + \cdots + r_{i-1,i}\mathbf{q_{i-1}})$ and $\mathbf{q_i} = \frac{1}{r_{ii}}\tilde{\mathbf{q_i}}$, where $\mathbf{a_i}, \tilde{\mathbf{q_i}}$ and $\mathbf{q_i}$ (with $i = 2, 3, \cdots, n$) are $n \times 1$ vectors.

III. The Proposed Watermarking Scheme

In this paper, we propose a new watermarking scheme base on improved QR decomposition which is presented in II-B and algorithm of Sun in [12].

A. Watermark embedding scheme

In the embedding process, the host color image is divided into 4×4 non-overlapping blocks at first. Then, the gray watermark image is transformed to binary sequence. Finally, the improved QR decomposition is performed on the image blocks in succession (respectively) and the watermark is embedded into R matrix. The proposed watermark embedding scheme can be summarized as follows:

Step 1) Divide the host color image I into 4×4 non-overlapping blocks. In this image, each pixel is represented by three components (R, G, B).

Step 2) Transform the gray watermark image to a one-dimensional array ($w = w_1, w_2, w_3, w_4$ and so on).

Step 3) Perform QR decomposition on one block based on II-B.

- Assign B components of the block to a 4×4 matrix (matrix A).

- Calculate A^T by transposing A

- Compute $M = A^T A$

- Find out R and Q by solving a set of equations as represented in II-B.

Step 4) Embed watermark to the triangular matrix R based on algorithm of Sun at [12].

- Get the first element of R matrix as $R(1,1)$

- Calculate $z = R(1,1)$ mod q (with q is a positive integer)

`Case` $w_i =$ "0"

$$R'(1,1) = \begin{cases} R(1,1) + q/4 - z & \text{if } z < 3 * q/4, \\ R(1,1) + (5 * q)/4 - z & \text{if } z > 3 * q/4. \end{cases} \quad (17)$$

`Case` $w_i =$ "255"

$$R'(1,1) = \begin{cases} R(1,1) - q/4 - z & \text{if } z < q/4, \\ R(1,1) + (3 * q)/4 - z & \text{if } z > q/4. \end{cases} \quad (18)$$

Step 5) Update A by formula (1): $A' = QR'$ and assign A' back to B components of the block.

Step 6) Repeat steps 3−5 until all blocks are embedded watermark values. Finally, the watermarked B components are reconstructed to obtain the watermarked image H'.

B. Watermark extraction scheme

Since the watermark is only embedded into R matrix in the embedding process, QR decomposition is not needed in the watermark extraction procedure. The main purpose of this extraction scheme is finding out the first element $R(1,1)$ of R matrix. The watermark extraction steps are described as follows.

Step 1) Divide the watermarked image H' into 4×4 non-overlapping blocks. In this image, each pixel is represented by three components (R, G, B).

Step 2) Assign B components of the block to a 4×4 matrix (matrix A).

Step 3) Get out the first element $R(1,1)$ of R matrix as follows: $R(1,1)$ = length of the first column vector of A matrix [10] or $R(1,1) = \sqrt{A(1,1)^2 + A(2,1)^2 + A(3,1)^2 + A(4,1)^2}$.

Step 4) Extract information of watermark based on algorithm of Sun in [12].

- Calculate $z = R(1,1)\ mod\ q$

- If $(z < q/2) \Rightarrow w =$ "0" else $w =$ "255"

Step 5) Repeat steps 2−4 until watermark values are extracted on all blocks. Finally, collect all extracted watermark values into an image.

IV. Experiment Result

The main performance of the watermarking methods under consideration is investigated by measuring their invisibility, robustness, execution time and embedding capacity. For proving the watermarking performance of the proposed method, all 24-bit 512×512 color images in the CVG-UGR image database are used as the host images [13], and a 4KB gray image as shown in Figure 1(h) is used as original watermark.

A. Invisibility

In order to evaluate the invisibility capability, not only is the peak signal-to-noise ratio (PSNR) used in this paper, but also the structural similarity (SSIM) index as a new method is also used to measure the similarity between the original color image H and the watermarked image H'. In general, a larger PSNR or SSIM means that the watermarking method has higher invisibility.

For relatively fair comparison, the propose method is compared with SunSVD [12] and SunQR [1.Introduction], which use singular value decomposition (SVD)-based watermarking

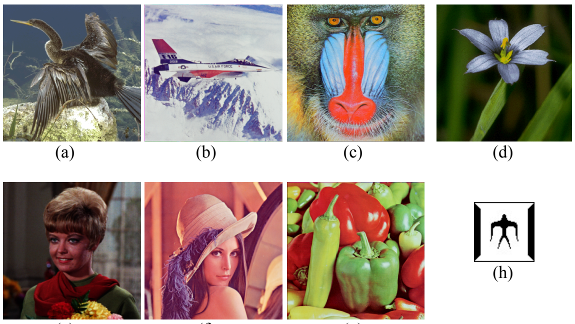


Fig. 1: Original host images: (a) anhinga, (b) avion, (c) baboon, (d) blueeye, (e) girl, (f) lena, (g) peppers; Original watermark image: (h) titech

TABLE I: The comparison of execution time between different methods (in second).

Method	Embedding time	Extraction time	Total
SunSVD [12]	0.826	0.603	1.429
SunQR [1.Introduction]	0.637	0.448	1.085
Proposed method	0.604	0.042	0.639

algorithm and QR decomposition by using Gram-Schmidt algorithm, respectively. Detail of the result is represented in Figure 2.

Figure 2 shows that the proposed method gives higher PSNR/SSIM values than SunSVD[12] and SunQR [1.Introduction]. This also means that the invisibility of watermarked image is better in the proposed scheme.

Theoretically, R is an upper triangular matrix with nonzero diagonal elements. According to (12) in II-B, the first diagonal element r_{11} is always positive because pixel value is also positive. However, in fact, the other diagonal ones can be zero or negative which base on (13), (14), and (15) in II-B.

Come back to (13) in II-B, we have $r_{22} = \sqrt{m_{22} - r_{12}^2}$. If $(m_{22} - r_{12}^2 = 0)$, $r_{22} = 0$. Thus, r_{23} and r_{24} are infinite. Otherwise, If $(m_{22} - r_{12}^2 < 0)$, r_{22} is also infinite.

It is similar to r_{33} and r_{44}. This leads to infinite values of Q and A' (the matrix A after embedding watermark). This is reason of reducing quality of watermarked image too.

Therefore, to solve this issue, we check the value below the square root before calculating $r_{ii} (i = 2, 3, 4)$. If this value is zero or negative, r_{ii} is set up to 1. This action not only gives out better invisibility of watermarked image, but also do not completely affect to embedding process as well as extraction because two processes only use the first diagonal element r_{11}.

B. Execution time

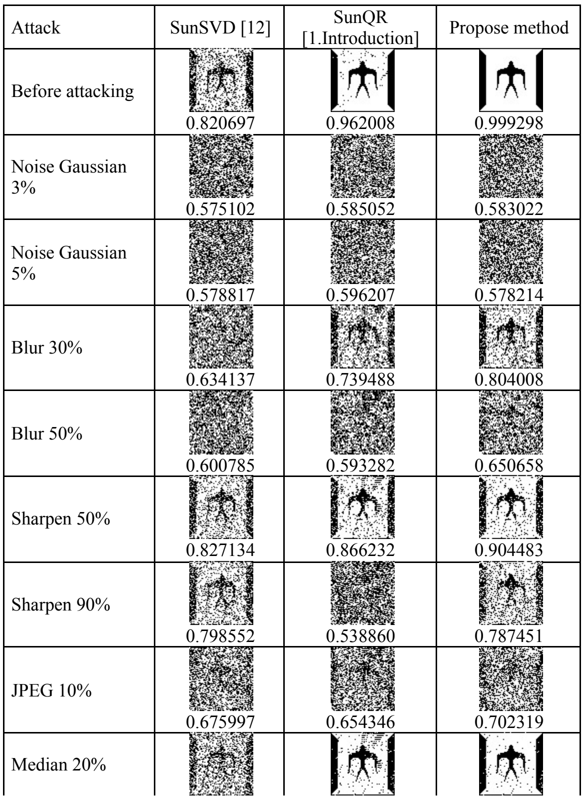
In these experiments, a laptop computer with Intel Core i3-2310M CPU at 2.10GHz, 3.00GB RAM, 32-bit OS and Visual Studio v11 is used as the computing platform. The embedding time and extraction time of the proposed methods is 0.597s, 0.042s, respectively. Table I shows a comparison of the execution time between different methods.

Fig. 2: The watermarked color images and PSNR/SSIM values without any attacks.

According to Table I, the total of execution time of the proposed method is the smallest which compares to SunSVD [12] and SunQR [1.Introduction]. For embedding time, the proposed algorithm is proximate SunQR[1.Introduction] because the both uses QR factorization; meanwhile, it is less than SunSVD [12] since SunSVD[12] takes SVD decomposition. For extraction time, the proposed method gives an effective result because it calculates length of the first column vector of A matrix to find out the first element $R(1, 1)$ of R matrix instead of using QR decomposition as SunQR[1.Introduction]. This demonstrates that the proposed scheme is extremely significant to improve speed of watermarking process, especially extraction time.

C. Analysis of robustness

In order to measure the robustness of the watermark, we use the normalized correlation (NC) between the original watermark W and the extracted watermark W', which is



Attack	SunSVD [12]	SunQR [1.Introduction]	Propose method
Before attacking	0.820697	0.962008	0.999298
Noise Gaussian 3%	0.575102	0.585052	0.583022
Noise Gaussian 5%	0.578817	0.596207	0.578214
Blur 30%	0.634137	0.739488	0.804008
Blur 50%	0.600785	0.593282	0.650658
Sharpen 50%	0.827134	0.866232	0.904483
Sharpen 90%	0.798552	0.538860	0.787451
JPEG 10%	0.675997	0.654346	0.702319
Median 20%			

Fig. 3: The extracted watermark and its NC values after some attacks via using different methods

denoted as follows.

$$NC = \frac{\sum_{j=1}^{4}\sum_{x=1}^{m}\sum_{y=1}^{n} W(x,y,j)W'(x,y,j)}{\sqrt{\sum_{j=1}^{4}\sum_{x=1}^{m}\sum_{y=1}^{n}(W(x,y,j))^2}\sqrt{\sum_{j=1}^{4}\sum_{x=1}^{m}\sum_{y=1}^{n}(W'(x,y,j))^2}}, \quad (19)$$

where $W(x,y,j)$ and $W'(x,y,j)$ present the value of pixel (x,y) in component j of the original watermark and the extracted one, and m,n denote the row and the column size of the original watermark image, respectively.

A higher NC indicates that the watermarking method has more robustness. According to experiment results, with bigger value q, the invisibility of watermarked image is worse whereas the watermark robustness is better. On the contrary, the result is in inverse order. Considering the tradeoff between the invisibility and the robustness of watermarking, the coefficient q is set to 10.

Figure 3 indicates that the proposed method has higher NC values as well as better extracted watermark than SunSVD [12] and SunQR [1.Introduction] after almost attacks except adding Gaussian noise with larger parameter. Consequently, the proposed scheme is more robust than scheme of Sun which based on SVD decomposition [12] and Gram-Schmidt algorithm for QR factorization [10]

V. Conclusion

Based on QR decomposition, an improved image watermarking scheme is proposed in this paper. For embedding watermark, the 24-bits color host image is divided into non-overlapping 4×4 pixel blocks and each pixel block is decomposed by improved QR decomposition where elements of Q and R are calculated separately where diagonal elements of R matrix are verified to guarantee their legality. After that, the watermark image is embedded into the color host image by modifying the first element of the upper triangular matrix R which relies on Sun algorithm [12]. In extraction process, only the first element of R matrix is needed, so computing Q and R matrixes must be not performed. This reduces significantly extracting time. Experimental results have shown that this proposed algorithm not only attains higher invisibility of watermarking, but also has effective execution time and stronger robustness in various distortion operations. The proposed scheme overcomes disadvantage of time complexity in SunSVD[12] and drawbacks of watermarked image invisibility as well as robustness of watermark in SunQR [1.Introduction].

References

[1] B.C.Mohan, S.Kumar, "A Robust Digital Image Watermarking Scheme Using Singular Value Decomposition", Journal of Multimedia 3(1), p.7-15, 2008.

[2] B. D. Moor, P. V. Dooren, "Generalizations of the singular value and QR decompositions", SIAM J Matrix Anal Appl 13(4), pp.993-1014,1992

[3] B. Raman,G. Gaurav,"A new robust reference watermarking scheme based on DWT-SVD", Computer Standards & Interfaces, pp.1002-1013, 2009.

[4] C.C Lai, "An improved SVD-based watermarking scheme using human visual characteristics", Optics Communucaion 284, pp. 938-944, 2011.

[5] G. Bhatnagar, B. Raman, "A new robust reference watermarking scheme based on DWT-SVD,, Computer Standards & Interfaces, Vol. 31(5), p.1002-1013, 2009.

[6] H. Chen, Y. Zhu, "A robust watermarking algorithm based on QR factorization and DCT using quantization index modulation technique", Journal of Zhejiang University SCIENCE C, pp. 573-584, 2012.

[7] K. L. Chung, W. N. Yang, Y. H. Huang, S. T. Wu, Y. C. Hsu, "SVD-based watermarking algorithm", Applied Mathematics and Computation, 188(1), 54-57, 2007.

[8] K. Mathew, "SVD based Image Watermarking Scheme", IJCA Special Issue on ECOT, 2010.

[9] [L. Vandenberghe, "6.QR factorization", ECE133A, pp.6-1-6-39, 2018.

[10] N. Yashar, H. K. Saied, "Fast and robust watermarking in still images based on QR decomposition", Multimedia Tools Appl, 72:2597-2618, 2014.

[11] O. Su, G. Wang, X. Zhang, G. Lv, B. Chen, "An improved color image watermarking algorithm based on QR decomposition," Multimedia Tools Appl, pp.1-23, 2015.

[12] R. Sun, H. Sun, T. Yao, "A SVD and quantization based semi-fragile watermarking technique for image authentication", Proc. Internat. Conf. Signal Process. 2, pp. 1952-1955, 2002.

[13] U. M. Gokhale, Y. V. Joshi, "A Semi Fragile Watermarking Algorithm Based on SVD-DWT for Image Authentication", International Journal of Advanced Research in Computer and Communication Engineering, Vol. 1, Iss. 4, 2012.

[14] University of Granada. Computer Vision Group. CVG-UGR Image Database. [2012-10-22]. http:decsai.ugr.escvgdbimagenesc512.php

[15] T. Ye, "A Robust Zero-Watermark Algorithm Based on Singular Value Decomposition and Discreet Cosine Transform",CCIS, pp. 1-8, 2011.

[16] W. Song, H. J. Jun, H. Li, L. Huang, "HouChaotic system and QR factorization based on robust digital image watermarking algorithm", J. Cent. South Univ. Technol. pp. 116-124, 2011.

[17] X. Zhu, J. Zhao, H. Xu, "A digital watermarking algorithm and implementation based on improved SVD", in Proc. 18th Int. Conf. Pattern Recognition (ICPR '06), vol. 3, pp. 651-656, 2006.