

# Performance Analysis of In-Band Full-Duplex Amplify-and-Forward Relay System with Direct Link

Ba Cao Nguyen\*, Xuan Nam Tran\*, Dinh Tan Tran<sup>†</sup>

\* Le Quy Don Technical University, Hanoi, Vietnam (e-mail: bacao.sqtt@gmail.com, namtx@mta.edu.vn)

<sup>†</sup> Telecommunication University, Nhatrang, Vietnam (e-mail:trdinhtan@gmail.com)

**Abstract**—In this paper, we analyze performance of the in-band full-duplex amplify-and-forward one-way relay system in which two terminal nodes operate in the half-duplex mode while the relay node in the full-duplex mode. Unlike previous works we consider the case of imperfect loop-interference cancellation at the full-duplex relay and assume that the direct link between the source and the destination nodes exists in the system. We show that the system performance in the case with the direct link is significantly improved over the case without this link. Using theoretical analysis we can obtain the closed-form of the approximate outage probability, throughput, achievable capacity and symbol error probability of the system. Based on this analysis an effective power allocation for the full-duplex mode is derived to improve the system performance. Numerical results are compared with simulated ones for different levels of residual loop-interference to verify the theoretical analysis.

**Index Terms**—In-band full-duplex relay, self-interference cancellation, amplify-and-forward, outage probability, symbol error probability.

## I. INTRODUCTION

With the potential to double the spectral efficiency, the in-band full-duplex (IBFD) communication has received great attention in recent years. However, the IBFD communication is significantly affected by the residual self-interference (RSI) due to imperfect self-interference cancellation (SIC) at the full-duplex operating node [1], [2], [3], [4]. In the cooperative communication system, the use of relay node which operates in the full-duplex mode allows improvement not only in signal coverage and transmission reliability but also spectral efficiency. As a result, researches on the IBFD cooperative communication systems have attracted great attention recently.

In the literature, many previous works have focused on the performance analysis of the IBFD cooperative communication systems in order to gain a helpful insight into their behavior. In [5] the authors analyzed the effect of relaying strategies on the spectral efficiency (SE) of the IBFD relay cooperative system using the amplify-and-forward (AF) and decode-and-forward (DF) relaying under assumption that there is no direct link between the source and the destination terminals. In [6] the closed-form expressions for the end-to-end capacity of the IBFD AF cooperative system were derived and compared with the half-duplex (HD) one. The cooperative system using the FD mode was shown to be superior to that with the HD mode within a RSI range. The work in [7] successfully derived the achievable capacity of the IBFD cognitive cooperative system using AF relaying. In [8] the outage probability (OP) of the IBFD AF relay system with the direct link and RSI was derived

by treating the RSI as a noise component. However, the closed-form expression for the OP was still not available. In [9] the authors considered a variable-gain IBFD AF cooperative system which has the direct link and is affected by the RSI. Maximal-ratio combining (MRC) was used for signal combination at the destination terminal. The exact and approximate expressions for the OP were obtained but not in the closed form. The authors in [10] proposed the use of a hybrid FD/HD relay which combines the opportunistic mode selection and transmit power adaptation. The relay node can operate in either FD or HD mode when considering the received signal via the direct link as noise or useful signal. This hybrid FD/HD relay was shown to be effective for the IBFD cooperative system with imperfect SIC. With the same assumption of the direct link, the work in [11] analyzed the OP of the IBFD AF cooperative system. The outage performance of the IBFD relay network with relay selection was analyzed in [12]. Dynamic switching between FD and HD modes at the relays was used to overcome the diversity limitation of the IBFD AF relay selection. In [13] the asymptotic expressions of the OP and symbol error rate (SER) were derived for the IBFD AF relay system by optimizing relay location and power allocation for the case without the direct link. In [14] hardware impairments were considered for the IBFD AF relay system with non-linear distortion of high power amplifiers. Other efforts were also paid for investigating performance of the IBFD two-way relay system under imperfect channel state information (CSI) and RSI as well as other parameters [15], [16], [17], [18], [19].

Although there have been great efforts in analyzing performance of the IBFD AF relay system there are still not enough researches for the case in which exists the direct link from the source to the destination terminal due to complex mathematical derivations. To the best of our knowledge, except the work in [9] the direct link was either neglected or only considered for the case of HD relays. However, the closed-form expression for the OP was not obtained in [9]. Thus, the achievable capacity and symbol error probability (SEP) have not also been derived. In order to tackle these problems, in this paper, we perform a detailed analysis of the IBFD AF relay system with the direct link and using a FD relay. In contrast to the previous works by using approximate functions we can obtain the closed-form expression for the OP. In addition to the OP, we can also obtain the throughput, achievable capacity and SEP of the system. Furthermore, we propose an optimal power allocation scheme for the IBFD mode based

on minimizing the system OP. The new contributions of this paper can be summarized as follows:

- The outage performance of the IBFD AF relay system using variable-gain with the direct link is analyzed with perfect CSI at the relay node. By using approximate functions, we can transform the OP of the system from the integral-form to an accurate closed-form expression. This makes the OP more tractable for calculation. From the derived OP, the closed-forms for the throughput, achievable capacity and SEP of the system are also obtained.

- The system performances in terms of the OP, throughput, achievable capacity and SEP are analyzed and evaluated for the case of imperfect SIC, fixed and varied RSI. Our analyses show that for the IBFD AF relay system with the direct link, the system quality is improved comparing with the no direct link. To reduce the impact of RSI at high signal to noise ratio (SNR) regime, an optimal power allocation scheme is proposed. Thanks to this scheme, the IBFD AF relay system achieves significant performance enhancement compared with the previous works. Theoretical analyses are verified by Monte-Carlo simulations.

The rest of this paper is organized as follows. Section II presents the system model. Performances of the system in terms of the OP, throughput, achievable and SEP are analyzed in Section III. Section IV presents the optimal power allocation scheme. Numerical results and discussions are given in Section V and finally, conclusions are drawn in Section VI.

## II. SYSTEM MODEL

In this section, we consider an IBFD AF one-way relay network as illustrated in Fig. 1. The source node  $S_1$  and the destination node  $S_2$  are equipped with a single antenna while the relay node R has two antennas, one for receiving and one for transmitting. The relay node working in the FD mode can transmit and receive signal simultaneously at the same frequency. Incoming data are transmitted from the source node  $S_1$  to the destination node  $S_2$  via two paths: (i) the relaying path using the relay node R, and (ii) the direct link from the source to the destination node (referred shortly as the direct link in the following).

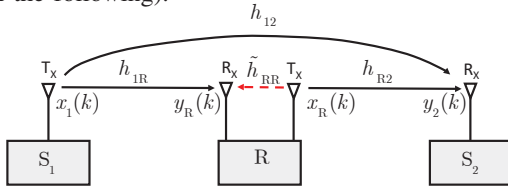


Fig. 1. System model of the IBFD AF one-way relay system with the direct link.

At the time slot  $k$ , the received signal at the relay node is given by

$$y_R(k) = h_{1R}x_1(k) + \tilde{h}_{RR}x_R(k) + z_R(k), \quad (1)$$

where  $x_1(k)$ ,  $x_R(k)$  are the transmitted signals of the nodes  $S_1$  and R, respectively.  $h_{1R}$  and  $\tilde{h}_{RR}$  are respectively the fading coefficients of the channels from  $S_1$  to R and from the

transmitting antenna to the receiving antenna of R.  $z_R(k)$  is the Additive White Gaussian Noise (AWGN) with zero-mean and variance of  $N_R$ ,  $z_R \sim \mathcal{CN}(0, N_R)$ . We assume that the delay due to signal processing at the relay equals one symbol period. Thus, the transmitted signal at the relay node is given by

$$x_R(k) = Gy_R(k-1), \quad (2)$$

where  $G$  is the relaying gain which is selected such that the transmit power of the relay node equals  $P_R$ , that is

$$\mathbb{E}\{|x_R(k)|^2\} = G^2\mathbb{E}\{|y_R(k-1)|^2\} = P_R, \quad (3)$$

where the  $\mathbb{E}\{\cdot\}$  denotes the expectation operator. Under assumption that the relay node has perfect knowledge of the fading coefficient  $h_{1R}$ , it can use a variable gain corresponding to the fading state, therefore, we have

$$G \triangleq \sqrt{\frac{P_R}{\rho_1 P_1 + I_R + N_R}} \quad (4)$$

where  $\rho_1 = |h_{1R}|^2$  is channel gain from  $S_1$  to R;  $P_1$  is the transmit power of the source node  $S_1$ ;  $I_R = \tilde{\Omega}_R P_R$  is the RSI at the relay node after self-interference cancellation. It is noted that the RSI after various SIC can be modeled by a complex Gaussian-distributed variable with zero mean and variance  $I_R$ . The loop interference due to the full duplex mode  $I_R$  is given in [9], [8], [19].

The received signal at the destination node  $S_2$  is given by

$$y_2(k) = h_{R2}x_R(k) + h_{12}x_1(k) + z_2(k), \quad (5)$$

where  $h_{R2}$ ,  $h_{12}$  are the fading coefficients of the links from R to  $S_2$  and from  $S_1$  to  $S_2$ , respectively;  $z_2(k)$  is the AWGN at the destination node,  $z_2 \sim \mathcal{CN}(0, N_2)$ . Combining (1), (2) and (5), the received signal at the destination node can be given by

$$y_2(k) = h_{R2}G \left[ h_{1R}x_1(k-1) + \tilde{h}_{RR}x_R(k-1) \right] + h_{R2}Gz_R(k-1) + h_{12}x_1(k) + z_2(k). \quad (6)$$

It is noted that, due to the processing delay at the relay node, the two replicas of the transmitted signal from  $S_1$  and R have some temporal separation. However, it is assumed that the destination node  $S_2$  can fully resolve the replicas [20]. As a result, the destination node  $S_2$  can appropriately co-phase and merge the two signal replicas using MRC. This assumption is appropriate when  $S_2$  uses some suitable receiver such as the ideal Rake one [9]. Then the end-to-end signal-to-interference-plus-noise ratio SINR of the system can be expressed as

$$\gamma = \gamma_{S_1 S_2} + \gamma_{S_1 R S_2}, \quad (7)$$

where  $\gamma_{S_1 S_2}$  and  $\gamma_{S_1 R S_2}$  are the SINR of the links from  $S_1 \rightarrow S_2$  and from  $S_1 \rightarrow R \rightarrow S_2$ , respectively.  $\gamma_{S_1 S_2}$  is given by

$$\gamma_{S_1 S_2} = \frac{|h_{12}|^2 P_1}{N_2} = \frac{\rho_3 P_1}{N_2}. \quad (8)$$

$\gamma_{S_1 R S_2}$  is determined as follows

$$\begin{aligned}\gamma_{S_1RS_2} &= \frac{|h_{1R}|^2|h_{R2}|^2P_1}{|h_{R2}|^2(I_R + N_R) + N_2/(G)^2} \\ &= \frac{\rho_1\rho_2P_1P_R}{(\rho_2P_R + N_2)(I_R + N_R) + \rho_1P_1N_2},\end{aligned}\quad (9)$$

where  $\rho_1 = |h_{1R}|^2$ ;  $\rho_2 = |h_{R2}|^2$ ;  $\rho_3 = |h_{12}|^2$ . When affected by the Rayleigh fading the magnitude of the random channel gains follows the Rayleigh distribution and the cumulative distribution function (CDF) of  $\rho_i = |h_i|^2$ ,  $i = 1 \div 3$ , is given by

$$F_{\rho_i}(x) = 1 - e^{-\frac{x}{\Omega_i}}, x \geq 0, \quad (10)$$

where  $\Omega_i = \mathbb{E}\{|h_i|^2\}$ .

### III. SYSTEM PERFORMANCE

In this section, we analyze the performance of the above system in term of the OP, throughput, achievable capacity and SEP.

#### A. Outage Probability

The OP of the IBFD AF relay system is defined as the probability that the achievable rate at the destination node is less than the minimum data rate required by the system. We assume that the minimum required data rate is  $R$  (bit/s/Hz). The OP can be defined as follows

$$P_{\text{out}} = \Pr\{C < R\}, \quad (11)$$

where  $C = \log_2(1 + \gamma)$  and  $\gamma$ ,  $R$  are SINR and the minimum required data rate at the destination node, respectively. Therefore, the outage occurs when

$$\log_2(1 + \gamma) < R. \quad (12)$$

As a result, we have:

$$\gamma < 2^R - 1. \quad (13)$$

Set  $x = 2^R - 1$ , we can determine the system OP as follows:

$$P_{\text{out}}(x) = \Pr\{\gamma < x\}. \quad (14)$$

*Theorem 1:* The OP expressions of the system is determined as follows:

$$\begin{aligned}P_{\text{out}}(x) &= \Pr\{\gamma < x\} = \Pr\{\gamma_{S_1S_2} + \gamma_{S_1RS_2} < x\} \\ &\approx \begin{cases} 1 - \frac{b}{b-a} \exp(-ax) - \frac{a}{a-b} \exp(-bx), & a \neq b, \\ 1 - bx \exp(-ax) - \exp(-bx), & a = b, \end{cases}\end{aligned}\quad (16)$$

where

$$a = \frac{I_R + N_R}{\Omega_1 P_1} + \frac{N_2}{\Omega_2 P_R}; b = \frac{N_2}{\Omega_3 P_1}.$$

*Proof:* By using the expression [21, Eq. 3.324.1] and Lemma 2 in [22], together with the approximation  $K_1(X) \approx \frac{1}{X}$  when  $X \ll 1$  [23], we can obtain the OP of the IBFD AF relay system in (15) which is shown in the next page. Here  $F_\rho(\cdot)$  is the CDF of  $\rho$  and  $f_\rho(\cdot)$  is the probability density

function (PDF) of  $\rho$ ;  $K_1(\cdot)$  denotes the first-order modified Bessel function of the second kind. After some mathematical manipulations, the integral-form of the OP in (15) reduces to the closed-form in (16).

#### B. Throughput

Besides the OP, throughput is another important performance factor that needs to be evaluated. The throughput  $\mathcal{T}$  of the system is defined as

$$\mathcal{T}(x) = R[1 - P_{\text{out}}(x)], \quad (17)$$

where  $R$  is the given transmission rate (bit/s/Hz),  $P_{\text{out}}(x)$  is the OP of the system, which is derived in (16).

#### C. Average Achievable Capacity

For the IBFD AF relay system, the achievable capacity is defined as follows

$$C_{\text{FD}} = \mathbb{E}\{\log_2(1 + \gamma)\} = \int_0^\infty \log_2(1 + \gamma) f_\gamma(\gamma) d\gamma, \quad (18)$$

where  $\gamma$  is the SINR of the considered system and  $f_\gamma(\gamma)$  is the PDF of  $\gamma$ .

*Theorem 2:* The achievable capacity of the IBFD AF relay system is determined as follows

$$C_{\text{FD}} = \begin{cases} \frac{1}{\ln 2} \left( \frac{b}{b-a} e^a E_1(a) + \frac{a}{a-b} e^b E_1(b) \right), & a \neq b, \\ \frac{1}{\ln 2} \left( \frac{b}{a} - b e^a E_1(a) + e^b E_1(b) \right), & a = b, \end{cases}\quad (19)$$

where  $E_1(\cdot)$  is the exponential integral which is defined as  $E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$ .

*Proof:* From (18), after some mathematical manipulations, we have

$$C_{\text{FD}} = \frac{1}{\ln 2} \int_0^\infty \frac{1 - P_{\text{out}}(x)}{1 + x} dx, \quad (20)$$

using [21, Eq. 3.532.4] it is straightforward to obtain the (19).

#### D. Symbol Error Probability

The SEP of the system is given by [4]:

$$\text{SEP} = \alpha \mathbb{E}\left\{Q\left(\sqrt{\beta\gamma}\right)\right\} = \frac{\alpha}{\sqrt{2\pi}} \int_0^\infty F\left(\frac{t^2}{\beta}\right) e^{-\frac{t^2}{2}} dt, \quad (21)$$

where  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$  is the Gaussian function;  $\gamma$  is the received SINR of the system;  $\alpha$  and  $\beta$  depends on the modulation types, e.g.,  $\alpha = 1, \beta = 2$  for the binary phase-shift keying (BPSK) modulation [4].  $F(x)$  is the CDF of the SINR and  $F(x) = P_{\text{out}}(x)$ , with  $P_{\text{out}}(x)$  being determined using equation (16).

$$\begin{aligned}
 P_{\text{out}}(x) &= \Pr \left\{ \frac{\rho_3 P_1}{N_2} + \frac{\rho_1 \rho_2 P_1 P_R}{(\rho_2 P_R + N_2)(I_R + N_R) + \rho_1 P_1 N_2} < x \right\}, \text{ set } X = x - \frac{\rho_3 P_1}{N_2}, \text{ thus} \\
 P_{\text{out}}(x) &= \int_0^{\frac{N_2 x}{P_1}} \left[ 1 - \int_0^{\infty} \left( 1 - F_{\rho_1} \left( \frac{[(z + \frac{N_2}{P_R} X) P_R + N_2](I_R + N_R) X}{P_1 P_R z} \right) \right) f_{\rho_2} \left( z + \frac{N_2}{P_R} X \right) dz \right] f_{\rho_3}(\rho_3) d\rho_3 \\
 &= \int_0^{\frac{N_2 x}{P_1}} \left[ 1 - \exp(-aX) 2 \sqrt{\frac{N_2(I_R + N_R)(X^2 + X)}{\Omega_1 \Omega_2 P_1 P_R}} K_1 \left( 2 \sqrt{\frac{N_2(I_R + N_R)(X^2 + X)}{\Omega_1 \Omega_2 P_1 P_R}} \right) \right] f_{\rho_3}(\rho_3) d\rho_3 \\
 &\approx \int_0^{\frac{N_2 x}{P_1}} [1 - \exp(-aX)] f_{\rho_3}(\rho_3) d\rho_3 = \int_0^{\frac{N_2 x}{P_1}} \left[ 1 - \exp\left(-a\left(x - \frac{\rho_3 P_1}{N_2}\right)\right) \right] \frac{1}{\Omega_3} \exp\left(-\frac{\rho_3}{\Omega_3}\right) d\rho_3.
 \end{aligned} \tag{15}$$

*Theorem 3:* The SEP of the IBFD AF system is determined as follows:

$$\text{SEP} = \begin{cases} \frac{\alpha\sqrt{\beta}}{2} \left( \frac{1}{\sqrt{\beta}} - \frac{b}{(b-a)\sqrt{\beta+2a}} - \frac{a}{(a-b)\sqrt{\beta+2b}} \right), & a \neq b, \\ \frac{\alpha\sqrt{\beta}}{2} \left( \frac{1}{\sqrt{\beta}} - \frac{1}{\sqrt{\beta+2b}} - \frac{b}{\sqrt{(\beta+2a)^3}} \right), & a = b. \end{cases} \tag{22}$$

*Proof:* From (21), set  $x = \frac{t^2}{\beta}$  we obtain the SEP as follows

$$\text{SEP} = \frac{\alpha\sqrt{\beta}}{2\sqrt{2\pi}} \int_0^{\infty} \frac{e^{-\beta x/2}}{\sqrt{x}} F(x) dx. \tag{23}$$

To determine the SEP, replacing  $F(x)$  in (23) by  $P_{\text{out}}(x)$  in (16) then using [21, Eq. 2.321.1] and [21, Eq. 3.361.1], we have (24) as follows

$$\begin{aligned}
 \text{SEP} &= \begin{cases} \frac{\alpha\sqrt{\beta}}{2\sqrt{2\pi}} \int_0^{\infty} \frac{e^{-\beta x/2}}{\sqrt{x}} \left( 1 - \frac{b}{b-a} e^{-ax} - \frac{a}{a-b} e^{-bx} \right) dx, & a \neq b \\ \frac{\alpha\sqrt{\beta}}{2\sqrt{2\pi}} \int_0^{\infty} \frac{e^{-\beta x/2}}{\sqrt{x}} (1 - bxe^{-ax} - e^{-bx}) dx, & a = b \end{cases} \\
 &= \begin{cases} \frac{\alpha\sqrt{\beta}}{2\sqrt{2\pi}} \left( \sqrt{\frac{2\pi}{\beta}} - \frac{b}{b-a} \sqrt{\frac{2\pi}{\beta+2a}} - \frac{a}{a-b} \sqrt{\frac{2\pi}{\beta+2b}} \right), & a \neq b \\ \frac{\alpha\sqrt{\beta}}{2\sqrt{2\pi}} \left( \sqrt{\frac{2\pi}{\beta}} - \sqrt{\frac{2\pi}{\beta+2b}} - b \sqrt{\frac{2\pi}{(\beta+2a)^3}} \right), & a = b. \end{cases} \tag{24}
 \end{aligned}$$

It is noted that the first line in (24) is obtained by using [21, Eq. 3.361.1] while the second line using [21, Eq. 2.321.1 and Eq. 3.361.1].

#### IV. OPTIMAL POWER ALLOCATION SCHEME

In this section, an optimal power allocation scheme is derived to minimize the OP and SEP of the system by reducing the RSI at the relay. As a consequence, the throughput and average achievable capacity will be maximized.

*Theorem 4:* The optimal power level is determined by

$$P_R^* = \sqrt{\frac{\Omega_1 P_1 N_2}{\Omega_2 \tilde{\Omega}_R}}. \tag{25}$$

*Proof:* The power optimization problem can be expressed as follows

$$P_R^* = \arg \min_{P_R} P_{\text{out}}(x). \tag{26}$$

For the case  $a \neq b$  we have

$$\begin{aligned}
 \frac{\partial P_{\text{out}}(x)}{\partial P_R} &= - \left( \frac{a' b e^{-ax}}{(b-a)^2} - \frac{a' b x e^{-ax}}{(b-a)} - \frac{a' b e^{-bx}}{(a-b)^2} \right) \\
 &= \frac{a' b e^{-ax}}{(b-a)^2} \left( -1 + (b-a)x + e^{(a-b)x} \right),
 \end{aligned} \tag{27}$$

where  $a' = \frac{\partial a}{\partial P_R} = \frac{\tilde{\Omega}_R}{\Omega_1 P_1} - \frac{N_2}{\Omega_2 P_R^2}$ ;  $b' = \frac{\partial b}{\partial P_R} = 0$ . Using the Taylor expansion

$$\begin{aligned}
 e^{(a-b)x} &= \sum_{n=0}^{\infty} \frac{[(a-b)x]^n}{n!} \\
 &= 1 + (a-b)x + \frac{[(a-b)x]^2}{2!} + \dots
 \end{aligned} \tag{28}$$

we can find only a critical point as given in (25) for (27) that is the root of  $a' = 0$ .

#### V. NUMERICAL RESULTS

In this section, we evaluate performance of the IBFD AF relay system using numerical results. In order to verify the theoretical analysis we also show the simulated results obtained by generating random channel gains  $\rho_i$  with means  $\Omega_i$ ,  $i = 1, 2, 3$ . In our evaluations, we set the average channel gains  $\Omega_1 = \Omega_2 = 1$  and the noise variances  $N_R = N_2 = 1$ . The system performance is evaluated for different SNR, with the average SNR is defined as  $\text{SNR} = P_1/N_R = P_R/N_2$ .

Fig. 2 illustrates the system OP versus the transmit powers  $P_1$ ,  $P_R$ , where  $P_1 = P_R$ , using equations (16). The minimum required rate used to obtain the OP is  $R = 2$  bit/s/Hz, and thus  $x = 3$ . Due to larger path loss, we assume that the average channel gain from the source node to the destination node is  $\Omega_3 = 0.1$  which is smaller than  $\Omega_1 = \Omega_2 = 1$ . The RSI power level is considered with  $I_R = -5, 5, 15, 25$  dB. Notice that, the RSI in this case is fixed when the transmit power at the relay node changed. As can be seen in the figure, when

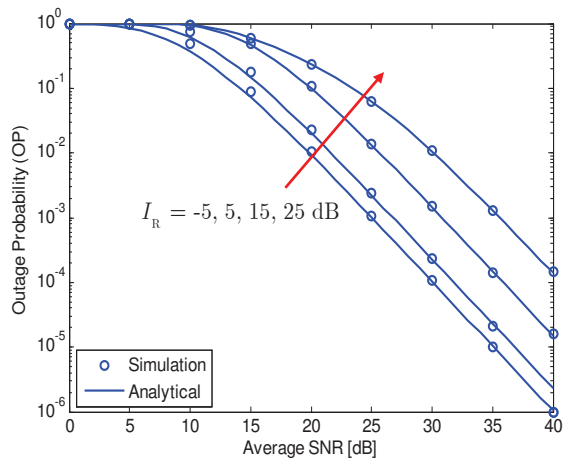


Fig. 2. The OP performance of the system versus the average SNR with fixed RSI,  $\Omega_3 = 0.1$ .

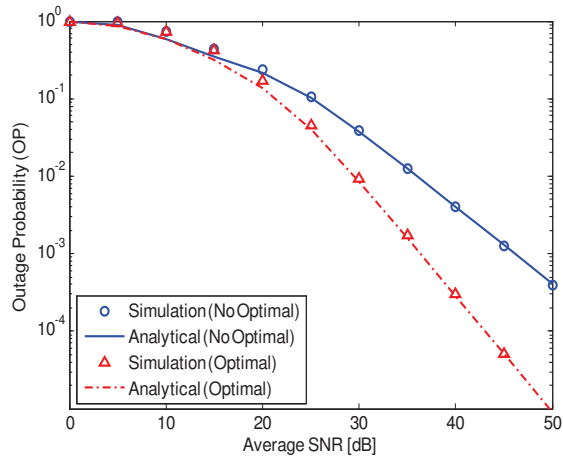


Fig. 3. The OP performance of the system versus the average SNR when the RSI is changed by transmit power at the relay node,  $\tilde{\Omega}_R = -10$  dB,  $\Omega_3 = 0.01$ . The optimal results obtained using (25) while no optimal using  $P_R = P_1$ .

the direct link exists, the OP performance of the system is significantly improved compared with those in [11], [12], [13]. Moreover, even affected by the RSI the OP of the system does not saturate to an irreducible floor. It is also noted that there is a good agreement between the analytical and simulation results.

Fig. 3 shows the OP performance in the case of varied RSI with  $\Omega_3 = 0.01$  and  $\tilde{\Omega}_R = -10$  dB. Other parameters used to obtain the results in Fig. 3 are the same with those in Fig. 2. It can be seen from the figure that the impact of the RSI increases with the transmit power due to the fact that  $I_R = \tilde{\Omega}_R P_R$ . The increase in the average SNR at the destination means that,  $P_R$  has been increased accordingly, leading to increase in  $I_R$ . Therefore, if the  $\Omega_3$  is very small, meaning the direct link from the source node to the destination node can be neglect, the OP will go to the outage floor at high SNR when  $P_R = P_1$ . To overcome this problem, the optimal power allocation will be

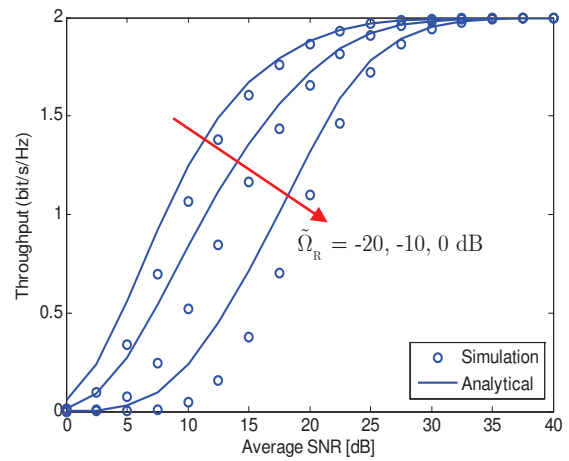


Fig. 4. Throughput of the system versus the average SNR at the relay node using the optimal power allocation with  $R = 2$ ,  $\Omega_3 = 0.01$ .

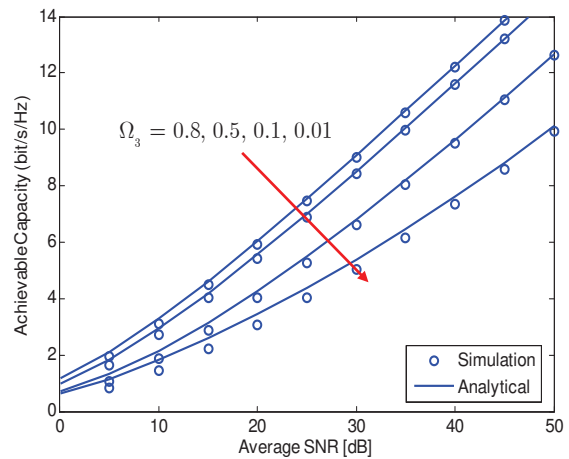


Fig. 5. The achievable capacity versus the average SNR at the relay node using the optimal power allocation,  $\tilde{\Omega}_R = -10$  dB.

applied. As shown in the figure, by using the optimization for the IBFD mode, the system quality is better significantly.

Fig. 4 illustrates the throughput of the system obtained for the case  $R = 2$  bit/s/Hz. The throughput of the system is shown to reach the target  $R$  when the transmit power is above 25 dB in the case with  $\tilde{\Omega}_R = -20$  dB,  $\Omega_3 = 0.01$ . If the RSI increases, i.e.  $\tilde{\Omega}_R = -10, 0$  dB, it requires an increase in the average SNR (higher than 25 dB) in order to achieve the target throughput.

Fig. 5 plots the achievable capacity of the system versus the average SNR at the relay node using power allocation with  $\tilde{\Omega}_R = -10$  dB. The theoretical curves are plotted by using expression (19). We change the channel gain of the direct link with  $\Omega_3 = 0.8, 0.5, 0.1, 0.01$  to consider. It is obvious that, the achievable capacity increases when the SNR increases combining with power allocation. On the other hand, increasing the channel gain of the direct link will increase the achievable capacity.

Finally, Fig. 6 shows the SEP performance of the system

versus the average SNR at the relay using BPSK modulation combining with power allocation. The theoretical SEP performance was obtained using equation (22) for two cases  $\tilde{\Omega}_R = -20$  dB,  $\Omega_3 = 0.1$  and  $\tilde{\Omega}_R = -10$  dB,  $\Omega_3 = 0.01$ . In this figure, we compare the SEP in the case of using power allocation ( $P_R$  is determined by (25)) with the SEP in the case of no optimization ( $P_R = P_1$ ) to one again see the gain of the optimal power allocation, especially in high SNR regime. As shown in the figure, when the SEP target is  $10^{-4}$ , the SEP performance using optimal has about 12.5 dB gain comparing with the case of no optimal, when  $\tilde{\Omega}_R = -10$  dB,  $\Omega_3 = 0.01$ . For the case  $\tilde{\Omega}_R = -20$  dB,  $\Omega_3 = 0.1$ , the gain achieve about 5 dB with SEP =  $10^{-6}$ . As a result, when the channel gain of the direct link is very small, by the using power allocation at the relay node, the SEP performance is more and more improvement. Therefore, depending on the requirement of the FD system, the RSI, and the average gain of the direct link, it is necessary to use the transmit power of the relay node suitably to improve the system performance.

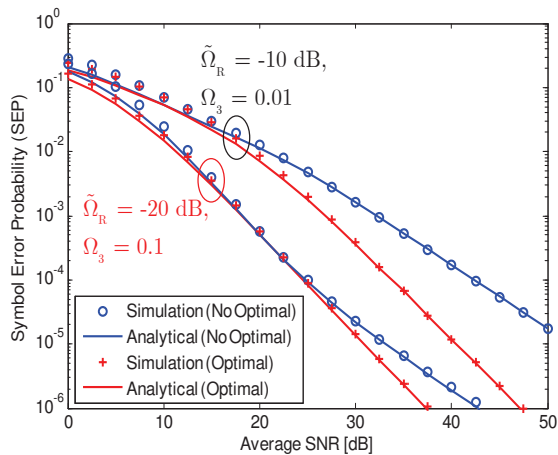


Fig. 6. The SEP performance of the system versus the average SNR at the relay node for the case with and without power allocation.

## VI. CONCLUSION

In this paper, we have studied performance of the IBFD AF relay system in the case of imperfect loop-interference cancellation. We have successfully obtained the closed-form expressions for the OP, throughput, achievable capacity and SEP. The system performance was also analyzed for the case of fixed RSI and varied RSI. To improve the system performance in the case the loop-interference is not perfectly canceled, we proposed an optimal power allocation scheme at the relay node. Numerical results showed that the system performance is significantly improved with the proposed power allocation scheme. Although the results in the current paper were obtained for the case of the IBFD AF relay system with the direct link, they can be easily applied to the case without direct link by setting the channel gain of the link equal zero.

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