



22nd International Conference on Knowledge-Based and Intelligent Information & Engineering Systems

Autodriver Autonomous Vehicles Control Strategy

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Abstract

Technically, a given geometric curve can be traced and recovered by its curvature center loci. We assume that a road is given mathematically and we are able to determine its curvature center. To follow a road, we make a vehicle to turn about the road curvature center at the right radius of curvature. We design an autonomous vehicle control that follows a given path by adjusting the vehicle rotation center to be on the road curvature center. The dynamics of the vehicle cause not to follow the road and the right path of motion. It means that its rotation center will move off from the road curvature center. We therefore develop and introduce an autodriver algorithm control to compensate the possible errors between the desired location on the road and the actual location of the vehicle.

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Selection and peer-review under responsibility of KES International.

Keywords: Autonomous vehicle, Vehicle Dynamics, Vehicle Control, Road Curvature Center.

1. Introduction

Autonomous vehicle is a hot topic in research environment both in industry and academia since several years ago [1]. Such study also includes better road design [2-7] as well as designing more intelligent and safer vehicles [8-21]. Also a vast amount of study are conducted about social impact, regulation, policies, and implementation methods of Autonomous Vehicles [22].

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Many algorithms have been introduced to control a vehicle autonomously. Some rely on sensory systems, camera, and vision processing which are well functional for searching in unknown environments equipped with smart decision making of some sort of obstacle avoidance strategy [1]. Generally speaking, roads are well defined in a global coordinate frame attached to the ground. Such roads as geometrical curves will have pre-determined curvature center and curvature radius. In 2010 a novel autodriver algorithm has been introduced to present the idea and mathematically and proved the concept of the current investigation, [23]. The algorithm has been adopted, applied, and developed by other investigators [14,16,24]. Recalling that that any vehicle on a road is always in a turn about the curvature center of the road at that instant, determines that there are three point of interest: 1-road curvature center, which is a road geometrical information regardless of any vehicle moving on the road, 2-kinematic rotation center that is supposedly a unique point at the intersection of perpendicular to all wheels of the vehicle. 3-dynamic turning point which is the actual point the vehicle in motion will turn about. The position of the dynamic turning point is a function of vehicle dynamic properties of the vehicle as well as vehicle velocity. The Auto-driver algorithm is about determining the required steer angle and vehicle velocity to adjust the dynamic turning point on the road curvature center at that instant.

In this study we determine the main errors in application of the Autodriver algorithm and introduce a control strategy to eliminate the differences between vehicle coordinates and the desired location on the road.

2. Road Curvature Center

Consider a parametric space curve where s is the road length measured from a fixed point on the road.

$$x = x(s), y = y(s), z = z(s) \quad (1)$$

$$x_0 = x(0), y_0 = y(0), z_0 = z(0) \quad (2)$$

At any point of the road there are three planes: the perpendicular plane to the road, the osculating plane, and the rectifying plane. The osculating plane is the plane that includes the tangent line and the curvature center of the road at the points of interest. The rectifying plane is perpendicular to both the osculating and normal planes [26]. The curvature κ of a road at a point P and radius of curvature ρ are defined by

$$\kappa = \frac{1}{\rho} = \frac{\left| \frac{d^2 \mathbf{r}}{ds^2} \right|}{\left| \frac{d\mathbf{r}}{ds} \right|} = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^2} \quad (3)$$

where \mathbf{r} , \mathbf{v} , \mathbf{a} are the position, velocity, and acceleration vectors of the vehicle respectively. The equations to describe a road are much simpler in case of a two dimensional road laying on the (X, Y) ground. Consider a road with a given equation $Y=f(X)$ in a global coordinate frame G . The road radius of curvature R_κ at point of the road is:

$$R_\kappa = \frac{(1+Y'^2)^{3/2}}{Y''}, \quad Y' = \frac{dY}{dX}, \quad Y'' = \frac{d^2Y}{dX^2} \quad (4)$$

Figure 1 illustrates an example of a two dimensional road with equation and its road center loci.

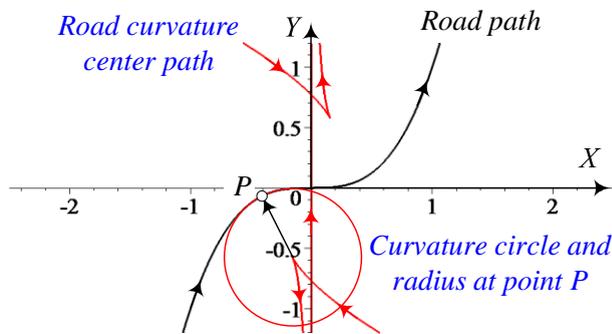


Figure 1. A curvature loci of a cubic curve .

3. Kinematic and Dynamic Vehicle Rotation Centers

Theoretically, the kinematic center of rotation of a front-wheel-steering vehicle should be on the extension of the rear axle. The kinematic center of rotation is the point that car tends to turn about at zero or very slow speed. The kinematic steering condition of a four-wheel front-wheel-steering vehicle is

$$\cot \delta_o - \cot \delta_i = \frac{w}{l} \tag{5}$$

where, δ_i and δ_o are the steer angle of the inner and outer wheels respectively. The distance between the steer axes of the steerable wheels is called the track w and the distance between the front and rear axles is called the wheelbase l [26, 14]. The kinematic radius of rotation of such vehicle, R , is measured as the distance between the kinematic center and the vehicle's mass center C where a_2 is the longitudinal distance between the rear axle and C , and δ is the cot-average of the inner and outer steer angles. The angle δ is the equivalent steer angle of a bicycle model of the vehicle having the same wheelbase l and radius of rotation R .

$$R = \sqrt{a_2^2 + l^2 \cot^2 \delta} \tag{6}$$

$$\cot \delta = \frac{\cot \delta_o + \cot \delta_i}{2} \tag{7}$$

Motion of a vehicle generates side slip that makes the vehicle to depart from its kinematic path. The departure depends on the speed, steer angle, geometric and dynamic properties of the vehicle, and tire-road interaction properties. However, keeping the steer angle and the speed of the vehicle constant cause the vehicle to follow another steady state circular path. The center of the steady state circle is the dynamic center of rotation [25, 20].

4. Dynamic Rotation Center

We attach a global coordinate frame G to the ground and a local coordinate frame B to the vehicle at the mass center C . The Z and z axes are assumed to remain parallel. The orientation of the frame B is indicated by the heading angle ψ between the x and X axes. The global position vector of the vehicle mass center is denoted by the location vector ${}^G\mathbf{d}$.

A planar vehicle model has three degrees of freedom: translation in the x and y directions, and a rotation about the z -axis. The equations of motion for a planar vehicle model in the body coordinate frame B are [26]:

$$\begin{aligned}\dot{v}_x &= \frac{F_x}{m} + r v_y \\ \dot{v}_y &= \frac{1}{m v_x} (-a_1 C_{\alpha f} + a_2 C_{\alpha r}) r - \frac{1}{m v_x} (C_{\alpha f} + C_{\alpha r}) v_y + \frac{1}{m} C_{\alpha f} \delta - r v_x \\ \dot{r} &= \frac{1}{I_z v_x} (-a_1^2 C_{\alpha f} - a_2^2 C_{\alpha r}) r - \frac{1}{I_z v_x} (a_1 C_{\alpha f} - a_2 C_{\alpha r}) v_y + \frac{1}{I_z} a_1 C_{\alpha f} \delta\end{aligned}\quad (8)$$

where $r = \dot{\psi} = \omega_z$, is the yaw rate of the vehicle. To simplify the analysis, we assume the vehicle to have a constant forward speed $v_x = \text{const}$, to make the steer angles as the only input of the dynamic system and the lateral speed v_y , yaw rate r are the output and traction force F_x as the outputs. By solving the the set of equations of motion we are able to determine v_y and r . Then by integrating v_x , v_y and r , we are able to calculate the position and orientation of the vehicle in both, body and global coordinate frames. The orientation of the vehicle is determined by integration the yaw rate

$$\psi = \int_0^t r dt \quad (9)$$

The velocity vector of the vehicle can be determined in the global coordinate frame using the rotation transformation matrix ${}^G R_B$.

$$\begin{bmatrix} v_X \\ v_Y \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} \quad (10)$$

The location of the vehicle is calculated by integration of the components of the global velocity plus the current location of the mass center.

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} + \begin{bmatrix} \int_0^t (v_x \cos \psi - v_y \sin \psi) dt \\ \int_0^t (v_x \sin \psi + v_y \cos \psi) dt \end{bmatrix} \quad (11)$$

The dynamic radius of curvature for the vehicle is:

$$R = \frac{v}{r} = \frac{\sqrt{v_x^2 + v_y^2}}{r} \approx \frac{v_x}{r} \sqrt{1 + \beta^2} \approx \frac{v_x}{r} \quad (12)$$

and the position of dynamic rotation center in the body and global coordinate frames are

$${}^B \mathbf{r}_c = \begin{bmatrix} R \sin \beta \\ R \cos \beta \end{bmatrix} \approx \begin{bmatrix} 0 \\ R \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} X_c \\ Y_c \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} + \begin{bmatrix} \int_0^t \left(v_x \cos \left(\int_0^t r dt \right) - v_y \sin \left(\int_0^t r dt \right) \right) dt \\ \int_0^t \left(v_x \sin \left(\int_0^t r dt \right) + v_y \cos \left(\int_0^t r dt \right) \right) dt \end{bmatrix} \quad (14)$$

The traction force as an output of the system is solely determined by the first equation of the motion employing the

outputs of the other two equations.

$$F_x = -m \frac{v_x}{r} \tag{15}$$

5. Autonomous Control

The planar bicycle model of a 4-wheel front-wheel-steering vehicle uses steer angle δ as the input to provide the lateral velocity v_y , yaw rate r , and traction force F_x as outputs. The equations of motion act as an interface box which uses the geometric characteristics of the vehicle a_1, a_2 , tire-ground dynamic parameters $C_{\alpha f}, C_{\alpha r}$, and forward velocity v_x as parameters. The outputs of the dynamic equations are being used in a set of integration and transformation to determine the actual location and orientation of the vehicle in on the ground. The outputs of the dynamic equations are being utilized in a set of integration and transformation (9) and (11) to determine the actual location and orientation of the vehicle in on the ground.

To control the vehicle, we must determine the required steer angle δ and speed v_x such that the vehicle actual position sits on the desired position on the road. The desired location coordinates will be fed backward into the set of kinematic transformation and integration to determine the associated vehicle kinematics variables r , and v_y . These variables are usually considered as outputs of the vehicle dynamics. They now must feed backward into the dynamic equation of the vehicle to determine the required input δ . In case the calculated δ is the right one then feeding it into the dynamic equations would locate the vehicle in the right position and the actual coordinates would be the desired ones. However, there is no straightforward step to solve differential equations backward. This situation is illustrated in Figure 2.

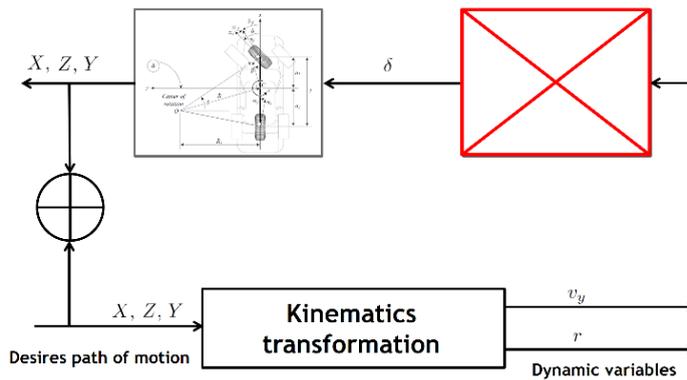


Figure 2. Reverse calculating the steer angle δ is not straightforward.

We need to introduce an approximation method to calculate δ . It has been shown that under normal conditions, vehicles are lazy and their transient response are not far from their steady state behavior [26,28,29]. Steady state condition of vehicles are governed by the same set of equations of motion after deleting all time derivatives operators. The result will be a set of algebraic equations that provide a quick way to calculate the steady state steer angle δ_{ss} . Therefore, we substitute the unknown box with the set of algebraic steady state equations to be able to calculate the steer angle to give us the required r and v_y assuming they are all at their steady state. The calculated steer angle will then be the input to the set of transient equations of motion to calculate the associated instant values of r and v_y . Then we use them to determine the actual position of the vehicle on the road. Considering the approximated value of the steer angle, we must compare the actual variables with the desired values and make a compensation control strategy [27,30].

Potentially there would be differences between X_d, X , and Y_d, Y as illustrated in Figure 3 for a lane change maneuver of the vehicle. The difference between lateral location needs an adjustment in the steer angle and the difference between longitudinal location needs an adjustment in the vehicle speed. The closed loop control of the system is illustrated in Figure 4.

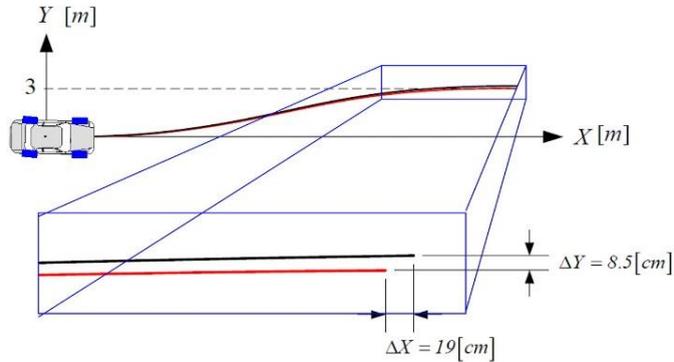


Figure 3. There are a longitudinal and a lateral position errors in any planar manoeuvre

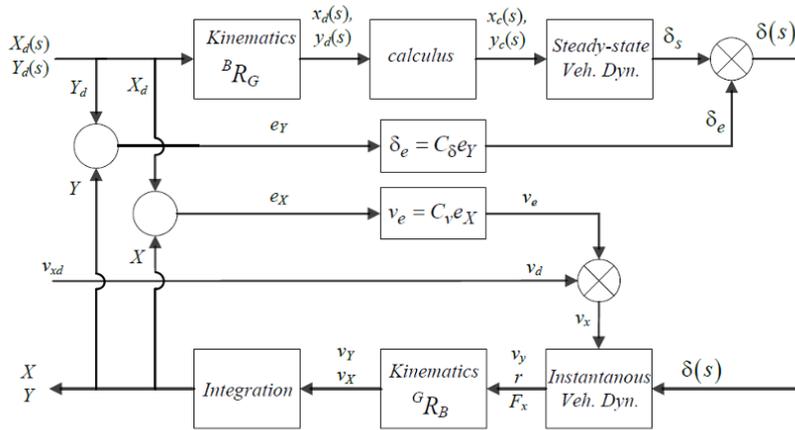


Figure 4. The control loop of the autodrivre autonomous vehicle.

6. Conclusion

Assuming a road can be expressed by a given mathematical function in a global coordinate frame attached to the ground, we can determine the road curvature center locations. The center is supposed to be the rotation center of the car. The actual turning point of a vehicle would be different than the kinematic and road curvature center due to dynamic behavior of the car.

This investigation presents the application of Autodrivre algorithm for autonomous vehicles. The theory was developed originally based on 4 wheel steering vehicles. However, the current investigation is applying the theory on front-wheel steering vehicles.

To follow a given path we adjust the center of rotation of the vehicle to coincide with the road curvature location. The side slip of the vehicle as a result of dynamic behavior of vehicles deviate the path of the vehicle from the expected path to be followed by the given steer angle and velocity.

The errors appeared by the dynamic effects of the vehicle have been determined and discussed to introduce a control strategy for elimination of the errors.

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