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Biomechanical Analysis of the Shooter-Weapon System Oscillation

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***Abstract*—The paper deals with analysis of shooter – weapon system oscillation when firing from a biomechanical point of view. The biomechanical model of shooter arms is developed by taking the muscles and tendons forces into consideration. Linking between shooter and the weapon is treated as elastic and viscose damping. The dynamic model developed for a gas operated submachine gun by applying Lagrange equation of motion has thirteen degrees of freedom. The shooter arms joint stiffness and damping coefficients are experimentally determined. The problem is numerically solved and experimentally verified for the 7.62 mm AKM. The method is able to analyze the shooter – weapon system parameters affecting the gas operated weapon firing stability.**

***Keywords*- biomechanics; shooter - weapon; firing stability, Lagrange eqation; automatic weapon; human arm; dynamics**

I. INTRODUCTION

The firing stability is one of the most important properties of any automatic weapon. In fact, the weapon can oscillate and move when firing. That causes the changes in aiming angle at the time when the projectile exits the barrel muzzle resulting in reduction of the probability of hitting a target [9]. Hence, the angle of the weapon jump in the vertical plane has been used as the main stability evaluation criterion. There are a large number of published works studying on the weapon firing stability. The most notable of them are presented in [2], [3], [9], [10] and [11]. In previous weapon stability studies the influence of shooter aims on the weapon oscillation is either neglected or partly taken into account independently of biomechanical properties of shooter aims.

In this paper, the dynamic model of shooter-weapon system, developed for a portable gas operated automatic weapon using Lagrange equation of motion, is supplemented by the biomechanical model of shooter left and right arm in which the muscles and tendons forces are converted to joint torques at the shooter shoulder, elbow and wrist through principle of virtual work.

II. BIOMECHANICAL MODEL OF SHOOTER’S ARM

A. Biomechanical Model of Joints and Humerus

The human arm (see Fig. 1) consists of three main segments, i.e. the upper arm, the lower arm and the hand that are connected by joints and together make the arm one of the most useful and complex tool of human body.

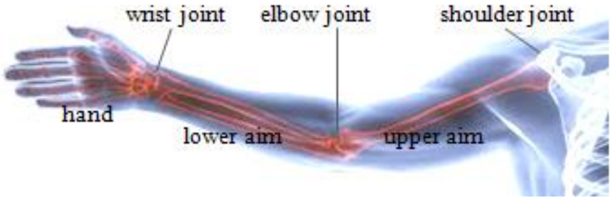

Figure 1. Human right arm segments

The main aim joints are shoulder joint, elbow joint and wrist joint. The arm segments can move under the control of the nervous system, muscle groups and tendons.

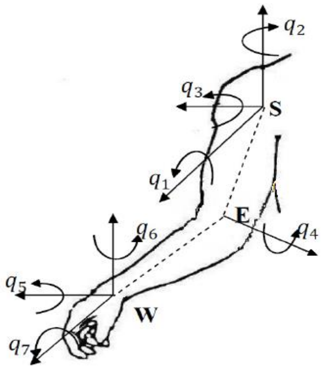

Figure 2. Degrees of freedom in human arm: S –shoulder, E – elbow, W – wrist

Arms are capable of moving in different directions to perform certain tasks, such as handling, holding and shooting with a gun. In performing a certain task, the human arm is considered to have seven degrees of freedom (Fig. 2). The shoulder and the wrist give pitch, yaw and roll. The elbow only allows for pitch.

B. Biomechanical Model of Muscles And Tendons

As a human body's part flexes, muscles are active elements producing power and movement. Conversely, during extension, muscles generate a passive tension. During movement bones and joints are passive elements controlled by the muscles.

There are several published studies, in which the muscle mathematical model is mentioned. For instance, the three-element Hill muscle model, constituted by a contractile element (CE) and two non-linear spring elements, one in series (SE) and another in parallel (PE) (Fig. 3), is presented in [6]. In this model, CE produces an active force generated by two kinds of protein: actin (thin fibers) and myosin (thick fibers). PE accounts for nonlinear elastic properties of muscles and it is responsible for the muscle passive behavior when muscle is stretched beyond its resting length. SE represents the ability of muscle immediate response and support for the damping element DE. The more comprehensive muscle model, provided in [8], takes the tendon force into consideration.

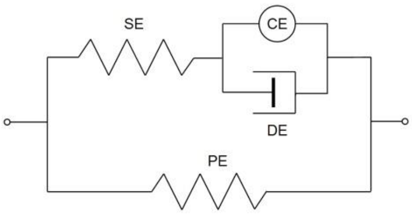

Figure 3. Three-element Hill muscle model

As it is described in [8, 10], the muscles and tendons forces can be replaced by the joint torques. Hence, the biomechanical model of the shooter's arm is developed and shown in Fig. 4.

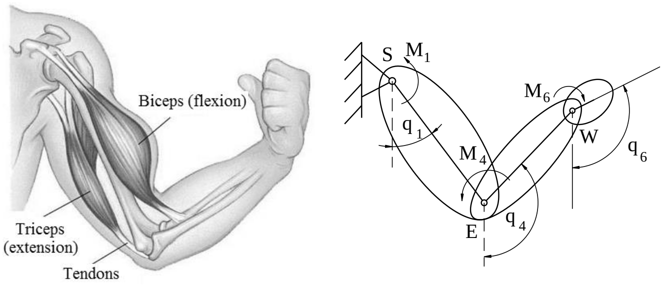

Figure 4. Biomechanical model of shooter's arm: S – shoulder joint, E – elbow joint, W – wrist joint; *M* – joint moment, *q* – angular displacement

We assume that the rotational motions around the shoulder, elbow and wrist joint are performed independently by specific groups of muscles. The muscle forces, thus, are converted to torques M_1, M_4 and M_6 at shoulder joint, elbow joint and wrist joint, respectively [7]. Their values are calculated as below

$$M_j = M_j^0 + k_j q_j + c_j \dot{q}_j, \text{ for } j = \{1; 4; 6\}, \tag{1}$$

where:

A_{0j} – cross-sectional areas of shoulder, elbow and wrist,

k_j – joint stiffness coefficient,

c_j – joint damping coefficient,

q_j – joint angular displacement,

$\dot{q}_j$ – joint angular velocity,

M_j^0 – initial joint torque (i.e. joint holding torque). Its value is given by formula

$$M_j^0 = \gamma A_{0j}, \tag{2}$$

in which γ is the constant defined at the static equilibrium of shooter – weapon system. The values of M_j^0, k_j, c_j can be also found experimentally [4], [5].

III. PROBLEM FORMULATION

A. Shooter – Weapon System Description

In this paper, the shooter – weapon system dynamic model is formulated for a gas – operated automatic weapon (see Fig. 5.).

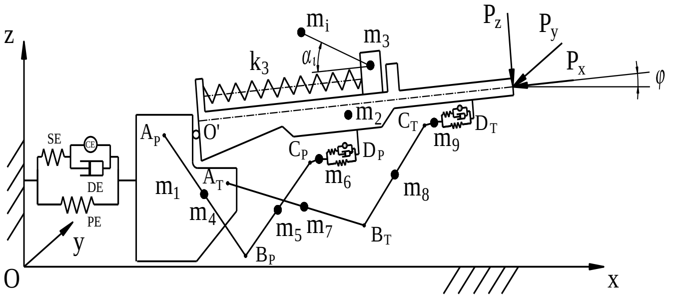

Figure 5. Dynamic model of shooter – gas operated automatic weapon system

The shooter – weapon system includes nine rigid bodies:

- body 1 – shooter shoulder of mass m_1,
- body 2 – weapon frame of mass m_2,
- body 3 – breech system of mass m_3,
- body 4 – upper right arm (A_PB_P) of mass m_4,
- body 5 – lower right arm (B_PC_P) of mass m_5,
- body 6 – right hand (C_PD_P) of mass m_6,
- body 7 – upper left arm (A_TB_T) of mass m_7,
- body 8 – lower left arm (B_TC_T) of mass m_8,
- body 9 – left hand (C_TD_T) of mass m_9 and

It is necessary to take into consideration the mass m_i of weapon moving parts.

In order to analyze the dynamics of the shooter – weapon system, Descartes coordinate systems have been established at the mass center of each bodies and the whole system as shown in Fig. 6 and Fig. 7, where:

O_0 represents the stationary coordinate system fixed on the ground,

O_k accounts for the local coordinate system established at the center of mass of *k*-th rigid body, for $k = \{1, 2,..., 9\}$.

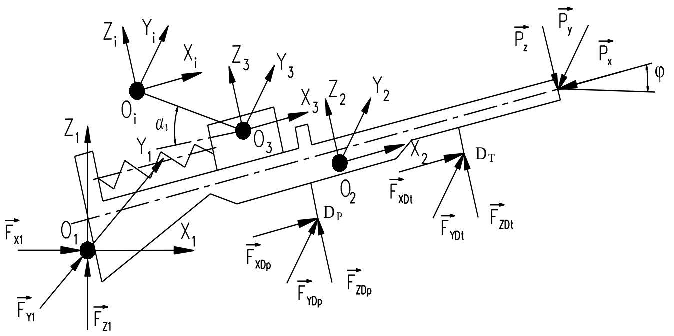

Figure 6. Coordinate systems at center of mass of bodies 1, 2, 3 and i

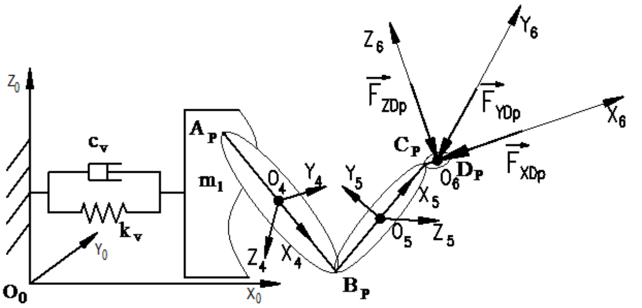

Figure 7. Coordinate systems at center of mass of bodies 4, 5 and 6

The coordinates of system rigid bodies are labeled as follows:

q_1 - longitudinal displacement of body 1 along X_0-axis,
q_2 - angular displacement of body 2 about X_1-axis,
q_3 - angular displacement of body 2 about Y_1-axis,
q_4 - angular displacement of body 2 about Z_1-axis,
q_5 - longitudinal displacement of body 3 along X_2-axis,
q_6 - angular displacement of body 4 about X_1-axis,
q_7 - angular displacement of body 4 about Y_1-axis,
q_8 - angular displacement of body 4 about Z_1-axis,
q_9 - angular displacement of body 5 about Z_4-axis,
q_{10} - angular displacement of body 7 about X_1-axis,
q_{11} - angular displacement of body 7 about Y_1-axis,
q_{12} - angular displacement of body 7 about Z_1-axis,
q_{13} - angular displacement of body 8 about Z_7-axis,
q_{14} - angular displacement of body 6 about X_5-axis,
q_{15} - angular displacement of body 6 about Y_5-axis,
q_{16} - angular displacement of body 6 about Z_5-axis,
q_{17} - longitudinal displacement of body 6 along X_2-axis,
q_{18} - angular displacement of body 9 about X_8-axis,
q_{19} - angular displacement of body 9 about Y_8-axis,
q_{20} - angular displacement of body 9 about Z_8-axis,
q_{21} - longitudinal displacement of body 9 along X_2-axis,
q_i - longitudinal displacement of body i along X_i-axis.

The examined shooter – weapon system has thirteen degrees of freedom and twenty two generalized coordinates, in which the independent generalized coordinates are from q_1 to q_{13}. The others are dependent generalized coordinates.

B. *Kinetic Energy*

Kinetic energy of the shooter – weapon system is equal to the sum of particular kinetic energy of the bodies, of which the system consists. The kinetic energy of *k*-th body of the system is generally given by formula

$$T_k = \frac{1}{2}\left(\dot{\vec{R}}_k^T \left[M\right]_k^{RR} \dot{\vec{R}}_k + \vec{\omega}_k^T \left[A\right]_{k0} \left[J\right]_k \left[A\right]_{k0}^T \vec{\omega}_k\right), \qquad (3)$$

where

$\vec{R}_k -$ position vector of the mass center of *k*-th body in O_0,

$\left[M\right]_k^{RR} -$ matrix of longitudinal displacement of *k*-th body,

$\vec{\omega}_k -$ angular velocity vector of *k*-th body in O_0,

$\left[A\right]_{k0} -$ matrix of absolute angular displacement of *k*-th body,

$\left[J\right]_k -$ matrix of inertia tensors of *k*-th body in O_k

For example, for $k = 4$, the kinetic energy of the upper right arm is

$$T_4 = \frac{1}{2}\left(\dot{\vec{R}}_4^T \left[M\right]_4^{RR} \dot{\vec{R}}_4 + \vec{\omega}_4^T \left[A\right]_{40} \left[J\right]_4 \left[A\right]_{40}^T \vec{\omega}_4\right)\cdot \qquad (4)$$

The matrices and vectors appearing in Eq. (4) are determined as follows

$$\left[J\right]_4 = \begin{bmatrix} J_4^{xx} & J_4^{xy} & J_4^{xz} \\ & J_4^{yy} & J_4^{yz} \\ Sym & & J_4^{zz} \end{bmatrix},$$

$$\left[M\right]_4^{RR} = \begin{bmatrix} m_4 & 0 & 0 \\ 0 & m_4 & 0 \\ 0 & 0 & m_4 \end{bmatrix},$$

$$\left[A\right]_{40} = \left[A\right]_{10} \left[A\right]_{41},$$

where:

$\left[A\right]_{10}$ is the matrix of absolute angular displacement of the upper right arm,

$$\left[A\right]_{41} = \left[A\right]_{41}^x \left[A\right]_{41}^y \left[A\right]_{41}^z,$$

in which:

$$\left[A\right]_{41}^x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\varphi_{4x} + q_6) & -sin(\varphi_{4x} + q_6) \\ 0 & sin(\varphi_{4x} + q_6) & cos(\varphi_{4x} + q_6) \end{bmatrix},$$

$$[A]_{41}^{y} = \begin{bmatrix} cos(\varphi_{4y}+q_7) & 0 & sin(\varphi_{4y}+q_7) \\ 0 & 1 & 0 \\ -sin(\varphi_{4y}+q_7) & 0 & cos(\varphi_{4y}+q_7) \end{bmatrix},$$

$$[A]_{41}^{z} = \begin{bmatrix} cos(\varphi_{4z}+q_8) & -sin(\varphi_{4z}+q_8) & 0 \\ sin(\varphi_{4z}+q_8) & cos(\varphi_{4z}+q_8) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $\varphi_{4x}, \varphi_{4y}$ and φ_{4z} are initial angular coordinates of X_4-axis, Y_4-axis and Z_4-axis in O_1 coordinate system, respectively.

C. Virtual Work and Generalized Force

The dynamical shooter – weapon system that is constructed from 9 rigid bodies has its configuration defined in terms of 13 generalized coordinates (i.e. degrees of freedom). Then, the virtual work for the system is given by

$$\delta W_{F_k,} = \vec{F}_k^T \delta\vec{R}_k, \tag{5}$$

where

$\vec{F}_k$ accounts for the vector of applied forces, F_k, acting on the k rigid bodies,

$\delta\vec{R}_k$ represents the virtual displacement of the k-th rigid body and it can be determined as

$$\delta\vec{R}_k = \sum_{j=1}^{13} \frac{\partial\vec{R}_k}{\partial\vec{q}_j}\delta\vec{q}_j, \tag{6}$$

If there are torques M_k, acting on the k-th rigid body, the virtual work done on the the system is

$$\delta W_{M_k} = \bar{M}_k^T \delta\vec{\theta}_k, \tag{7}$$

where

$\bar{M}_k^T$ represents the matrix of torques, M_k, acting on the k-th rigid body,

$\delta\vec{\theta}_k$ is the virtual angular motion of the k-th rigid body and it is determined as bellow

$$\delta\vec{\theta}_k = \sum_{j=1}^{13} \frac{\partial\vec{\theta}_k}{\partial\vec{q}_j}\delta\vec{q}_j. \tag{8}$$

From Eq. (5) and Eq. (7) we are able to derive expressions for the generalized forces of the system, Q_j.

Finally, we apply the Lagrange equations in order to describe the motion of the shooter – weapon system, it yields

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \text{ for } j = \{1,...,13\}. \tag{9}$$

Where:

T – total kinetic energy of the whole system,

q_j – independent generalized coordinate,

Q_j– generalized force corresponding to the generalized coordinate q_j.

IV. PROBLEM SOLUTION

The above developed biomechanical model describing shooter – weapon system oscillation when firing is validated for the AKM automatic rifle.

The stiffness and damping coefficients of shooter's shoulder joint, elbow joint and wrist joint have been determined experimentally. Particular views of the experiment setup are shown in Fig. 8, where are used seven stiffness sensors. On each shooter arms, there are three sensors located at the shoulder, elbow and wrist. The last one is located on the barrel. Experimental results are shown in Tab. 1.

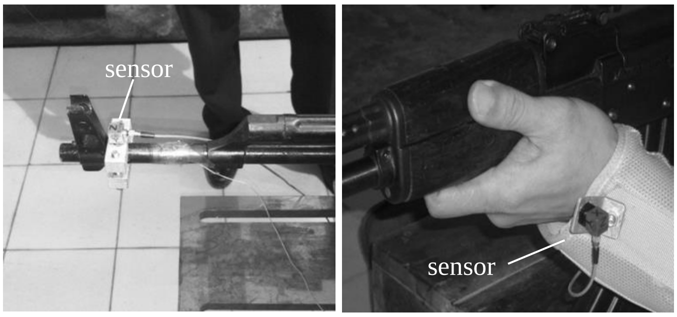

Figure 8. Locations of sensors on AKM barrel (left) and shooter left arm wrist (right)

TABLE I. EXPERIMENTAL RESULTS

	Stiffness (N.m.rad^{-1})	**Damping** (N.m.rad^{-1}s^{-1})
Shoulder	20 ÷ 30	2 ÷ 3
Elbow	70 ÷ 80	2 ÷ 4
Wrist	5 ÷ 10	2 ÷ 3

Two shooters selected for the experiment have the same physical conditions: 165 cm in height and 66 kg in mass, based on which we are able to estimate the shooter shoulder mass and arms parameters [4], [5]. The oscillation of shooter – weapon system is examined for two selected shooters corresponding to two different values of initial holding torques at the shooter – weapon system static equilibrium, represented by γ = 3400 N/m (for shooter No. 01) and γ = 3450 N/m (for shooter No. 02).

The force of shot and force of shot impulse diagrams are determined based on the knowledge of firing force acting on the barrel and the gas chamber, and the functional diagram of the weapon. The firing force is determined by known pressure curves in the barrel and gas chamber which are results of solution of the interior ballistics [1]. Then, the functional diagram is determined using known pressure curves.

System of differential equations written in form of Eq. (8), supplemented by calculation of the force of shot and its impulse, has been solved by numerical integration method.

In order to record the location of the gun barrel during single shots, we use two high-speed cameras Fastcam SA1.1, model 675K - C1 with the maximum frame rate 675 000 fr.s^{-1}. The used frequency of imaging 5400 fr.s^{-1} made it possible to obtain the record with the resolution 1024×1024 pixels. The first camera is located on the site to record the barrel vertical displacement (Fig. 9). The other one is to record the horizontal displacement. Its location is shown in Fig. 10.

Figure 9. Location of camera for barrel vertical displacement record

Figure 10. Location of camera for barrel horizontal displacement record

The experiment has been performed under temperature of 30°C and humidity of 70%. The measured data were processed using DASYLab motion analyzing system. Based on knowledge of the total length of the weapon, and the measured data of vertical and horizontal displacement of the gun barrel, we able to calculate the change in evaluation and traverse angle of the gun barrel, respectively.

The results of solution of evaluation and traverse angle in the case of single shot are compared with experimentally obtained data. The comparison is shown in Fig. 11, Fig. 12, Fig. 13 and Fig. 14. It can be seen that the presented mathematical model provides good agreement with experimental data.

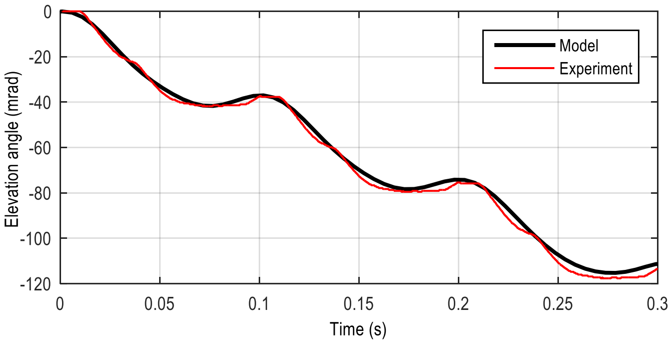

Figure 11. Evaluation angle vs. time for γ = 3400 N/m

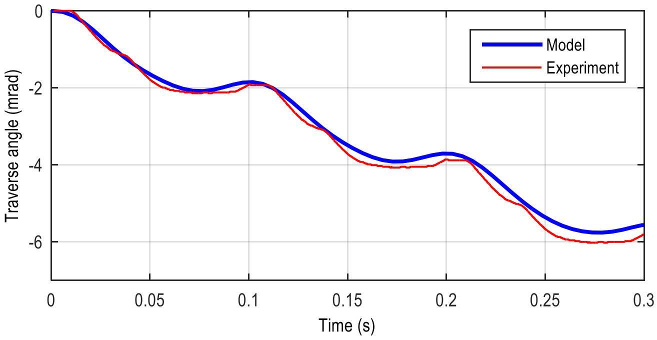

Figure 12. Traverse angle vs. time for γ = 3400 N/m

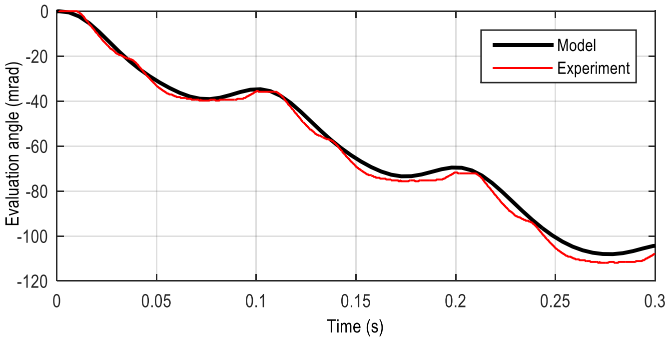

Figure 13. Evaluation angle vs. time for γ = 3450 N/m

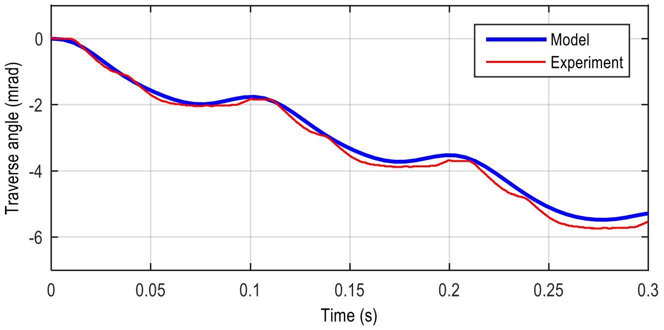

Figure 14. Traverse angle vs. time for γ = 3450 N/m

V. CONCLUSIONS

In this paper, the shooter – weapon system oscillation was analyzed by taking biomechanical model of shooter arms into consideration. The arms muscles and tendons forces are converted to joint torques at the shooter shoulder, elbow and wrist. Linking between shooter and the weapon is elastic and viscose damping. The dynamic model is formulated for a portable gas operated submachine gun and has thirteen degrees of freedom. Dynamics of the shooter – weapon system is examined in horizontal and vertical planes. Presented theory was solved using a numerical method and experimentally verified for the 7.62 mm AKM. The method can be used to analyze the shooter – weapon system parameters influencing on the weapon firing stability.

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