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Linear network coding using channel quantization combining with SIC based estimation for multiple antenna systems

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ABSTRACT

This paper proposes a network coding method with linear mapping and low-complexity estimation for the two-way relay non-reciprocal channel using the *M*-QAM modulation. The proposed network coding is developed by combining the channel quantization method and linear estimation based on successive interference cancellation. The effect of residual interference due to channel quantization on the signal decision is analyzed in detail in order to understand the system behavior. Analytical and simulation results show that the proposed network coding method can achieve high SER performance and up-link throughput while requiring low computational complexity.

1. Introduction

The fourth industrial evolution has opened up a new era of wireless communications which needs to support mass deployment of short-range low-power Internet of Things (IoT) devices. In order to meet this challenge advanced wireless communication systems such as Long-Term Evolution (LTE) or the premature fifth generation (5G) mobile communications have all included relaying in their standards for range extension and cooperative communications. Data terminals such as mobile devices in LTE networks or wireless sensors used in IoT systems usually have limited power capacity, which limits the network coverage. In order to expand the communication range and increase the transmission efficiency, the two-way relay networks have received many attentions, recently [1–5].

In a typical half-duplex two-way relay network (TWRN), two single-antenna terminal nodes S_1, S_2 transmit their information to each other via one multi-antenna relay node R as shown in Fig. 1. The transmission process can be carried out in either two phases [6,7] or three phases [8–10], and the system with two-phase process was proved to achieve better spectral efficiency than the other one. In this system, during the first phase, the two terminal nodes simultaneously transmit their signals x_1 and x_2 to the relay node. In order to obtain the CSI, during the handshaking stage, the system can utilize a pilot-assisted channel estimation scheme [11] for static channels. In practical systems such as cellular communications or ad hoc networks the channels are often time varying due to user mobility. In such a case, an adaptive channel estimation scheme such as proposed in [12,13] can be used to better estimate the CSI. The received signal at the relay node y_1 and y_2 is then processed by an appropriate processing method to form the so-called network coded symbols x_R. During the second phase, the relay node broadcasts the network coded symbols to the two terminal nodes. Depending on the transmitted signals in the first phase and the processing method at the relay, each terminal node can use a suitable detection algorithm to estimate the transmitted signal from its counterpart.

1.1. Related works

In order to process the received signals at the relay node, various methods haven been considered, including amplify-and-forward (AF) [14–16], decode-and-forward (DF) [17–19], analog network coding (ANC) [10,14,20,21], digital network coding (NC) and physical-layer network coding (PNC) [6,7,22–35]. Although NC requires more computational complexity than the ANC does,

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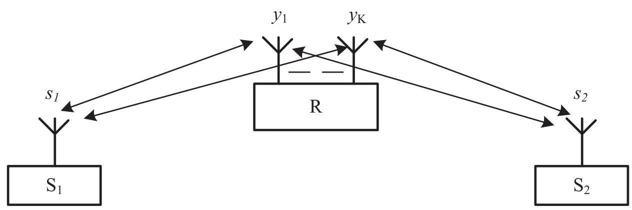

Fig. 1. System model of two-way relay network.

it can eliminate the noise effect at the relay node and thus attains better error performance. In the NC system, the relay node estimates individual symbols x_1 and x_2 from the two terminal nodes and then combine them together as a network coded symbol $x_R = x_1 \odot x_2$, where $\odot$ denotes a general network coding operator. In contrast, the PNC system estimates the network coded symbol x_R directly from the received signal [23]. The network coding operator can be performed by the simple XOR for the NC or the XOR mapping for the PNC with BPSK and 4-QAM constellations [22,24]. It can also be the modulo operation for higher-order modulations [25–29] or the combined XOR and modulo operation [30]. The encoded symbol is then mapped into a modulation constellation before retransmitting to the terminal nodes. Depending on the relation between the transmitted-signal constellation used by the terminal nodes and constellation by the relay node, the mapping can be categorized as linear or non-linear one. In the linear-mapping TWRN [28,29], the two mapping constellations are exactly the same. In contrast, the nonlinear-mapping system often has the constellation size at the relay node larger than the original one at the terminal nodes. While the NC mapping is always linear [26,31], the PNC can be either linear [23,25,27–29,31] or non-linear [30,32–34]. It is well known that the non-linear PNC system achieves better SER performance than the NC and linear PNC system [29]. However, the non-linear PNC system results in spectrum expansion due to enlarged constellation size [30,32–34]. Moreover, the system structure for the non-linear PNC is also more complex in both encoding and decoding processing.

The linear PNC system achieves a higher performance than the NC since the encoded symbol is the linear combination of both information symbol and the channel state information (CSI) of the two forward links from the terminal nodes to the relay node [23,25,27–29]. However, the sooner requires more processing complexity and the terminal nodes face difficulty in decoding received symbols if they do not have the CSI of the forward links. This situation happens in the realistic system where the forward and backward channels are non-reciprocal and there is no CSI feedback from the relay to the terminal nodes.

The NC system using maximum likelihood (ML) estimation achieves high SER performance and the simplest network encoding [26]. However, this system places high complexity at the relay node, which increases logarithmically with the constellation size. Other researches also focused on solutions to achieve the optimal SER performance [32,33]. However, these schemes still impose high processing complexity at both the relay node and terminal nodes and nonlinear NC was used instead. To overcome these limitations, recent researches proposed two possible effective solutions. The first solution is to use nonlinear PNC coding with a try in reducing the processing complexity at the relay node [30]. The channel quantization method used in [30] allows for simple signal estimation and achieves the same error performance as of the NC using ML while requiring the equivalent computational complexity as of the conventional Vertical-Bell Laboratories Layered Space-Time (V-BLAST). The second solution accepts high processing complexity in order to avoid nonlinear PNC. In [25,27–29], linear PNC was carried out by linear combination of the transmit information and the forward-channel CSI. The proposed mapping allows these systems to keep the original constellation [27]. In other efforts, to achieve high capacity of TWRN, the channel code is used in combination with linear PNC in [25]. In order to reduce the complexity compared with the systems in [25,27], the Pulse Amplitude Modulation (PAM) was used in the linear PNC system for the real channels in [28]. This system is then extended to the M-QAM modulation for complex channels in [29]. However, compared with [30], these systems still exhibit high complexity because they have to estimate two symbols (w_{S_1}, w_{S_2}) and search for the optimal coefficients $(\alpha_{opt}, \beta_{opt})$ simultaneously. The precoding two-way relaying scheme using zero forcing (ZF) estimation in [36] does not require the codeword to contain the CSI while still ensuring linear mapping. However, the transmitter of this scheme needs to know the CSI of the forward link from it to the relay.

In summary, these PNC schemes have disadvantages of requiring either high processing complexity or nonlinear network coding. Furthermore, they also require the terminal nodes to have the CSI of both the forward and backward channels, which is not always easy to attain in the practical systems.

1.2. Contributions of the paper

In this paper, we are interested in the NC system because of its advantages in utilizing linear network coding, having low complexity as well as without necessity of the forward CSI at the terminal nodes. Specifically, we propose an enhanced network coding system which is developed based on the channel quantization in [30] and successive interference cancellation (namely QSIC) to take advantage of its low complexity property. However, our proposed system uses different encoding and signal estimation methods. The main contributions of our paper versus the previous works are summarized as follows:

- Firstly, although the same idea of channel quantization is applied, our proposed system uses the linear NC while the non-linear PNC was utilized in [30].
- Secondly, while both the proposed system and that in [30] use the similar quantization approaches, the number of quantization levels differ from each other. Moreover, the estimation method in [30] could apply only to the case of the QPSK modulation, whereas our proposed method can be used for various modulation schemes such as PAM, BPSK and M-QAM. In addition, we also provide detailed analysis of the residual interference existence for different signal constellations.
- Thirdly, the PNC symbols generated in [27–30] contain the forward CSI, whereas those in our scheme are independent of the forward CSI. Therefore, our proposal can be applied to those systems with non-reciprocal channel or affected by the varying fading.
- Finally, compared with the method in [26,30], our the proposed network coding method does not require a function for adding two symbols but it uses the linear multiplication of the real part or imaginary part, respectively. In addition, our NC scheme maintains the same modulation constellation. Therefore, it does not require the high complexity demodulation such as the ML in [26,30].

The rest of this paper is organized as follows. Section 2 introduces the system model of the two-way relay communication network. The proposed NC using the channel quantization and the QSIC-based estimation method is presented in Section 3. Analysis of the residual interference effect on the signal decision discussed in Section 4. Simulation results and performance evaluations are shown in Section 5. Finally, Section 6 concludes the paper.

Throughout this paper, we will use the following mathematical notation. Bold lower-case letter presents a vector, bold upper-case letter is used for matrix and italic normal letter is for a variable. Notations $()^T, ()^*, |\,|$ are for transpose, conjugate transpose, and absolute value respectively.

2. System model

We consider the general model of the TWRN as shown in Fig. 1. Two single-antenna terminal nodes $S_m, (m = 1, 2)$ communicate with each other via the help of a relay node R with $K, (K \geqslant 1)$ antennas. It is assumed that there is no direct link between the two terminal nodes due to long distance. The communication in the TWRN consists of two phases, i.e. the multiple access (MA) phase and the broadcast (BC) phase. Since the operation of the TWRN in the BC phase is the same as that in a point-to-point system, we will consider only the MA phase in the sequel. Two terminal nodes $S_m, (m = 1, 2)$ transmit their message $s_m = (1/\sqrt{\mu})x_m$ to the relay node simultaneously, where x_m is carved from an M-QAM signal constellation and $1/\sqrt{\mu}$ is the normalized power factor to ensure $E(|s_m|^2) = 1$. The received signal at the k-th antenna $(k = 1, \ldots, K)$ of the relay node is given by

$$y_k = h_{k,1}s_1 + h_{k,2}s_2 + n_k = h_{k,1}\frac{1}{\sqrt{\mu}}x_1 + h_{k,2}\frac{1}{\sqrt{\mu}}x_2 + n_k$$
$$= \bar{h}_{k,1}x_1 + \bar{h}_{k,2}x_2 + n_k, \tag{1}$$

where $h_{k,m}$ is the channel coefficient of the link between the m-th source node and k-th antenna of the relay node; $\bar{h}_{k,m} = h_{k,m}/\sqrt{\mu}$ denotes the respective equivalent channel coefficient. All channels are assumed to be affected by flat quasi-static Rayleigh fading. Thus the channel coefficients are invariant during each phase but varying independently from phase to phase (non-reversible channel). Each coefficient $h_{k,m}$ can be modeled as a complex Gaussian random variable with zero mean and unit variance, i.e. $h_{k,m} \sim \mathcal{N}_c(0, 1)$. The CSI is assumed available at the receive node but not at the transmit one. n_k denotes the additive white Gaussian noise with zero mean and σ_n^2 variance, $n_k \sim \mathcal{N}_c(0, \sigma_n^2)$.

The received signal vector $\mathbf{y} = [y_1, \ldots, y_K]^T$ at the relay node R can be expressed as follows:

$$\mathbf{y} = \begin{bmatrix} \bar{h}_{1,1} & \bar{h}_{1,2} \\ \vdots & \vdots \\ \bar{h}_{K,1} & \bar{h}_{K,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_K \end{bmatrix} = \mathbf{Hx} + \mathbf{n}, \tag{2}$$

where

$$\mathbf{H} = \begin{bmatrix} \bar{h}_{1,1} & \bar{h}_{1,2} \\ \vdots & \vdots \\ \bar{h}_{K,1} & \bar{h}_{K,2} \end{bmatrix}, \mathbf{x} = [x_1, x_2]^T, \mathbf{n} = [n_1, \ldots, n_K]^T.$$

3. Network coding based on combined channel quantization and SIC estimation

3.1. QSIC-NC for single-antenna relay node

We first consider a simple case where the relay node has only one antenna, i.e. $K = 1$. The received signal at the relay node in the MA phase can be given by

$$y_1 = \bar{h}_{1,1}x_1 + \bar{h}_{1,2}x_2 + n_1. \tag{3}$$

In order to generate the network coded symbol x_R, the relay node performs three processing steps, including channel quantization, SIC loop back based estimation and network coding.

3.1.1. Channel quantization

Let us define quantization step by

$$|h| = \max\{|\bar{h}_{1,1}|, |\bar{h}_{1,2}|\}. \tag{4}$$

For ease of presentation suppose that $|h| = |\bar{h}_{1,1}|$. The case $|h| = |\bar{h}_{1,2}|$ can be followed in a similar way. Dividing both sides of Eq. (3) by h and notice that:
$\bar{h}_{1,2}/\bar{h}_{1,1} = (h_{1,2}/\sqrt{\mu})/(h_{1,1}/\sqrt{\mu}) = h_{1,2}/h_{1,1}$, results in [30]:

$$w_1 = \frac{y_1}{\bar{h}_{1,1}} = x_1 + \frac{h_{1,2}}{h_{1,1}}x_2 + \frac{n_1}{\bar{h}_{1,1}} = x_1 + Lx_2 + lx_2 + \frac{n_1}{\bar{h}_{1,1}}, \tag{5}$$

where L is the nearest complex integer to $h_{1,2}/h_{1,1}$, l is the rounding residual (also known as the residual interference) of $h_{1,2}/h_{1,1}$, specifically,

$$L = \text{round}\left(\frac{h_{1,2}}{h_{1,1}}\right) = \arg\min_{x \in (\mathbb{Z}+j\mathbb{Z})}\left|\frac{h_{1,2}}{h_{1,1}} - x\right|, \tag{6}$$

$$l = \text{res}\left(\frac{h_{1,2}}{h_{1,1}}\right) = \frac{h_{1,2}}{h_{1,1}} - L. \tag{7}$$

where round$(\cdot)$ and res$(\cdot)$ denote the rounding operation and the residual operation, respectively.

Lemma 1. *If* $|h_{1,1}| = max\{|h_{1,1}|, |h_{1,2}|\}$ *then* $L \in \{0, \pm 1, \pm j, \pm 1 \pm j\}$.

Proof. See Appendix A. □

Note that the expression of quantization value L in (6) looks similar to that used in [30]. However, since our quantization step $|h|$ is selected based on the maximum magnitude of $\bar{h}_{1,1}$ and $\bar{h}_{1,2}$, the value L belongs to the set given in Lemma 1. This set differs from that in [30], which leads to a different estimation method allowing us to avoid non-linear mapping and remove CSI from the network-coded symbols.

3.1.2. SIC-based estimation

The conventional SIC method considers x_1 and $(L + l)x_2$ in Eq. (5) as two separate signals. Firstly, the signal with higher power will be detected by treating the remaining signal as interference. Then, we detect the signal with lower power by canceling the previous signal has been detected. When the power of x_1 is equivalent to that of $(L + l)x_2$, this detection method will not be effective. In this paper, we improve the classical SIC by considering $(x_1 + Lx_2)$ and lx_2 as two separate signals and applying the SIC method until all the signals are successfully estimated. In order to estimate $(x_1 + Lx_2)$ the relay needs to have full information about its constellation. This requirement can be easily met by using a pre-stored constellation library at the relay node. Given the employed modulation for x_1 and x_2, and calculated L the relay can have full information about the constellation of $(x_1 + Lx_2)$ and thus can easily estimate the sum signal $(x_1 + Lx_2)$ for arbitrary modulation. The proposed SIC method can be summarized as follows:

- Step 1: Estimation of $(x_1 + Lx_2)$. From Eq. (5), it is clear that $|l|^2$ is much smaller than 1, so $(x_1 + Lx_2)$ can be estimated directly from w_1 as follows:

$$\overline{x_1 + Lx_2} = \hat{Q}(w_1) = \hat{Q}\left((x_1 + Lx_2) + lx_2 + \frac{n_1}{\bar{h}_{1,1}}\right), \tag{8}$$

where $\hat{Q}(\cdot)$ is the linear decision function of the signal constellation $(x_1 + Lx_2)$. Denote the signal constellation of $(x_1 + Lx_2)$ by $\mathcal{A}_{\bar{L}}$, where $\bar{L} \in \{0, 1, 1 + j\}$. Details of the constellation $\mathcal{A}_{\bar{L}}$ and the decision function $\hat{Q}(\cdot)$ are provided in Section 3.1.4.

- Step 2: Estimation of x_2 from lx_2. x_2 can be estimated from lx_2 by removing the estimate $(\overline{x_1 + Lx_2})$ in Eq. (5) as follows:

$$\hat{x}_{2l} = Q\left(\frac{w_1 - (\overline{x_1 + Lx_2})}{l}\right), \tag{9}$$

where $\hat{x}_{2l}$ represents the component part of x_2 in lx_2, $Q(\cdot)$ is the decision function for the M-QAM signal constellation.
- Step 3: Remove $Lx_2 + lx_2$ in Eq. (5) to estimate x_1 as follows:

$$\hat{x}_1 = Q(w_1 - (L + l)\hat{x}_{2l}). \tag{10}$$

- Step 4: Finally, remove x_1 in Eq. (5) to estimate x_2 as follows:

$$\hat{x}_2 = Q\left(\frac{w_1 - \hat{x}_1}{L + l}\right). \tag{11}$$

3.1.3. Linear NC

The purpose of the linear NC is to generate the network coded symbol x_R from the estimated symbols at the relay node. In [26,30], the simple modulo operation was used as the linear operation for the two estimated symbols. However, this method requires the ML estimation at the terminal nodes for decoding, which imposes increased computational complexity. Linear encoding and decoding were proposed in [25,27–29] to reduce the system complexity. However, the symbol encoding requires the CSI of both the forward links from the two terminal nodes to the relay. This requirement becomes difficult if the channel is not reciprocal, i.e. if the forward and backward channel are not the same. More importantly, one terminal node needs to know the CSI of the forward link from the other to the relay, which requires the feedback information from the relay. In order to facilitate these difficulties, we propose the following simple linear encoding and decoding method. This method allows the terminal nodes to refrain from the CSI feedback for decoding. The proposed method allows each terminal node to simply estimate the real and the imaginary part of the network coded symbol x_R for decoding. The network coded symbol x_R is created by the following linear combination:

$$x_R = \text{sign}(\hat{x}_{1r}\hat{x}_{2r})(|\hat{x}_{1r}\hat{x}_{2r}| \bmod 8) + j\,\text{sign}(\hat{x}_{1i}\hat{x}_{2i})(|\hat{x}_{1i}\hat{x}_{2i}| \bmod 8), \tag{12}$$

where $\text{sign}(\cdot)$ denotes the sign function, $a \bmod b$ is modulo operation which results in the remainder after division of a by b; $\hat{x}_{kr}$ and $\hat{x}_{ki}$ denote the real and the imaginary parts of $\hat{x}_k$, respectively.

In the BC phase, the received signal at the i-th terminal node $y^{(i)}, (i = 1, 2)$ can be expressed as follows:

$$y^{(i)} = g^{(i)}x_R + z^{(i)}, \tag{13}$$

where $g^{(i)}$ is the channel coefficient between the antenna of the relay node and that of the i-th terminal node, $z^{(i)}$ is the additive noise. The estimate of x_R is given by

$$\tilde{x}_R = Q\left(\frac{y^{(i)}}{g^{(i)}}\right). \tag{14}$$

Each terminal node will use its transmitted symbol to decode the counter-partner's one from the received signal x_R. For example, node S_1 performs the following operation to obtain the estimate of the transmitted symbol x_2 from node S_2:

$$\tilde{x}_2 = \text{sign}(x_{1r}x_{Rr})(|x_{1r}x_{Rr}| \bmod 8) + j\,\text{sign}(x_{1i}x_{Ri})(|x_{1i}x_{Ri}| \bmod 8). \tag{15}$$

3.1.4. Constellation $\mathcal{A}_{\bar{L}}$ and decision function $\hat{Q}(\cdot)$

Since L varies depending on the channel gains, the receiver needs to store information about the signal constellations of $(x_1 + Lx_2)$ for estimation. This requirement can be easily met by using additional memory at the relay, which results in only a small hardware modification. If the quantization step is fixed, for example to $\bar{h}_{1,1}$, then the received quantization levels of L in Eq. (6) have many different values. Therefore, it is difficult to estimate the signal constellation of $(x_1 + Lx_2)$. However, when $|L|$ is a large value, we can first estimate the component x_2 in Lx_2, and then remove it from $(x_1 + Lx_2)$ to estimate x_1.

Lemma 2. *The estimation of x_2 from Lx_2 in Eq. (5) will be linear if it satisfies the following condition: $|L| > (\sqrt{M} - 1)\sqrt{2}$.*

Proof. See Appendix B. □

In the case of 4-QAM modulation, i.e. $M = 4$, when $|L| > \sqrt{2}$ it is possible to estimate the transmitted symbols by the following simple SIC method:

$$\begin{cases} \hat{x}_2 = Q\left(\frac{w_1}{L}\right), \\ \hat{x}_1 = Q(w_1 - (L + l)\hat{x}_2). \end{cases} \tag{16}$$

For $M > 4$, since there are different integer values of L_r, L_i that satisfy the condition: $\sqrt{2} < |L| \leqslant (\sqrt{M} - 1)\sqrt{2}$. As a result, the receiver needs larger memories for storing constellations for each pair of $(|L_r|, |L_i|)$. In order to reduce the amount of memory storage, it is necessary to choose the maximum quantization step equal $\max\{|h_{1,1}|, |h_{1,2}|\}$.

Denote Ω the constellation of the M-QAM modulation. Since $x_1, x_2 \in \Omega$ and $|L_r|, |L_i| \in [0, 1]$, the constellation that is generated by $(x_1 + Lx_2)$ has the following properties:

Lemma 3. *The signal constellation of $(x_1 + Lx_2)$ does not depend on the sign of L_r, L_i.*

Proof. See Appendix C. □

Lemma 4. *If $|L_r| = |L_i|$ then the signal constellations of $(x_{a1} + |L_r|x_{a2})$ and $(x_{b1} + j|L_i|x_{b2})$ are equivalent, where $x_{a1}, x_{a2}, x_{b1}, x_{b2} \in \Omega$.*

Proof. See Appendix D. □

Using Lemma 3 and Lemma 4, we can determine the constellation $\mathcal{A}_{\bar{L}}$ as follows:

- Case 1: $L = 0$, the constellation $\mathcal{A}_{\bar{L}}$ of $(x_1 + Lx_2)$ is similar to that of the M-QAM. Denote this case as $\bar{L} = 0$.
- Case 2: $L \in [\pm 1, \pm j]$, the constellation $\mathcal{A}_{\bar{L}}$ is the same as that of $(x_1 + x_2)$. Denote this case as $\bar{L} = 1$.
- Case 3: $L \in [\pm 1 \pm j]$, the constellation $\mathcal{A}_{\bar{L}}$ is the same as that of $(x_1 + (1 + j)x_2)$. Denote this case as $\bar{L} = 1 + j$.

Fig. 2 shows examples of constellation of $(x_1 + Lx_2)$ for $M = 4$ and $L \in \{0, \pm 1, \pm j, \pm 1 \pm j\}$.

Note that since the constellation points are symmetric about the coordinate axes and the origin, it is possible to reduce the number of constellation points that need to be stored in the memory of the relay node. Specifically, the relay node only needs to store the signal constellation points of $(x_1 + \bar{L}x_2)$, denoted by $\mathcal{A}_{\bar{L} \geqslant 0}$, which belongs to the first quadrant of the coordinate axes. In order to determine the constellation points in the first quadrant of $(x_1 + \bar{L}x_2)$, the paper proposes Algorithm 1 presented below. Note that this algorithm can be used by the manufacturer to determine the constellation $\mathcal{A}_{\bar{L} \geqslant 0}$ and store it into the receiver memory. Upon having estimated the channel, the receiver can calculate L from the received signal. Depending on the value of L, the receiver can

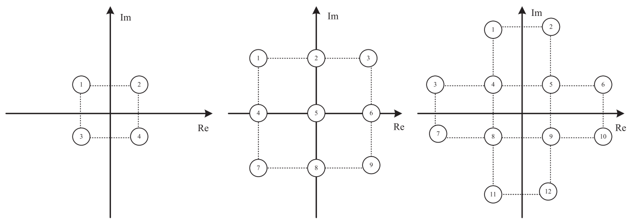

Fig. 2. Examples of constellation of $(x_1 + Lx_2)$ with $M = 4$ and $L \in \{0, \pm 1, \pm j, \pm 1 \pm j\}$.

decide the constellation being used and read the constellation values for signal estimation.

Algorithm 1. Determine the constellation points in the first quadrant $\mathcal{A}_{\bar{L}\geqslant 0}$ of $(x_1 + \bar{L}x_2)$

Input: $\bar{L} \in \{0, 1, 1 + j\}, (s_1, s_2) \in M$-QAM;
Output: First quadrant constellation points $\mathcal{A}_{\bar{L}\geqslant 0}$
for all $\bar{L}$ **do**
  for all (s_1, s_2) **do**
    Compute the constellation of $(x_1 + \bar{L}x_2)$:
    $\mathcal{A}_{\bar{L}} = s_1 + \bar{L}s_2$
  end for
  Search for all points in the first quadrant of the constellation of $(x_1 + \bar{L}x_2)$:
  $\mathcal{A}_{\bar{L}\geqslant 0} \leftarrow \mathcal{A}_{\bar{L}}(\mathcal{A}_{\bar{L},r} \geqslant 0, \mathcal{A}_{\bar{L},i} \geqslant 0)$
  Let $c_{\bar{L}\geqslant 0}$ be the total constellation points in the first quadrant $\mathcal{A}_{\bar{L}\geqslant 0}$
  Save $\mathcal{A}_{\bar{L}\geqslant 0}$ and $c_{\bar{L}\geqslant 0}$ into memory
end for

Table 1 shows the total number of constellation points in the first quadrant $c_{\bar{L}\geqslant 0}$ for $M = 4, 16, 64$.

In order to estimate a signal point in the constellation, instead of estimating the whole signal constellation $(x_1 + \bar{L}x_2)$, we only need to estimate one-fourth of the total number of constellation points. The decision function $\hat{\mathrm{Q}}(\cdot)$ in Eq. (8) carries out the following two steps:

- Step 1. Estimation of the first-quadrant constellation points:

$$x \triangleq \hat{\mathrm{Q}}(w_1) = \arg\min_{x\in\mathcal{A}_{\bar{L}\geqslant 0}} ||w_{1r}| + j|w_{1i}| - x|^2. \tag{17}$$

- Step 2. Determine the sign of x as follows:

$$\overline{x_1 + Lx_2} = x' \triangleq \hat{\mathrm{Q}}(w_1) = \arg\min_{x'\in\{\pm x_r \pm jx_i\}} |w_1 - x'|^2. \tag{18}$$

For example, [30] shows that in the case of the 4-QAM and $L \in \{\pm 1 \pm j\}$, the signal constellation $(x_1 + Lx_2)$ has 12 symmetric points. For this constellation we only need to estimate 7 points in the constellation while the non-linear PNC in [30] and the ML-NC in [26] need to estimate 12 points and 16 points, respectively. It is clear that the proposed estimation method requires a small number of points than the previous methods.

Table 1
The total constellation points $c_{\bar{L}\geqslant 0}$ in the first quadrant $\mathcal{A}_{\bar{L}\geqslant 0}$.

	$M = 4$	$M = 16$	$M = 64$
$c_{\bar{L}\geqslant 0}$ when $\bar{L} = 0$	1	4	16
$c_{\bar{L}\geqslant 0}$ when $\bar{L} = 1$	4	16	64
$c_{\bar{L}\geqslant 0}$ when $\bar{L} = 1 + j$	3	19	93

At the relay node, we propose the following adaptive algorithm (Algorithm 2) to find the quantization step L and the constellation $\mathcal{A}_{\bar{L}\geqslant 0}$ for the above estimation steps as follows:

Algorithm 2. Adaptive algorithm to determine L and estimate the constellation $\mathcal{A}_{\bar{L}\geqslant 0}$ for $K = 1$

Given $h_{1,1}, h_{1,2}$
Compute the quantization values:
$L_1 = \mathrm{round}(h_{1,2}/h_{1,1}),\ L_2 = \mathrm{round}(h_{1,1}/h_{1,2})$
Compute the quantization error:
$l_1 = h_{1,2}/h_{1,1} - L_1, l_2 = h_{1,1}/h_{1,2} - L_2$
if all $|L_u| \leqslant \sqrt{2}$ where $(u = 1, 2)$ **then**
  Select: $u \leftarrow \min\{|l_1|,\ |l_2|\}$
  Set: $h_{\max} \leftarrow h_{1,u}, L \leftarrow L_u, l \leftarrow l_u$
else
  Select: $u \leftarrow |L_u| \leqslant \sqrt{2}$
  Set: $h_{\max} \leftarrow h_{1,u}, L \leftarrow L_u, l \leftarrow l_u$
end if
Read memory location $\bar{L} = |L_r| + j|L_i|$ to determine the first quadrant of constellation:
$\mathcal{A}_{\bar{L}\geqslant 0} \leftarrow \mathrm{Memory}(\mathcal{A}_{\bar{L}\geqslant 0})$ and
$c_{\bar{L}\geqslant 0} \leftarrow \mathrm{Memory}(c_{\bar{L}\geqslant 0})$

Upon having achieved the quantization step $h_{\max}$ and parameters $\mathcal{A}_{\bar{L}\geqslant 0}, c_{\bar{L}\geqslant 0}, L,\ l$ it is possible to estimate and encode x_R as presented above.

3.2. QSIC-NC for the multiple-antenna relay

This section considers the system in which the relay node is equipped with $K \geqslant 2$ antennas. In the MA phase, the received signal at the relay node is similar with that in Eq. (2) in the Section 2.

In this case, the equivalent channel $\mathbf{H}$ in Eq. (2) can be decomposed by the QR method as follows [30]:

$$\mathbf{H} = \mathbf{QR}', \tag{19}$$

where $\mathbf{Q}_{K\times K}$ is the unitary matrix with $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$ and $\mathbf{R}' = \begin{bmatrix} \mathbf{R}_{2\times 2} \\ \mathbf{0}_{(K-2)\times 2} \end{bmatrix}$ is an upper triangle matrix.

Multiplying both sides of Eq. (2) by $\mathbf{Q}^H$, we obtain the following equation:

$$\mathbf{Q}^H\mathbf{y} = \begin{bmatrix} \mathbf{R}_{2\times 2} \\ \mathbf{0}_{(K-2)\times 2} \end{bmatrix}\mathbf{x} + \mathbf{Q}^H\mathbf{n} = \begin{bmatrix} \mathbf{w}_{2\times 1} \\ \mathbf{0}_{(K-2)\times 1} \end{bmatrix}. \tag{20}$$

Similar to [30], by expanding $\mathbf{w}_{2\times 1}$ in Eq. (20), we obtain

$$\begin{aligned} w_1 &= r_{1,1}x_1 + r_{1,2}x_2 + u_1, \\ w_2 &= r_{2,2}x_2 + u_2, \end{aligned} \tag{21}$$

where $\mathbf{u} = [u_1\ u_2]^T = \mathbf{Q}^H\mathbf{n}$.

3.2.1. Channel quantization

Dividing both sides of the first equation in (21) by $r_{1,1}$, we obtain the following equation:

$$w_{1a} \triangleq \frac{w_1}{r_{1,1}} = x_1 + Lx_2 + lx_2 + \frac{u_1}{r_{1,1}}, \tag{22}$$

where

$$L = \mathrm{round}\left(\frac{r_{1,2}}{r_{1,1}}\right),\ l = \frac{r_{1,2}}{r_{1,1}} - L. \tag{23}$$

Without loss of generality, we assume that $L \in \{0, \pm 1, \pm j, \pm 1 \pm j\}$. For other cases, we can swap two columns in $\mathbf{H}$ and then decompose it again. An adaptive algorithm is proposed to find the quantization step L and the constellation $A_{\bar{L}\geqslant 0}$ as given in the following algorithm (Algorithm 3).

Algorithm 3. Adaptive algorithm to determine L and estimate the constellation $\mathcal{A}_{\bar{L}\geqslant 0}$ for $K \geqslant 2$

Given $\mathbf{y}, \mathbf{H}$
Decompose $\mathbf{H}$ using the QR method:
  $[\mathbf{Q^1}, \mathbf{R^1}] = \mathrm{qr}(\mathbf{H})$
Let $\mathbf{H}'$ be the matrix in which the two columns of $\mathbf{H}$ are swapped: $[\mathbf{Q^2}, \mathbf{R^2}] = \mathrm{qr}(\mathbf{H}')$
Compute the quantization values:
  $L_1 = \mathrm{round}(r^1_{1,2}/r^1_{1,1}), L_2 = \mathrm{round}(r^2_{1,2}/r^2_{1,1})$
Compute the quantization error:
  $l_1 = r^1_{1,2}/r^1_{1,1} - L_1, l_2 = r^2_{1,2}/r^2_{1,1} - L_2$
if all $|L_u| \leqslant \sqrt{2}$ where $(u = 1, 2)$ **then**
  Select: $u \leftarrow \min\{|l_1|, |l_2|\}$
  Set: $L \leftarrow L_u, l \leftarrow l_u, \mathbf{R} \leftarrow \mathbf{R^u}$
  Compute $\mathbf{w}_{2\times 1}$ in the Eq. (20): $\mathbf{Q}_u^H \mathbf{y}$
else
  Select: $u \leftarrow |L_u| \leqslant \sqrt{2}$
  Set: $L \leftarrow L_u, l \leftarrow l_u, \mathbf{R} \leftarrow \mathbf{R^u}$
  Compute $\mathbf{w}_{2\times 1}$ in the Eq. (20): $\mathbf{Q}_u^H \mathbf{y}$
end if
Read memory location $\bar{L} = |L_r| + j|L_i|$ to determine the first quadrant of the constellation:
  $\mathcal{A}_{\bar{L}\geqslant 0} \leftarrow \mathrm{Memory}(\mathcal{A}_{\bar{L}\geqslant 0})$ and
  $c_{\bar{L}\geqslant 0} \leftarrow \mathrm{Memory}(c_{\bar{L}\geqslant 0})$

The results $\mathcal{A}_{\bar{L}\geqslant 0}, c_{\bar{L}\geqslant 0}, L, l, \mathbf{R}, \mathbf{w}_{2\times 1}$ will be used for signal estimation as follows.

3.2.2. SIC-based signal estimation

The signal estimation in this case is the same with the case $K = 1$. However, the reliability of $(x_1 + Lx_2)$ estimation is increased due to cancel lx_2 that is estimated from w_2 in Eq. (21).

- Step 1. Soft estimation of x_2 in the second equation of (21) using w_2. For the 4-QAM constellation, the soft estimation is performed individually for the real and imaginary part as follows [35]:

$$\widehat{x}_2 = E\{x_2|w_2\} = \tanh\left(\frac{r_{2,2}w_2}{\sigma^2}\right). \tag{24}$$

  For other cases, i.e. $M > 4$, the soft estimation is performed for the real and imaginary part as follows [37]:

$$\widehat{x}_{2r} = \begin{cases} \mathrm{sign}\left(\frac{w_{2r}}{r_{2,2}}\right)(\sqrt{M}-1), & \text{if } \left|\frac{w_{2r}}{r_{2,2}}\right| > (\sqrt{M}-1) \\ \frac{w_{2r}}{r_{2,2}}, & \text{if } \left|\frac{w_{2r}}{r_{2,2}}\right| \leqslant (\sqrt{M}-1) \end{cases} \tag{25}$$

- Step 2. Cancellation of the residual interference lx_2 in (22). Using the interference cancellation methods by V-BLAST and Sorted V-BLAST [23], the element $\widehat{x}_2$ is estimated from w_2 and used to cancel $(L+l)x_2$ from w_{1a}. If the estimate $\widehat{x}_2$ is correct, it will not affect the estimate $\widehat{x}_1$ in (22). In contrast, if $\widehat{x}_2$ is incorrect, the estimate $\widehat{x}_1$ will be affected. The level of effect depends on the value of $|L+l|$. Therefore, we can apply the idea in [30] by removing the residual part in Eq. (22), i.e. lx_2, as follows:

$$w_{1b} \triangleq w_{1a} - l\widehat{x}_2 = x_1 + Lx_2 + l(x_2 - \widehat{x}_2) + \frac{u_1}{r_{1,1}}. \tag{26}$$

- Step 3. Estimation of $(x_1 + Lx_2)$. Depending on the value of $|L_r| + j|L_i| \in \{0, 1, 1+j\}$, we can obtain the constellation of $(x_1 + Lx_2)$ using the similar method in Section 3.1.4. The value of $\overline{x_1 + Lx_2}$ is given by

$$\overline{x_1 + Lx_2} = \hat{Q}(w_{1b}). \tag{27}$$

- Step 4. Partial estimation of x_2. Since the element w_{1a} in Eq. (22) and w_2 in Eq. (21) contains the element x_2, after canceling the element $(x_1 + Lx_2)$ in (22), we can use the Maximum Ratio Combining (MRC) to combine them together as follows:

$$x_{2,\mathrm{MRC}} \triangleq \frac{l^*(w_{1a} - \overline{(x_1 + Lx_2)}) + r^*_{2,2}w_2}{|l|^2 + |r_{2,2}|^2}. \tag{28}$$

  The decision of $\widehat{x}_{2l}$ is given by

$$\widehat{x}_{2l} = Q(x_{2,\mathrm{MRC}}). \tag{29}$$

- Step 5. Full estimation of x_1. The estimate $\widehat{x}_{2l}$ in Eq. (29) will be used to cancel the element x_2 in w_{1a} of Eq. (22):

$$\hat{x}_1 = Q\Big(w_{1a} - (L+l)\widehat{x}_{2l}\Big). \tag{30}$$

- Step 6. Full estimation of x_2. Having canceled the element x_1, we can use MRC again to estimate all components x_2 in w_1 and w_2 as follows:

$$\hat{x}_2 = Q\left(\frac{r^*_{1,2}(w_1 - r_{1,1}\hat{x}_1) + r^*_{2,2}w_2}{|r_{1,2}|^2 + |r_{2,2}|^2}\right). \tag{31}$$

3.2.3. Linear NC

Linear NC for the case of the multiple antenna system is similar to that in Section 3.1.3.

In the BC phase, the encoded symbol x_R is broadcast to the terminal nodes. Since the system operation in this phase is the same with that in a point-to-point system, the transmission from the relay node to the terminal nodes can be performed using various methods such as orthogonal space-time block code (OSTBC) [38], quasi-orthogonal space-time block code (QOSTBC) [39] to achieve diversity gain, spatial division multiplexing (SDM) to achieve multiplexing gain [40], or spatial modulation to improve the spectral efficiency [41]. The relay node can use one random antenna for transmission as in the case $K = 1$ of Section III.A.3. For reciprocal channels, the max-min antenna selection with binary network coding or transmit beamforming [42] can be used to achieve high performance.

4. Effect of the residual interference lx_2 on the signal decision $(x_1 + Lx_2)$

Recall from the previous section that the signal processing at the relay node during the MA phase involves three steps including channel quantization, signal estimation and linear NC. However, it is only the estimation step that affects the system performance. In this paper, we will focus our attention on the effect of lx_2 on the estimation of $(x_1 + lx_2)$ because this estimation has significant influence on the next step.

4.1. Case $K = 1$

Because the elements x_1, x_2 in Eq. (5) are symbols that belong to the M-QAM constellation, the decision borders are perpendicular to the co-ordinate axes. Moreover, since the real and imaginary value of L are integer numbers $[0, 1]$, the constellation of $(x_1 + Lx_2)$ also has the decision borders similar to that of the M-QAM constellation. Therefore, the estimation of the signal symbol $(x_1 + Lx_2)$ can be done separately for the real and the imaginary part. Because the analysis for the real and the imaginary parts are equivalent we will only perform it for the real part and then deduce the result for the imaginary part. Let $x_a = x_{ar} + jx_{ai}, x_b = x_{br} + jx_{bi}, (x_a, x_b \in \mathcal{A}_{\bar{L}})$. Without loss of generality, let us assume that $x_{ar} \neq x_{br}$. The distance between the real parts of the two nearest points in the constellation $(x_1 + Lx_2)$ can be given by Eq. (32).

$$\begin{aligned} d_r &= |x_{ar} - x_{br}| \\ &= |(x_{1ar} + L_r x_{2ar} - L_i x_{2ai}) - (x_{1br} + L_r x_{2br} - L_i x_{2bi})| \\ &= |(x_{1ar} - x_{1br}) + L_r(x_{2ar} - x_{2br}) - L_i(x_{2ai} - x_{2bi})| \end{aligned} \quad (32)$$

Since all $x_{1ar}, x_{1br}, x_{2ar}, x_{2br}, x_{2ai}, x_{2bi}$ are integer, the odd numbers that belong to the range $[-\sqrt{M}+1, \sqrt{M}-1]$, where M is the order of the M-QAM constellation. On the other hand, since the difference between two odd numbers is an even number, d_r is certainly an even number. Therefore, the minimum distance between the two points having different real parts in the constellation $(x_1 + Lx_2)$ is given by $d_{r\min} = 2$.

On other hand, from the Eq. (5), if $(x_1 + Lx_2)$ is the desired estimation, the remaining part $(lx_2 + n)$ can be considered the interference. For simplicity, we can ignore the effect of the noise. Therefore, there is only the interference component lx_2 that affects the signal decision $(x_1 + Lx_2)$. From [30] $|l_r| < 0.5, |l_i| < 0.5$, thus the magnitude of the real part of lx_2, denoted by d_{r_lx}, is presented as follows:

$$d_{r\_lx} = |l_r x_{2r} - l_i x_{2i}| \leqslant |l_r x_{2r}| + |l_i x_{2i}| < \begin{cases} 0.5 \times 1 + 0.5 \times 1 = 1 & \text{if 4-QAM} \\ 0.5 \times 3 + 0.5 \times 3 = 3 & \text{if 16-QAM} \\ 0.5 \times 7 + 0.5 \times 7 = 7 & \text{if 64-QAM} \end{cases} \quad (33)$$

From Eq. (33), it is clear that in the case of the 4-QAM constellation, since the value of d_{r_lx} is less than that of $d_{r\min}/2$, the decision of $(x_1 + Lx_2)$ does not cause error and thus the system can obtain high performance. In the case of higher modulation order $(M > 4)$, since the value of d_{r_lx} is larger than $d_{r\min}/2$, the estimation in this case is not effective. Therefore, there is an irreducible error floor at high SNR range.

4.2. Case $K \geqslant 2$

Different from the case of the single-antenna relay node, in the case of the multiple-antenna relay, we can see from (26) that the interference component lx_2 is removed before estimating the component $(x_1 + lx_2)$. Denote $\delta_{x_2} \triangleq (x_2 - \widehat{x}_2)$ the decision error. As shown in [30], the decision error is less than $1/(1+\gamma) \approx \sigma^2/(\sigma^2 + |r_{2,2}|^2)$. Therefore, the value of the real part in the component $l(x_2 - \widehat{x}_2)$ is less than $0.5\sigma^2/(\sigma^2 + |r_{2,2}|^2) < 1/2 < d_{\min}/2$. When the number of received antennas increases, the decision error δ_{x_2} decreases significantly, leading to notable improvement in SER performance. Specifically, in the case of the constellation with $M > 4$, the performance of the system with a multiple-antenna relay node outperforms the system with single-antenna relay and there is no error floor as in the latter system.

5. Simulation results and performance evaluation

This section presents the performance evaluation of the proposed two-way QSIC-NC system in which the relay node has K antennas. The system performance in terms of SER, uplink throughput and computational complexity are evaluated at the relay node. It is assumed that upon having the network coded symbol x_R, the relay node can use the individual symbols $x_i, (i = 1, 2)$ for estimation. All channels are assumed to undergo flat Rayleigh fading and we assume that the channel gains are invariant during one transmission phase, but varies from phase to phase. For other channel models such as the multi-path fading channels a complex equalizer can be used for ISI mitigation before signal estimation. The M-QAM modulation is used for all evaluations. Moreover, the system performances of the VBLAST-NC, Sorted VBLAST-NC [23], ML-NC [26], and nonlinear PNC in [30] are also shown for comparison.

5.1. Comparison the SER performance

Fig. 3 compares the SER performance at the relay node of the proposed QSIC-NC with other related schemes including the ML-NC in [26], the VBLAST-NC and Sorted VBLAST-NC in [23] for the case using $K = 1, 2, 3$ and $M = 4$. It can be seen that the proposed system achieves the same performance of the ML-NC, and outperforms the VBLAST-NC and Sorted VBLAST-NC. Specifically, for $K = 2$ and at SER $= 10^{-3}$, the QSIC-NC scheme obtains an SNR gain of approximately 10.5 dB as compared with the VBLAST-NC and 6.5 dB with the Sorted VBLAST-NC. When $K = 3$ the QSIC-NC scheme achieves about 3.0 and 1.0 dB gain, respectively. More SNR gain can be obtained at lower SER.

For higher modulation, such as $M = 16$ and $M = 64$, we still observe that the proposed QSIC-NC can achieve the same performance of the ML-NC and outperform the other two schemes for large K $(K > 2)$. For $K = 2$, although it still outperforms the VBLAST-NC and the Sorted VBLAST-NC, the QSIC-NC loses a small gain at the high SNR region compared with the ML-NC. Note that to simplify the presentation we have not included herein the simulated results for these two cases.

5.2. Comparison the uplink throughput

The system throughput at the relay node can be calculated using the following equation: $(1 - \text{FER})R\log_2 M$, where M is the constellation size, $R = 2$ is the symbol rate at the relay, and FER

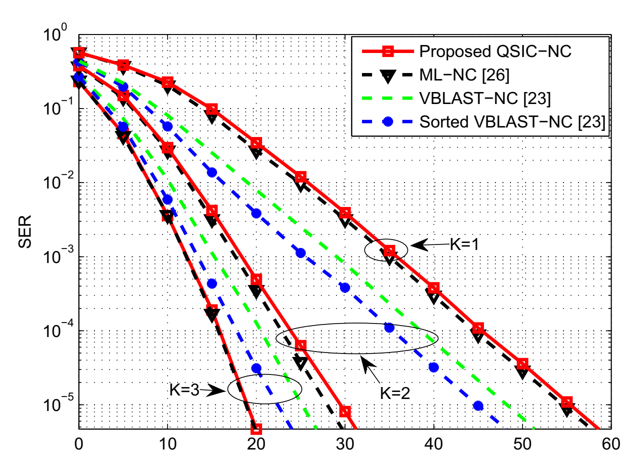

Fig. 3. Comparison the SER performance in case $M = 4$.

is the frame error rate. In the simulation, it is assumed that each frame has 20 symbols.

Fig. 4 compares the uplink throughput of the proposed QSIC-NC scheme with the other schemes in the case of the 4-QAM modulation. As can be seen when the number of relay antennas increases, the throughput of the QSIC-NC scheme is close to that of the ML-NC. Compared with other schemes, the QSIC-NC outperforms them significantly. For example, when $K = 2$ and at SNR = 10 dB, the throughput of the QSIC-NC scheme is higher than that of the VBLAST-NC and the Sorted VBLAST-NC about 1.25 bits/time slot and 0.7 bits/time slot, respectively. Similar trend still applies to the case $M = 16$ and $M = 64$, but not shown here to simplify the presentation.

5.3. Comparison of the different mapping methods

Fig. 5 compares the SER performance of the proposed QSIC-NC using the linear mapping and the CQ-PNC method [30] using nonlinear mapping with the 4-QAM modulation. It is clear that the proposed QSIC-NC using the linear network coding achieves the similar performance of the nonlinear method, especially for large K.

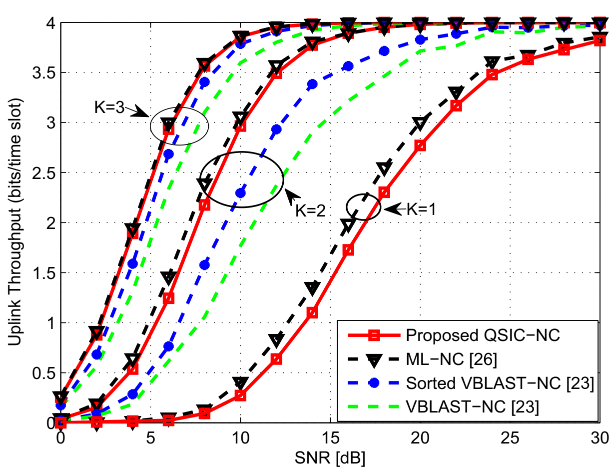

Fig. 4. The uplink throughput in case $M = 4$.

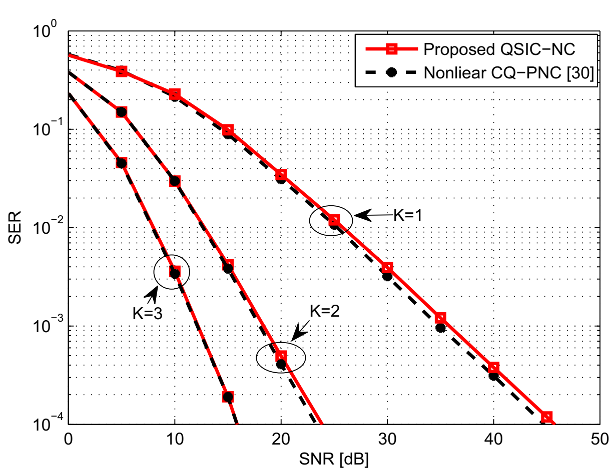

Fig. 5. Performance comparison of the proposed linear mapping with the nonlinear mapping in work [30] using 4-QAM modulation.

5.4. Complexity comparison

5.4.1. Qualitative comparison

It clear that the ML-NC method in [26] has the complexity order $\mathcal{O}\left(M^2\right)$ and the VBLAST-NC and Sorted VBLAST-NC [23] have the complexity order $\mathcal{O}(M)$ for the M-QAM constellation. The proposed QSIC-NC has also the complexity order $\mathcal{O}(M)$ and therefore the same with that of the two VBLAST schemes.

5.4.2. Quantitative comparison

This section presents the comparison of the estimated complexity at the relay node. The complexity unit used for analysis is flop (FLoating-point OPeration) count. A complex addition or subtraction is counted as 2 flops, a complex multiplication requires 6 flops, a complex division requires 11 flops, a complex and real multiplication 2 flops, a complex and real addition 1 flop and finally both a multiplication and an addition of real numbers only 1 flop.

Firstly, the decision function $\hat{Q}(\cdot)$ in Eq. (8) with $K = 1$ or (27) with $K \geqslant 2$ is performed in Eqs. (17) and (18). Note that the complexity for calculating (17) depends on $\mathcal{A}_{\bar{L}\geqslant 0}$. Moreover, each $\mathcal{A}_{\bar{L}\geqslant 0}$ has the number of the points $c_{\bar{L}\geqslant 0}$ as shown in Table 1. Therefore, we need to calculate both $(\overline{\mathcal{A}}_{\bar{L}\geqslant 0}, \bar{c}_{\bar{L}\geqslant 0})$ for estimating the complexity of (17), where $\overline{\mathcal{A}}_{\bar{L}\geqslant 0}$ is an average of all $\mathcal{A}_{\bar{L}\geqslant 0}$, and $\bar{c}_{\bar{L}\geqslant 0}$ is an average number of points in $\overline{\mathcal{A}}_{\bar{L}\geqslant 0}$. Let $P_r(\bar{L} = 0), P_r(\bar{L} = 1), P_r(\bar{L} = 1 + j)$ denote the probabilities that the three respective cases $\bar{L} = 0, \bar{L} = 1$ and $\bar{L} = 1 + j$ occur. Let us further assume that $P_r(\bar{L} = 0) = P_r(\bar{L} = 1) = P_r(\bar{L} = 1 + j) = 1/3$ for equal probability. The average number of the points $\bar{c}_{\bar{L}\geqslant 0}$ of the constellation $\overline{\mathcal{A}}_{\bar{L}\geqslant 0}$ are shown in Table 2. For example, when $M = 4$ the number of points is calculated as follows: $\bar{c}_{\bar{L}\geqslant 0} = 1 \times 1/3 + 4 \times 1/3 + 3 \times 1/3 = 2.67$ points. Moreover, d denotes the required number of flops for the decision function $Q(x)$. The values of d for different cases of M are given in Table 2.

In order to calculate the processing complexity at the relay we need to count the number of flops in the adaptive algorithm (for example Algorithm 2 in the case $K = 1$, Algorithm 3 in the case $K \geqslant 2$), the number of flops in the channel quantization, SIC-based estimation and linear NC.

For $K = 1$, the proposed QSIC-NC scheme requires 47 flops to perform Algorithm 2 and channel quantization in Eq. (5). The complexity for the SIC-based estimation from Eqs. (8)–(11) is $56 + 9\bar{c}_{\bar{L}\geqslant 0} + 3d$ flops. The complexity to perform encoding operation in Eq. (12) is 6 flops. Therefore, the total processing complexity at the relay of the proposed scheme is $9\bar{c}_{\bar{L}\geqslant 0} + 3d + 109$ flops. Meanwhile, the number of flops required by the ML-NC scheme in [26] is $19M^2 + 6$ flops.

For $K \geqslant 2$, the number of flops required for decomposing the complex matrix $\mathbf{H} = \mathbf{QR}'$ of dimension $K \times 2$ is $16K^2 + 12K$ flops [43]. The number of flops for multiplying $\mathbf{Q}^H$ of dimension $K \times K$ by $\mathbf{y}$ of dimension $K \times 1$ in the Eq. (20) is $7K^2 - K$ flops. Thus, the number of flops required by Algorithm 3 and Eq. (22) is $39K^2 + 23K + 26$ flops. The complexity of the SIC-based estimation in Eqs. (24)–(31) is $9\bar{c}_{\bar{L}\geqslant 0} + 3d + 88$ flops. Therefore, the total processing complexity at the relay of the proposed scheme is

Table 2
The total constellation points $c_{\bar{L}\geqslant 0}$ in the first quadrant $\mathcal{A}_{\bar{L}\geqslant 0}$ and the number of flops for the decision function $Q(\cdot)$.

	$M = 4$	$M = 16$	$M = 64$
$c_{\bar{L}\geqslant 0}$	2.67	13	57.67
d	2	16	32

Table 3
Comparison of the processing efficiency at the relay node [flops/bit].

Schemes	$K=1$	$K=2$			$K=3$		
	$M=4$	$M=4$	$M=16$	$M=64$	$M=4$	$M=16$	$M=64$
Proposed QSIC-NC	70	173	241	466	282	350	570
ML-NC [26]	155	315	4995	79875	475	7555	120835
VBLAST-NC [23]		68	82	98	131	145	161
Sorted VBLAST-NC [23]		113	127	143	222	236	252
CQ-PNC [30]	39	89			153		

$39K^2+23K+9\bar{c}_{\bar{L}\geqslant 0}+3d+114$ flops. The complexity for other schemes is given as follows: $M^2(20K-1)+6$ flops for the ML-NC [26], $23K^2+11K+2d+18$ flops for the VBLAST-NC [23], $39K^2+23K+2d+20$ flops for the Sorted VBLAST-NC scheme [23].

Table 3 summarizes the processing efficiency in terms of flop per bit at the relay node of the different schemes under consideration. The table shows that the proposed scheme has much higher processing efficiency compared with the ML-NC but slightly lower than the VBLAST-NC and the Sorted VBLAST-NC. The CQ-PNC [30] is shown to achieve the highest processing efficiency, but it is worth noting again that it cannot be used for modulation schemes with $M>4$.

6. Conclusions

This paper proposed a low-complexity linear network coding scheme based on linear estimation for the two-way relay non-reciprocal channel with the multiple antenna relay node. The estimation method uses the channel quantization in combination with the successive interference cancellation. The proposed scheme allows for both linear coding and signal estimation. As a result, it achieves equivalent SER performance but much lower computational complexity compared with the ML-NC at the cost of additional memory at the relay for storing the signal constellations. The proposed scheme also overcomes the necessity to map the network coded symbol x_R to a higher-order modulation at the relay as in the previous works [30,32–34]. In addition, the network coded symbol does not contain the CSI of the forward channels from the terminal to the relay node and thus it is more suitable for the non-reciprocal channels as compared with the schemes in [27–29].

Appendix A

A.1. Proof of Lemma 1

Consider the ratio:

$$\frac{h_{1,2}}{h_{1,1}}=\frac{h_{1,2}h_{1,1}^*}{|h_{1,1}|^2}=\frac{h_{1,2r}h_{1,1r}+h_{1,2i}h_{1,1i}+j(h_{1,2i}h_{1,1r}-h_{1,2r}h_{1,1i})}{h_{1,1r}^2+h_{1,1i}^2}$$
$$=\frac{h_{1,2r}h_{1,1r}+h_{1,2i}h_{1,1i}}{h_{1,1r}^2+h_{1,1i}^2}+j\frac{h_{1,2i}h_{1,1r}-h_{1,2r}h_{1,1i}}{h_{1,1r}^2+h_{1,1i}^2}, \tag{34}$$

where $h_{1,mr}$ and $h_{1,mi}$ are the real and the imaginary parts of $h_{1,m}$; $j^2=-1$.

Because $|h_{1,2}|\leqslant|h_{1,1}|\iff h_{1,2r}^2+h_{1,2i}^2\leqslant h_{1,1r}^2+h_{1,1i}^2$, by applying the CauchySchwarz inequality for the real and the imaginary part of Eq. (34) we have

$$\begin{cases}\left|\frac{h_{1,2r}h_{1,1r}+h_{1,2i}h_{1,1i}}{h_{1,1r}^2+h_{1,1i}^2}\right|\leqslant\frac{\sqrt{(h_{1,2r}^2+h_{1,2i}^2)(h_{1,1r}^2+h_{1,1i}^2)}}{|h_{1,1r}^2+h_{1,1i}^2|}\leqslant 1\\ \left|\frac{h_{1,2i}h_{1,1r}-h_{1,2r}h_{1,1i}}{h_{1,1r}^2+h_{1,1i}^2}\right|\leqslant\frac{\sqrt{(h_{1,2r}^2+h_{1,2i}^2)(h_{1,1r}^2+h_{1,1i}^2)}}{|h_{1,1r}^2+h_{1,1i}^2|}\leqslant 1\end{cases}. \tag{35}$$

On the other hand, since L is a complex integer it is inferred from (35) that

$$\begin{cases}|L_r|=\text{Round}\left(\left|\frac{h_{1,2r}h_{1,1r}+h_{1,2i}h_{1,1i}}{h_{1,1r}^2+h_{1,1i}^2}\right|\right)\leqslant 1\\ |L_i|=\text{Round}\left(\left|\frac{h_{1,2i}h_{1,1r}-h_{1,2r}h_{1,1i}}{h_{1,1r}^2+h_{1,1i}^2}\right|\right)\leqslant 1\end{cases}, \tag{36}$$

therefore, $L\in\{0,\pm1,\pm j,\pm1\pm j\}$.

Appendix B

B.1. Proof of Lemma 2

When $|L|>(\sqrt{M}-1)\sqrt{2}$, dividing both sides of Eq. (5) by L, we get

$$\frac{w_1}{L}=\left(\frac{x_1}{L}+x_2\right)+\frac{lx_2}{L}+\frac{n_1}{L\bar{h}_{1,1}}. \tag{37}$$

Because the value of L is large the value of $(lx_2/L+n_1/L\bar{h}_{1,1})$ is small. Therefore, we can ignore the effect of this component. Eq. (37) is then rewritten as

$$\frac{w_1}{L}=\frac{L_rx_{1r}+L_ix_{1i}+j(L_rx_{1i}-L_ix_{1r})}{|L|^2}+x_{2r}+jx_{2i}. \tag{38}$$

From Eq. (38), if x_2 is the desired estimate, the remaining part can be considered the interference. On the other hand, since x_2 is the signal symbol in the M-QAM constellation, we can estimate the real and the imaginary part of x_2 separately. Because the decision border of the M-QAM constellation is unit, the estimate of x_2 has no error if the real or the imaginary part satisfies the following conditions:

$$\left|\frac{L_rx_{1r}\pm L_ix_{1i}}{|L|^2}\right|_{\max}<1=\frac{d_{\min}}{2} \tag{39}$$

or

$$\left|\frac{L_rx_{1r}\pm L_ix_{1i}}{|L|^2}\right|_{\max}=\frac{(\sqrt{M}-1)(|L_r|+|L_i|)}{|L|^2} \tag{40}$$

and

$$\frac{(\sqrt{M}-1)(|L_r|+|L_i|)}{|L|^2}\leqslant\frac{(\sqrt{M}-1)\sqrt{2}}{|L|}<1, \tag{41}$$

therefore $|L|>(\sqrt{M}-1)\sqrt{2}$.

Appendix C

C.1. Proof of Lemma 3

Two constellations A and B are alike if for any point in the constellation A we always find a point in the constellation B such that they have the same real and imaginary part, and vice versa.

Consider two constellations of the signal $(x_{a1}+(L_r+jL_i)x_{a2})$ and $(x_{b1}+(|L_r|+j|L_i|)x_{b2})$, where

$$\begin{cases}(x_{a1}+(L_r+jL_i)x_{a2})=(x_{a1r}+L_rx_{a2r}-L_ix_{a2i})+j(x_{a1i}+L_rx_{a2i}+L_ix_{a2r})\\(x_{b1}+(|L_r|+j|L_i|)x_{b2})=(x_{b1r}+|L_r|x_{b2r}-|L_i|x_{b2i})\\\qquad\qquad\qquad\qquad+j(x_{b1i}+|L_r|x_{b2i}+|L_i|x_{b2r})\end{cases} \tag{42}$$

The two constellations are the same if

$$\begin{cases}(x_{b1r}+|L_r|x_{b2r}-|L_i|x_{b2i})=(x_{a1r}+L_rx_{a2r}-L_ix_{a2i})\\(x_{b1i}+|L_r|x_{b2i}+|L_i|x_{b2r})=(x_{a1i}+L_rx_{a2i}+L_ix_{a2r})\end{cases} \tag{43}$$

Because $x_{a1r}x_{a1i},x_{b1r},x_{b1i}$ belong to the M-QAM constellation, the real and the imaginary value are symmetric with each other about the coordinate origin. Therefore, Eq. (43) has at least one root. In fact, we always find pairs $(x_{a1r}x_{a1i}),(x_{b1r},x_{b1i}),(x_{a2r}x_{a2i}),(x_{b2r},x_{b2i})$ that satisfy

$$\begin{cases}x_{b1r}=x_{a1r}\\|L_r|x_{b2r}=L_rx_{a2r}\\|L_i|x_{b2i}=L_ix_{a2i}\\x_{b1i}=x_{a1i}\\|L_r|x_{b2i}=L_rx_{a2i}\\|L_i|x_{b2r}=L_ix_{a2r}\end{cases} \tag{44}$$

Thus with any one point in the constellation $(x_{a1}+(L_r+jL_i)x_{a2})$ we always find a point in the constellation $(x_{b1}+(|L_r|+j|L_i|)x_{b2})$ and vice versa.

Appendix D

D.1. Proof of Lemma 4

Two constellations $(x_{a1}+|L_r|x_{a2})$ and $(x_{b1}+j|L_i|x_{b2})$ are equivalent if they satisfy the following condition:

$$(x_{a1}+j|L_r|x_{a2})=(x_{b1}+|L_i|(jx_{b2})). \tag{45}$$

It is similar to the Proof in Lemma 3 if L_r,L_i satisfy the condition $|L_r|=|L_i|$ then we always find two pairs (x_{a2r},x_{b2i}) and (x_{a2i},x_{b2r}) for that

$$\begin{cases}|L_r|x_{a2r}=|L_i|x_{b2i}\\|L_r|x_{a2i}=|L_i|x_{b2r}\end{cases}. \tag{46}$$

Appendix E. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.aeue.2018.08.002.

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