Output feedback Controller using High-Gain Observer in Multi-Motor Drive Systems

Dao Phuong Nam, Pham Tuan Thanh, Tran Xuan Tinh

***Abstract*— Multi-motor drive systems have been widely used in many modern industry. It is a nonlinear, multi-input multi-output (MIMO) and strong-coupling complicated system, including the effect of friction and elastic. The control law for this dive system based on the determining of the tension. However, it is hard to obtain this tension in practice by using a load cell or a pressure meter due to the accuracy of sensors or external disturbance. In order to solve this problem, a high-gain observer is proposed to estimate the state variables in this drive system, such as speeds, tension. An emerging proposed technique in the control law is the use of high-gain observers together with state feedback control to obtain a separation principle for the stabilization of whole system. The theory and simulation results point out the good performance of the proposed output feedback for the drive system.**

***Keywords* – High-gain observer, Multi-motor drive systems, Sliding mode control, Tension, Output feedback Controller.**

I. INTRODUCTION

Multi-motor drive systems have been researched by many researchers in the recent times. The Control law based on neural network technique have been proposed by Yaoji Me et al. (2013) (see [1], [2], [3], [4], for examples). However, it is hard to find the corresponding networks as well as learning rules. Besides, the model of this system is approximately described as a linear system to use the transfer function to design the control law. Furthermore, the tracking ability or the stabilization of the whole system are not still solved under the effects of observer using neural network technique. In the multi-motor drive control systems, it is necessary to obtain the belt tension to design the suitable state feedback control law. However, it is hard to measure this belt tension by using sensors, … and the observer based on high gain technique is proposed in our work. Besides, the state feedback control design based on sliding mode control technique enables to remove efficient of disturbance and uncertainties. Therefore, a high-gain observer is proposed to estimate the tension in this system and combine with the state feedback controller to obtain the output feedback control law satisfied the separation principle. The stability of whole system is obtained by the output feedback control law and verified by theory, simulations.

This work is composed of five sections. In Section II, the problem statements is shown. Section III describes the output feedback control design. Then the high gain observer for multi – motor system is explained. Next, the sliding mode control of this system and the ability to satisfy the separation principle of output feedback controller are discussed. In Section IV, simulation results are shown. The conclusion is summarized in Section V.

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II. PROBLEM STATEMENTS

In [1], the multi-motor system (in Fig. 1) using two induction motor is described by the following dynamic equation (1) and the nomenclatures used in these equations are summarized in Table 1:

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{n_{p1}}{J_1}[(\mathrm{u}_1 - x_1)\dfrac{n_{p1}T_{r1}}{L_{r1}}\varphi_{r1}^2 - (\mathrm{T}_{L1} + \mathrm{r}_1 .x_3)] \\ \dfrac{dx_2}{dt} = \dfrac{n_{p2}}{J_2}[(\mathrm{u}_2 - x_2)\dfrac{n_{p2}T_{r2}}{L_{r2}}\varphi_{r2}^2 - (\mathrm{T}_{L2} - \mathrm{r}_2 .x_3)] \\ \dfrac{dx_3}{dt} = \dfrac{K}{T}(\dfrac{1}{n_{p_1}}\mathrm{r}_1\mathrm{k}_1 x_1 - \dfrac{1}{n_{p_2}}\mathrm{r}_2\mathrm{k}_2 x_2) - \dfrac{x_3}{T} \end{cases} \tag{1}$$

Where:

$x = \left[x_1, x_2, x_3\right]^T = \left[\omega_{r1}, \omega_{r2}, F\right]^T$ is state variable;

$u = (\mathrm{u}_1, \mathrm{u}_2) = (\omega_1, \omega_2)$ is control variable;

$y = \omega_{r1}$ is output variable;

The control objective is to find the synchronous speeds $u = (\mathrm{u}_1, \mathrm{u}_2) = (\omega_1, \omega_2)$ to obtain that the desired value are tracked by each motor speeds and tensions in presence of friction and elastic.

TABLE I. DYNAMIC PARAMETER

$K = E/V$	Transfer function
E	Young's Modulus of belt
V	Expected line velocity
$T = \dfrac{L_0}{AV}$	Time constant of tension variation
L_0, A	Distance between racks, Section area (m^2)
n_{pi}	Number of pole-pairs in the ith motor
J	Inertia moment of rotor (kgm^2)
T_L, φ_r	Load torque (Nm), Flux of rotor (Wb)
L_r	Self-induction of rotor (H)
$\mathrm{r}, k, \omega_r, \omega, F$	Radius of roller, velocity ratio, electric angle velocity of rotor, angle velocity of stator, belt tension

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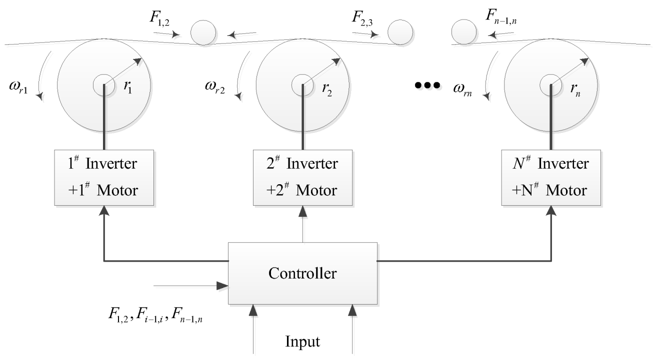

Figure 1. The Control System of Multi-Motor Drives [2]

III. OUTPUT FEEDBACK CONTROL DESIGN

In this section, a new scheme is proposed to design a output feedback controller involving a high gain observer and a sliding mode control law. Moreover, the ability to satisfy the separation principle is pointed out in multi-motor control system.

A. High Gain Observer of Multi-Motor Systems

As mentioned above, the main motivation of the work is to find a suitable high gain observer for the class of multi motor systems. In the following, one will present the proposed high gain observer to estimate the tension in this system and provide a full analysis of observation error convergence.

Consider MISO systems are described as follows:

$$\begin{cases} \dfrac{d}{dt}x = Ax + \gamma(x,u,y) + \varphi(u,y) \\ y = c^T x + \xi(u) \end{cases} \tag{2}$$

Where $\gamma(x,u,y)$ satisfy the global Lipschitz condition $\left|\gamma(x,u) - \gamma(\hat{x},u)\right| \le \alpha \left|x - \hat{x}\right|$ and $A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & & 0 \\ & & & \dots & \vdots \\ & & & 0 & 1 \\ 0 & 0 & & \dots & 0 \end{bmatrix}$,

$c^T = [1, \ 0, \ \dots \ 0]$

Lemma 1 [5]: The classical high gain observer is pointed out by the following equations:

$$\frac{d}{dt}\hat{x} = A\hat{x} + L\left(y - c^T\hat{x}\right) + \gamma(\hat{x},u) \ (3) \text{ where } L = \begin{bmatrix} h_1\varepsilon^{-1} \\ \vdots \\ h_n\varepsilon^{-n} \end{bmatrix} \text{ and}$$

ε is a small enough positive number and $h_n, h_{n-1}, \dots, h_1$ are coefficients of a Hurvith polynomial (4)

$$P(s) = h_n + h_{n-1}s + \dots + h_1 s^{n-1} + s^n \tag{4}$$

Remark 1:
The classical high gain observer is the next development of Lipschitz observer with the additional contents of the coefficient ε to obtain $a < \dfrac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}$ without solving the LMIs problem.

Lemma 2. We obtain the high-gain observer for multi-motor systems (1) based on the equation (3) (in Lemma1) with:

$$\gamma(x,u) = \begin{bmatrix} a_{11} & a_{12}-1 & a_{13} \\ a_{21} & a_{22} & a_{23}-1 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} x + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \\ 0 & 0 \end{bmatrix} u + \begin{bmatrix} k_1 \\ k_2 \\ 0 \end{bmatrix} \tag{5}$$

$$\begin{aligned} & a_{11} = -\frac{n_{p1}^2 T_{r1}\varphi_{r1}^2}{L_{r1}}, a_{12} = a_{21} = 0, a_{13} = -\frac{n_{p1}r_1}{J_1}, \\ & a_{22} = \frac{n_{p2}^2 T_{r2}\varphi_{r2}^2}{L_{r2}}, a_{23} = \frac{n_{p1}r_1}{J_1}, a_{31} = \frac{r_1 k_1 K}{n_{p1}T}, \\ & a_{32} = -\frac{r_2 k_2 K}{n_{p2}T}, a_{33} = -\frac{1}{T}, b_{11} = \frac{n_{p1}^2 T_{r1}\varphi_{r1}^2}{L_{r1}}, \\ & b_{22} = \frac{n_{p2}^2 T_{r2}\varphi_{r2}^2}{L_{r2}}, k_1 = -\frac{n_{p1}T_{L1}}{J_1}, k_2 = -\frac{n_{p2}T_{L2}}{J_2} \end{aligned} \tag{6}$$

is proposed by Khalil [5] with the advantages in selecting the convergence speed of observer errors.

Proof:

The multi-motor system (1) is one of the system satisfied the dynamic equation (2). Moreover, the global Lipschitz condition is obtained because there exists $\alpha \ge 0$ satisfy:

$$\left|\gamma(x,u) - \gamma(\hat{x},u)\right| = \left\| \begin{bmatrix} a_{11} & a_{12}-1 & a_{13} \\ a_{21} & a_{22} & a_{23}-1 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} (x - \hat{x}) \right\| \le \alpha \left|x - \hat{x}\right|$$

B. Sliding Mode Control of Multi-Motor Systems

In this section, the main work is to find a state feedback control law based on the sliding mode control technique for the class of multi motor systems.

Consider nonlinear systems are described as follows:

$$\frac{d}{dt}x = Ax + B\left(u + u_d(x,t)\right) \tag{7}$$

Where $u_d(x,t)$ is the nonlinear term in system.

Lemma 3 [6]: The sliding mode controller is described as follows $u = -\left[S.Ax + \beta \operatorname{sgn}(\sigma)\right]$ (8)

based on the sliding surface:
$$\begin{aligned} & \{x : \sigma = Sx = 0\}, \\ & S = \left(B^T X^{-1} B\right)^{-1} B^T X^{-1} X \end{aligned} \tag{9}$$

With X is satisfied the LMI problem as follows:

$$\Pi^T \left(AX + XA^T\right)\Pi < 0, X > 0, \Pi = [1 \ \ 0 \ \ 0]^T \tag{10}$$

Remark 2. We obtain the sliding mode control for multi-motor systems (1) based on Lemma 3 because it belong to the class of systems (7)

C. Output Feedback Control Design of Multi-Motor Systems

After we obtain the output feedback control law combined between sliding mode controller and high gain observer, the main work is to point out the ability to obtain the separation principle of the proposed solution.

Consider the nonlinear systems:

$$\begin{cases} \dfrac{d}{dt}x = Ax + f(x,u,t) \\ y = Cx \end{cases} \tag{11}$$

With $f(x,u,t)$ satisfies the global Lipschitz condition

$$\left|f(x,u,t) - f(x',u,t)\right| \le a\left|x - x'\right|\left(\forall x, x', u\right) \tag{12}$$

Lemma 4 [5]: If there exists a control Lyapunov function $V(x)$ and the corresponding control input $u = r(x)$ satisfy

$$\left|\frac{\partial V}{\partial \underline{x}}\left[f(\underline{x},u,t) - f(\underline{x}',u,t)\right]\right| \le b\left|x - x'\right|^2, \forall x, x' > 0 \tag{13}$$

Then the output feedback control law using the observer (14, 15) and the state feedback controller $u = r(x)$ is described as above:

$$\frac{d\hat{x}}{dt} = A\hat{x} + f(\hat{x},u,t) + L(y - C\hat{x}) \tag{14}$$

Where L is the matrix satisfies all the real parts of eigenvalues of $(A - LC)$ is negative and matrices P, Q satisfy the Lyapunov equation

$$(A - LC)^T P + P(A - LC) = -Q \tag{15}$$

and
$$a < \frac{\lambda_{\min}(\mathrm{Q}) - \mathrm{b}}{2\lambda_{\max}(P)} \tag{16}$$

Theorem 1. The whole system (Fig. 1) is asymptotically stable by the output feedback control law with the high-gain observer (3, 4) and the nonlinear state feedback controller (8).

Proof: Using the Lyapunov candidate function $V(x) = x^T Px$, we obtain the inequality (13) based on x is the state trajectory of multi motor system (1).

Remark 3. This result is a development from the results in [1] – [4], because the separation principle of output feedback controller have not been implemented in previous researches.

IV. SIMULATION RESULTS

In this section, we consider several simulation results to demonstrate the ability of the proposed output feedback control law based on the multi motor system as table 2:

TABLE II. MULTI-MOTOR SYSTEM PARAMETER

n_{p1}	4
J_1	50 Kgm^2
L_{r1}	0,2 H
T_{L1}	30 Nm
n_{p2}	4
J_2	55 Kgm^2
L_{r2}	0,3 H
T_{L2}	25 Nm

A. Simulation Results for High Gain Observer

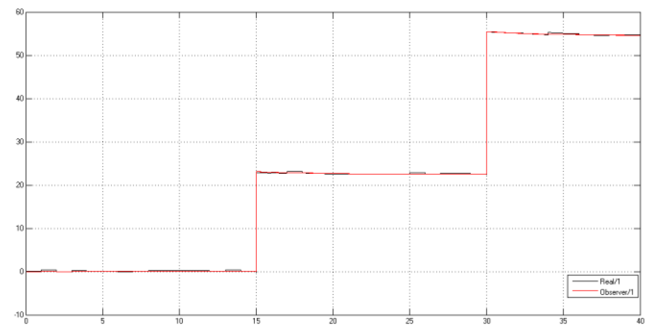

Figure 2. Transient response of the error of speed (m/s) vs. time (s) of the first motor using high gain observer

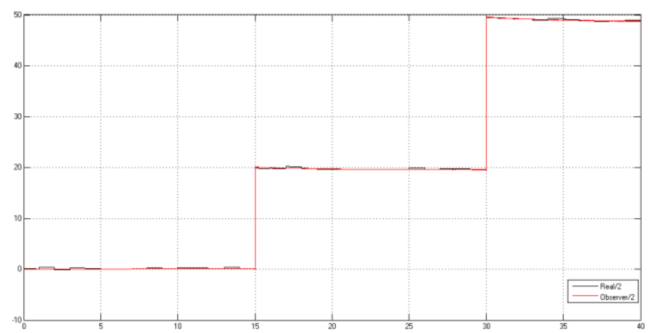

Figure 3. Transient response of the error of tension (N) vs. time (s) of the second motor using high gain observer

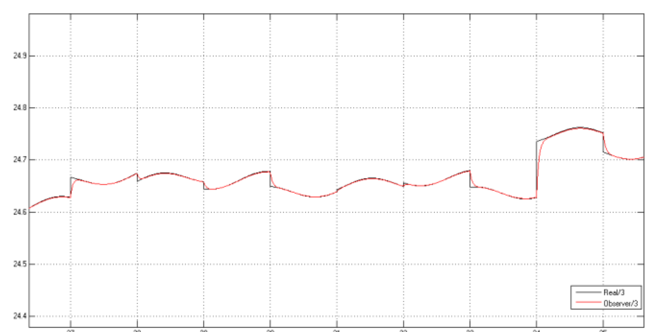

Figure 4. Transient response of the error of tension (N) vs. time (s) of the second motor using high gain observer

Fig. 2,3 show the high performance behaviour of the proposed high gain observer. Moreover, we obtain the high tracking perfomance of tension in presence of friction and elastic (fig. 4).

B. Simulation Results for Output Feedback Controller

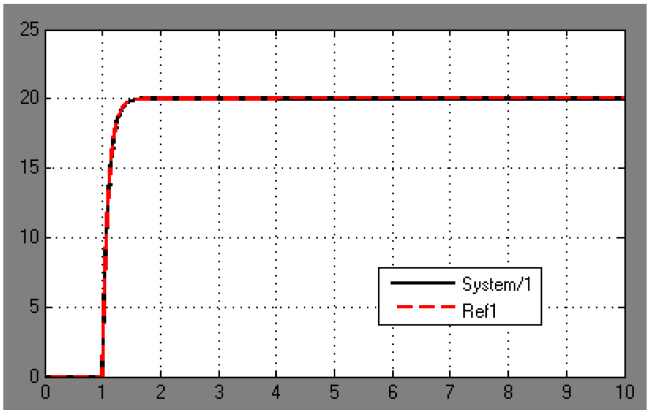

Figure 5. Transient response of the speed (rad/s) vs. time (s) of the first motor using the proposed output feedback control

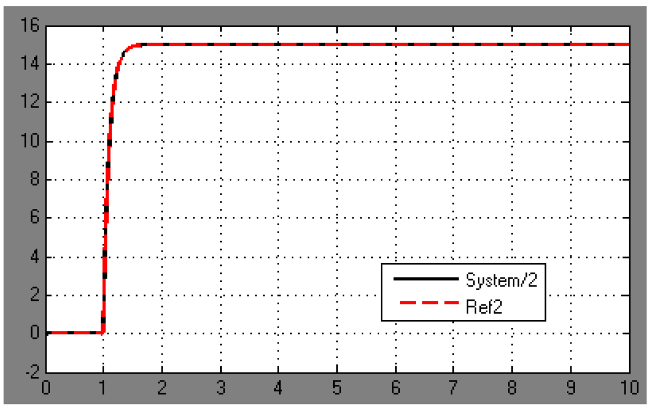

Figure 6. Transient response of the speed (rad/s) vs. time (s) of the second motor using the proposed output feedback control

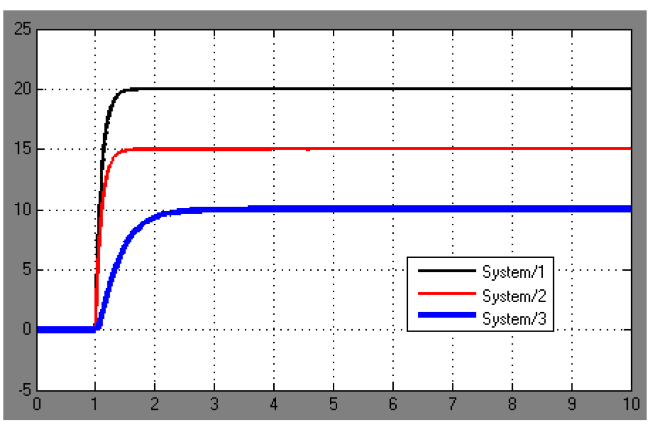

Figure 7. Transient response of tension (N) vs. time using the proposed output feedback control

Based on the proposed output feedback control law, we obtain the good behavior of the speeds and tension (Fig. 5,6,7). Fig.7 shows the tension response although the speed difference between the two motors.

V. CONCLUSION

This paper described an output feedback control law based on the combination between high gain observer and sliding mode control. The proposed control law enables to obtain the separation principle in presence of friction and elastic. The effectiveness of the proposed control scheme was pointed out by theory and simulation results.

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