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Cognitive two-way relay systems with multiple primary receivers: exact and asymptotic outage formulation

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Abstract: This study is concerned with outage performance analysis of a cognitive two-way relay system having a primary network of one transmitter and multiple receivers over Nakagami-*m* fading channels. In the considered cognitive radio system, the secondary network facilitates information exchange between source nodes with the aid of multiple decode-and-forward two-way relays. In the presence of multiple relays, opportunistic relay selection is employed to obtain a relay with the best channel quality for reliable information recovery. To analyse the performance of the considered cognitive radio system, exact and asymptotic closed-form expressions of outage probability (OP) are derived in this study. The derived theoretical OP expressions are in good agreement with related empirical values which show that using relays can enhance the performance of secondary networks without sacrificing that of the primary network.

1 Introduction

Wireless multiple access has been extended from the frequency dimension in the first generation (1G) of mobile communications to the space dimension in 4G [1]. For 5G, a new multiple access dimension has not been standardised yet and existing ones will be exploited in more flexible and efficient fashions. Over the frequency dimension, a more dynamic and intelligent alternative of frequency division multiple access (FDMA) and orthogonal FDMA is the principle of cognitive radio communications, where multiple secondary (unlicenced) networks and a primary (licenced) network can simultaneously access a common frequency band [2].

To maintain co-channel interference power below a tolerable level in the licenced network, transmit signal powers in unlicenced networks are limited to pre-determined thresholds. This in turn reduces the coverage of unlicenced networks [3]. To extend the coverage of unlicenced networks without sacrificing the licenced network's performance, relaying nodes can be employed for adding more space diversity to wireless links between source nodes [3, 4]. In particular, one-way relays can be used in secondary networks and the resulting diversity gains have been verified by theoretical and empirical values of outage performance [5]. However, diversity gains of adding one-way relays to secondary networks come at the expense of doubling the number of time slots used for one round of information exchange between source nodes. This results in a significant spectral efficiency loss in secondary networks.

To alleviate the spectrum efficiency loss while maintaining diversity gain, two-way relaying [6–10] can be deployed in secondary networks as an alternative of one-way relaying. In particular, Hatamnia *et al.* [6] considered a cognitive radio system having a secondary network with a single two-way relay node. Exact closed-form expressions of outage probability (OP)/symbol error rate and their lower bounds were derived to analyse the performance of the considered cognitive system over Nakagami-*m* fading channels. Unlike [6] considering a single two-way relay, Duy and Kong [7] addressed a cognitive radio system using multiple two-way relay nodes in the secondary network. Opportunistic relay selection is used to obtain a relay with the best channel quality for information recovery in the system. The system outage performance was analysed by using the derived closed-form OP expression and its related empirical values [7]. Addressing the problem of power allocation for multiple two-way relays, Ubaidulla and Aissa [8] proposed an optimal scheme for power allocation and relay selection that maximises the system throughput under constraints on the secondary transmit power and primary users' interference. Unlike [7, 8] analysing performance metrics of OP and system throughput, Ben Fredj and Aissa [9] derived the closed-form expression of average bit error rate in a cognitive two-way relay system over Rayleigh fading channels.

Different from [6–9] considering only co-channel interference from secondary networks to primary receivers, Zhang *et al.* [10] investigated a cognitive two-way relay system with co-channel interference from a primary transmitter to secondary receivers. More specifically, exact and asymptotic expressions of the system OP were derived to analyse the effect of the mutual interference between the primary and secondary networks on the diversity order. Considering a more general model than [10–12] evaluated the system quality having many primary transceivers and derived an optimal factor that creates zero-forcing beamforming [11, 12]. However, these analyses are only valid when the numbers of the secondary relay nodes are more than the number of the primary receivers. Considering a two-way cognitive system in the presence of multiple primary users [13], the authors derive closed-form expressions for the user OP. In [14], Nguyen *et al.* analyse the impact of transceiver impairments on OP two-way cognitive decode-and-forward (DF) relay network, in which the relay node is self-powered by harvesting energy from the transmitted signals of two terminal nodes.

Unlike existing studies [6–14] on performance analysis of cognitive relay systems, the main contribution of this manuscript is the consideration of multiple secondary two-way relays and opportunistic relay selection in formulating the outage performance of a secondary network over Nakagami-*m* fading channels. The considered model in this paper can be used for a cognitive radio system consisting of (i) multiple unlicenced networks and (ii) a licenced cellular network of one base station and multiple mobile stations (receivers) simultaneously sharing a common spectrum band. More specifically, this paper derives exact and asymptotic OP expressions in the secondary network having multiple two-way relays over Nakagami-*m* fading channels under a given constraint on co-channel interference power from the secondary network to

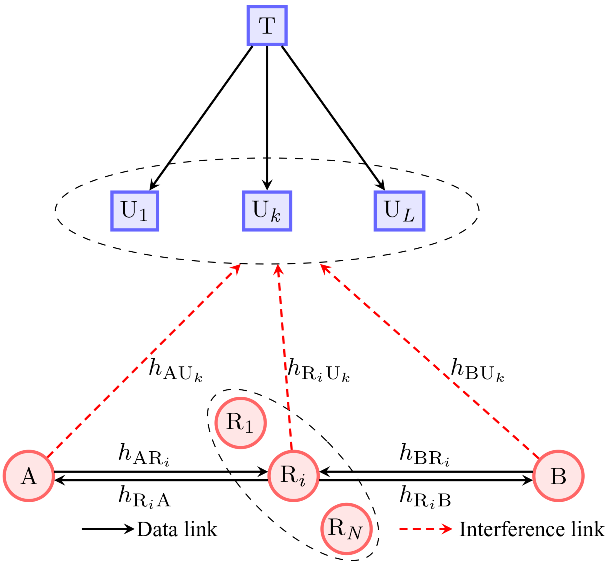

Fig. 1 *Cognitive two-way relay network with multiple primary receivers*

multiple receivers in the primary network. Moreover, the outage analysis in this manuscript is valid without any restriction on the number of secondary relay nodes or primary receivers. The derived theoretical OP expressions and related empirical values are in good agreement which reveals several insights into effects of network settings and power constraints on the performance of cognitive two-way relay systems.

The remaining parts of this paper are organised as follows. Section 2 introduces the considered cognitive two-way relay system model. Detailed derivations of exact and asymptotic OP expressions for analysing the considered system performance are presented in Section 3. Section 4 provides numerical results of the derived theoretical OP and related empirical ones to evaluate the outage performance of the considered cognitive radio system. Finally, Section 5 concludes this paper.

2 System model

In the considered cognitive radio system, the secondary network consists of two source nodes (denoted by A and B) and N relays (denoted by R_i with $i = 1, 2, \ldots, N$). Sharing the same frequency band with the secondary network, the primary network has a primary transmitter denoted as T and L receivers denoted by U_k with $k = 1, 2, \ldots, L$ as illustrated in Fig. 1. Like [15–17], this manuscript assumes the distance between the primary transmitter and the secondary network is far enough, so that the interference from the primary network to the secondary network can be negligible. In the considered system, it is assumed that there is no direct link between node A and node B due to severe shadowing and channel path loss. This paper assumes that each node in the system is equipped with a single antenna.

Wireless channels are assumed to be Nakagami-m flat fading. h_{XY} represents the channel gain of the $\mathsf{X} \to \mathsf{Y}$ link, where $\mathsf{X} \in \{\mathrm{A}, \mathrm{B}, \mathrm{R}_i\}$ and $\mathsf{Y} \in \{\mathrm{A}, \mathrm{B}, \mathrm{R}_i, \mathrm{U}_k\}$. More specifically, h_{XY} is a random variable having gamma distribution with parameters m_{XY} and Ω_{XY}. As a result, the cumulative distribution function (CDF) and probability density function (PDF) of $|h_{\mathsf{XY}}|^2$ can be written, respectively, by

$$F_{|h_{\mathsf{XY}}|^2}(z) = 1 - \frac{\Gamma(m_{\mathsf{XY}}, \alpha_{\mathsf{XY}} z)}{\Gamma(m_{\mathsf{XY}})} \tag{1}$$

and

$$f_{|h_{\mathsf{XY}}|^2}(z) = \frac{\alpha_{\mathsf{XY}}^{m_{\mathsf{XY}}}}{\Gamma(m_{\mathsf{XY}})} z^{m_{\mathsf{XY}} - 1} \mathrm{e}^{-\alpha_{\mathsf{XY}} z}, \tag{2}$$

where $\alpha_{\mathsf{XY}} = m_{\mathsf{XY}} / \Omega_{\mathsf{XY}}$.

Powers of transmit signals from nodes in the secondary network are properly adjusted, so that co-channel interference powers at primary receivers U_k do not exceed the peak power denoted by $\widetilde{I}_{\mathrm{p}}$. It is assumed that the maximum allowable transmit signal power at nodes in the secondary network is denoted by $\widetilde{\mathscr{P}}_{\mathrm{m}}$. The powers of transmit signals from node A, node B and relay R_i are denoted by P_{A}, P_{B} and P_{R_i}, respectively. Under the power constraints, the transmit signal powers of node X, $\mathsf{X} \in \{\mathrm{A}, \mathrm{B}, \mathrm{R}_i\}$, in the secondary network can be determined as follows:

$$P_{\mathsf{X}} = \min\left(\widetilde{\mathscr{P}}_{\mathrm{m}}, \frac{\widetilde{I}_{\mathrm{p}}}{\max_{k = 1, 2, \ldots, L} |h_{\mathsf{X}\mathrm{U}_k}|^2}\right). \tag{3}$$

In the considered secondary network, one round of information exchange between node A and node B consists of three phases. In the first phase, node A transmits its signal o all relay nodes. In the second phase, node B transmits its signal to all relay nodes. The instantaneous signal-to-noise ratios (SNRs) at the ith relay node in the first and second phases can be written, respectively, by

$$\gamma_{\mathrm{AR}_i} = |h_{\mathrm{AR}_i}|^2 \min\left(\mathscr{P}_{\mathrm{m}}, \frac{I_{\mathrm{p}}}{\max_{k = 1, 2, \ldots, L} |h_{\mathrm{AU}_k}|^2}\right), \tag{4}$$

and

$$\gamma_{\mathrm{BR}_i} = |h_{\mathrm{BR}_i}|^2 \min\left(\mathscr{P}_{\mathrm{m}}, \frac{I_{\mathrm{p}}}{\max_{k = 1, 2, \ldots, L} |h_{\mathrm{BU}_k}|^2}\right), \tag{5}$$

where $\mathscr{P}_{\mathrm{m}} = \widetilde{\mathscr{P}}_{\mathrm{m}} / \sigma^2$, $I_{\mathrm{p}} = \widetilde{I}_{\mathrm{p}} / \sigma^2$ and σ^2 is the variance of noise at the receivers.

At the end of the second phase, all relays will decode the received signals from A and B using selective DF. The set of relays successfully decoding signals received from A is denoted by $\mathscr{R}_{\mathrm{A}}$ and that from B is denoted by $\mathscr{R}_{\mathrm{B}}$. The set of relays successfully decoding signals received from both A and B is denoted by $\mathscr{R} = \mathscr{R}_{\mathrm{A}} \cap \mathscr{R}_{\mathrm{B}}$.

In the third phase, the SNRs at nodes A and B, denoted by $\gamma_{R_i\mathrm{A}}$ and $\gamma_{R_i\mathrm{B}}$, respectively, can be determined as follows:

$$\gamma_{\mathrm{R}_i\mathrm{A}} = \frac{|h_{R_i\mathrm{A}}|^2 P_{\mathrm{R}_i}}{\sigma^2} \tag{6}$$

and

$$\gamma_{\mathrm{R}_i\mathrm{B}} = \frac{|h_{R_i\mathrm{B}}|^2 P_{\mathrm{R}_i}}{\sigma^2}. \tag{7}$$

In the presence of multiple relaying nodes, the motivation of selecting the most suitable relay with the best channel quality leads to the principle of an opportunistic relay selection scheme which provides the system with additional diversity gain. As compared to exhaustive relay selection which finds an optimal subset of relay nodes among all existing relays for optimising a certain system performance metric (e.g. capacity, OP, signal-to-interference-plus-NR, …) [18, 19], opportunistic relay selection offers a much lower complexity at the expense of poorer performance.

Using an opportunistic relay selection scheme, a selected relay denoted by R_{i^*} will broadcast its encoded signal toward nodes A and B. In particular, the selected relay R_{i^*} is obtained as follows [10]:

$$i^* = \arg \max_{i = 1, 2, \ldots, n} \gamma_{R_i}, \tag{8}$$

where n is the cardinality of $\mathscr{R}$ (i.e. $n = |\mathscr{R}|$) and

$$\gamma_{R_i} = \min\left(\gamma_{R_i\mathrm{A}}, \gamma_{R_i\mathrm{B}}\right). \tag{9}$$

Using the fact that γ_{R_i} are independent on each other, we can write $\gamma_{R_{i^*}}$ as

$$\gamma_{R_{i^*}} = \max_{i=1,\ldots,n} \gamma_{R_i}. \tag{10}$$

In the considered cognitive radio system, it is assumed that channel state information (CSI) of wireless links is perfectly known and perfect CSI can be exchanged among nodes in the considered system via error-free zero-latency connections. Relay selection can be performed by a distributed or centralised manner [20, 21]. This paper considers a distributed process of relay selection as follows. On the basis of the known CSI, all n relay nodes calculate the channel quality in the third time slot. In particular, the ith node will calculate $\gamma_{R_i} = \min(\gamma_{R_i\mathrm{A}}, \gamma_{R_i\mathrm{B}})$. Then, in each relay node, a countdown timer is activated from an initial value which is determined by $T_i = 1/\gamma_{R_i}$. As a result, for a relay node having better channel quality, its initial value T_i is smaller and its countdown timer quickly approaches to zero value. When a countdown timer of a relay reaches to zero value that relay will transmit signal to the other relays, so that they know that the secondary network has already selected the best relay. At the same time, other relays are switched to a receive mode for the next selection process. For a mathematical representation, the opportunistic relay selection can be expressed by (8) and (9).

In a cognitive relay system, one round of information exchange between nodes A and B in a secondary network with conventional one-way relays has four time slots. Unlike the conventional one-way relays, two-way relays for an information exchange round between A and B using three time slots are considered in this paper. Under the considered opportunistic relay selection, during each time slot (either in the conventional one-way relaying or in the considered two-way relaying), only one node (A or B or R_i) transmits signal whose power is bounded, so that corresponding co-channel interference powers at primary receivers do not exceed the peak power $\tilde{I}_\mathrm{p}$. As a result, adding conventional one-way or two-way relay nodes in the secondary network does not affect the performance of the primary network. In addition, adding relays to the secondary network produces an additional diversity gain which translates into performance enhancement of the secondary network. Hence, conventional one-way or two-way relays can enhance the performance of the secondary networks without degrading that of the primary network.

3 OP formulation

3.1 Exact expression of OP

This section derives a closed-form expression of OP in the secondary network over Nakagami-m fading channels. It is noted that the number of successfully decoding nodes in the first phase is a discrete random number. As a result, the probability of the event $|\mathscr{R}_\mathrm{A}| = n_\mathrm{A}$ with $n_\mathrm{A} \in \{0, 1, 2, \ldots, N\}$ is computed as follows:

$$\begin{aligned}\Pr(|\mathscr{R}_\mathrm{A}| = n_\mathrm{A}) \\ = \binom{N}{n_\mathrm{A}} \Pr\left[\bigcap_{i\in\mathscr{R}_\mathrm{A}} (\gamma_{\mathrm{AR}_i} \ge \gamma_\mathrm{th}), \bigcap_{i\notin\mathscr{R}_\mathrm{A}} (\gamma_{\mathrm{AR}_i} < \gamma_\mathrm{th})\right].\end{aligned} \tag{11}$$

Since all wireless channel links are assumed to be statistically independent, one can have

$$\begin{aligned}\Pr(|\mathscr{R}_\mathrm{A}| = n_\mathrm{A}) \\ = \binom{N}{n_\mathrm{A}} \left[1 - F_{\gamma_{\mathrm{AR}_i}}(\gamma_\mathrm{th})\right]^{n_\mathrm{A}} \left[F_{\gamma_{\mathrm{AR}_i}}(\gamma_\mathrm{th})\right]^{N-n_\mathrm{A}},\end{aligned} \tag{12}$$

where γ_th is the outage SNR threshold and $F_{\gamma_{\mathrm{AR}_i}}(\gamma)$ denotes the CDF of γ_{AR_i}.

Similar to the above computations for the first phase, $\Pr(|\mathscr{R}_\mathrm{B}| = n_\mathrm{B})$ for the second phase can be determined by

$$\begin{aligned}\Pr(|\mathscr{R}_\mathrm{B}| = n_\mathrm{B}) \\ = \binom{N}{n_\mathrm{B}} \Pr\left[\bigcap_{i\in\mathscr{R}_\mathrm{B}} (\gamma_{\mathrm{BR}_i} \ge \gamma_\mathrm{th}), \bigcap_{i\notin\mathscr{R}_\mathrm{B}} (\gamma_{\mathrm{BR}_i} < \gamma_\mathrm{th})\right] \\ = \binom{N}{n_\mathrm{B}} \left[1 - F_{\gamma_{\mathrm{BR}_i}}(\gamma_\mathrm{th})\right]^{n_\mathrm{B}} \left[F_{\gamma_{\mathrm{BR}_i}}(\gamma_\mathrm{th})\right]^{N-n_\mathrm{B}}.\end{aligned} \tag{13}$$

At the end of the second phase, a selected relay is obtained from the intersection of two sets $\mathscr{R}_\mathrm{A}$ and $\mathscr{R}_\mathrm{B}$. Let us define $n = |\mathscr{R}|$, then we have $n \le n_\mathrm{A}$, $n \le n_\mathrm{B}$. For the sake of simplicity and without loss of generality, it is assumed that

$$\mathscr{R}_\mathrm{A} = \{R_1, R_2, \ldots, R_n, R_{n+1}, \ldots, R_{n+t_\mathrm{A}}\}, \tag{14}$$

$$\mathscr{R}_\mathrm{B} = \{R_1, R_2, \ldots, R_n, R_{n+t_\mathrm{A}+1}, \ldots, R_{n+t_\mathrm{A}+t_\mathrm{B}}\}, \tag{15}$$

$$\mathscr{R} = \{R_1, R_2, \ldots, R_n\}, \tag{16}$$

where $0 \le t_\mathrm{A} \le N - n$ and $0 \le t_\mathrm{B} \le N - n - t_\mathrm{A}$.

For a given combination of $(n, t_\mathrm{A}, t_\mathrm{B})$, we shall have $\binom{N}{n}\binom{N-n}{t_\mathrm{A}}\binom{N-n-t_\mathrm{A}}{t_\mathrm{B}}$ instances having n relay nodes decoding successfully both from node A and node B, where t_A is the number of nodes successfully decoding from only node A and t_B is that from only node B. On the basis of the theorem of total probability, we have

$$\mathrm{OP} = \sum_{n=0}^{N} \Pr(|\mathscr{R}| = n) \Pr\left(\gamma_{R_{i^*}} < \gamma_\mathrm{th}\right). \tag{17}$$

The following theorem provides a closed-form expression of OP.

Theorem 1: Over Nakagami-m fading channels, the system OP is expressed by a closed-form expression as follows:

$$\begin{aligned}\mathrm{OP} = \sum_{n=0}^{N} \sum_{t_\mathrm{A}=0}^{N-n} \sum_{t_\mathrm{B}=0}^{N-n-t_\mathrm{A}} \binom{N}{n}\binom{N-n}{t_\mathrm{A}}\binom{N-n-t_\mathrm{A}}{t_\mathrm{B}} \\ \times \left[1 - F_{\gamma_{\mathrm{AR}_i}}(\gamma_\mathrm{th})\right]^{n+t_\mathrm{A}} \left[F_{\gamma_{\mathrm{AR}_i}}(\gamma_\mathrm{th})\right]^{N-n-t_\mathrm{A}} \\ \times \left[1 - F_{\gamma_{\mathrm{BR}_i}}(\gamma_\mathrm{th})\right]^{n+t_\mathrm{B}} \left[F_{\gamma_{\mathrm{BR}_i}}(\gamma_\mathrm{th})\right]^{N-n-t_\mathrm{B}} \\ \times \left[F_{\gamma_{R_i}}(\gamma_\mathrm{th})\right]^{n},\end{aligned} \tag{18}$$

where $F_{\gamma_{\mathrm{AR}_i}}(\gamma)$, $F_{\gamma_{\mathrm{BR}_i}}(\gamma)$ and $F_{\gamma_{R_i}}(\gamma_\mathrm{th})$ are given by (19)–(21), respectively.

$$\begin{aligned}F_{\gamma_{\mathrm{AR}_i}}(\gamma) = F_{X_{ai}}\left(\frac{\gamma}{\mathscr{P}_m}\right) F_{X_{ak}}(\varepsilon) + 1 - F_{X_{ak}}(\varepsilon) \\ - \sum_{u_1=0}^{L-1} \binom{L-1}{u_1} (-1)^{u_1} \sum_{\substack{l_1, l_2, \ldots, l_{m_{ak}} \ge 0 \\ l_1 + l_2 + \cdots + l_{m_{ak}} = u_1}} \frac{u_1!}{l_1! l_2! \ldots l_{m_{ak}}!} \\ \times \prod_{\mathrm{w}_1=0}^{m_{ak}-1} \left(\frac{\alpha_{ak}^{\mathrm{w}_1}}{\mathrm{w}_1!}\right)^{l_{\mathrm{w}_1+1}} \frac{L\alpha_{ak}^{m_{ak}}}{\Gamma(m_{ak})} \sum_{s_1=0}^{m_{ai}-1} \frac{(\alpha_{ai}\gamma/I_\mathrm{p})^{s_1}}{s_1!} \\ \times \frac{\Gamma\left(m_{ak} + \tilde{l}_{ak} + s_1, ((u_1+1)\alpha_{ak} + \alpha_{ai}\gamma/I_\mathrm{p})\varepsilon\right)}{\left[(u_1+1)\alpha_{ak} + \alpha_{ai}\gamma/I_\mathrm{p}\right]^{(m_{ak} + \tilde{l}_{ak} + s_1)}}.\end{aligned} \tag{19}$$

$$F_{\gamma_{\mathrm{BR}_i}}(\gamma) = F_{X_{bi}}\left(\frac{\gamma}{\mathscr{P}_{\mathrm{m}}}\right)F_{X_{bk}}(\varepsilon) + 1 - F_{X_{bk}}(\varepsilon) - \sum_{u_2=0}^{L-1}\binom{L-1}{u_2}(-1)^{u_2}\sum_{\substack{l_1, l_2, \ldots, l_{\mathrm{m}_{bk}} \geq 0 \\ l_1 + l_2 + \cdots + l_{\mathrm{m}_{bk}} = u_2}}\frac{u_2!}{l_1!l_2!\ldots l_{\mathrm{m}_{bk}}!} \times \prod_{\mathrm{w}_2=0}^{m_{bk}-1}\left(\frac{\alpha_{bk}^{\mathrm{w}_2}}{\mathrm{w}_2!}\right)^{l_{\mathrm{w}_2+1}}\frac{L\alpha_{bk}^{m_{bk}}}{\Gamma(m_{bk})}\sum_{s_2=0}^{m_{bi}-1}\frac{(\alpha_{bi}\gamma/I_{\mathrm{p}})^{s_2}}{s_2!} \times \frac{\Gamma\left(m_{bk} + \tilde{l}_{bk} + s_2, \left((u_2+1)\alpha_{bk} + \alpha_{bi}\gamma/I_{\mathrm{p}}\right)\varepsilon\right)}{\left[(u_2+1)\alpha_{bk} + \alpha_{bi}\gamma/I_{\mathrm{p}}\right]^{(m_{bk} + \tilde{l}_{bk} + s_2)}}. \tag{20}$$

(see (21)) .

Proof: To obtain OP, we need to calculate $F_{\gamma_{\mathrm{AR}_i}}(\gamma_{\mathrm{th}})$, $F_{\gamma_{\mathrm{BR}_i}}(\gamma_{\mathrm{th}})$ and $F_{\gamma_{R_i}}(\gamma_{\mathrm{th}})$. We first consider $F_{\gamma_{\mathrm{AR}_i}}(\gamma)$ which can be written as

$$F_{\gamma_{\mathrm{AR}_i}}(\gamma) = \Pr\left(\gamma_{\mathrm{AR}_i} < \gamma\right) = \Pr\left[\left|h_{\mathrm{AR}_i}\right|^2 \min\left(\mathscr{P}_{\mathrm{m}}, \frac{I_{\mathrm{p}}}{\max_{k=1,2,\ldots,L}\left|h_{\mathrm{AU}_k}\right|^2}\right) < \gamma\right]. \tag{22}$$

For notational simplicity, we introduce $X_{ai} = \left|h_{\mathrm{AR}_i}\right|^2$, $X_{ia} = \left|h_{R_i\mathrm{A}}\right|^2$, $X_{ak} = \max_{k=1,2,\ldots,L}\left|h_{\mathrm{AU}_k}\right|^2$, $X_{ip} = \max_{k=1,2,\ldots,L}\left|h_{R_i\mathrm{U}_k}\right|^2$, $\varepsilon = (I_{\mathrm{p}}/\mathscr{P}_{\mathrm{m}})$, the CDF of γ_{AR_i} can be rewritten as

$$F_{\gamma_{\mathrm{AR}_i}}(\gamma) = \Pr\left(X_{ai} < \frac{\gamma}{\mathscr{P}_m}, X_{ak} < \varepsilon\right) + \Pr\left(X_{ai} < \frac{\gamma X_{ak}}{I_{\mathrm{p}}}, X_{ak} > \varepsilon\right). \tag{23}$$

It is noted that X_{ak} is the maximum of L independent and identically distributed gamma random variables. The following corollary will be useful in deriving the CDF and PDF of γ_{AR_i}.

Corollary 1: Let Z be the maximum of L i.i.d. gamma random variables Z_i, $(i = 1, 2, \ldots, L)$ with parameters m_z and α_z. The CDF and PDF of the random variable Z are given, respectively, by

$$F_Z(z) = \sum_{u=0}^{L}\binom{L}{u}(-1)^u\sum_{\substack{l_1, l_2, \ldots, l_{\mathrm{m}_z} \geq 0 \\ l_1 + l_2 + \cdots + l_{\mathrm{m}_z} = u}}\frac{u!}{l_1!l_2!\ldots l_{\mathrm{m}_z}!} \times \prod_{\mathrm{w}=0}^{m_z-1}\left(\frac{\alpha_z^{\mathrm{w}}}{\mathrm{w}!}\right)^{l_{\mathrm{w}+1}} z^{\tilde{l}_z}\mathrm{e}^{-\alpha_z u z}, \tag{24}$$

and

$$f_Z(z) = \sum_{u=0}^{L-1}\binom{L-1}{u}(-1)^u\sum_{\substack{l_1, l_2, \ldots, l_{\mathrm{m}_z} \geq 0 \\ l_1 + l_2 + \cdots + l_{\mathrm{m}_z} = u}}\frac{u!}{l_1!l_2!\ldots l_{\mathrm{m}_z}!} \times \prod_{\mathrm{w}=0}^{m_z-1}\left(\frac{\alpha_z^{\mathrm{w}}}{\mathrm{w}!}\right)^{l_{\mathrm{w}+1}} z^{m_z + \tilde{l}_z - 1}\mathrm{e}^{-(u+1)\alpha_z z}, \tag{25}$$

where $\tilde{l}_z = \sum_{\mathrm{w}=0}^{m_z-1}\mathrm{w}l_{\mathrm{w}+1}$, $\binom{n}{k} \triangleq n!/(k!(n-k)!)$ and $0 \leq k \leq n$.

Proof: See [22]. □

On the basis of (22) and Corollary 1, we have

$$\begin{aligned} F_{\gamma_{\mathrm{AR}_i}} &= F_{X_{ai}}\left(\frac{\gamma}{\mathscr{P}_m}\right)F_{X_{ak}}(\varepsilon) + \int_{\varepsilon}^{\infty}\int_{0}^{(\gamma/I_{\mathrm{p}})x_{ak}} f_{X_{ai}}\left(\frac{\gamma x_{ak}}{I_{\mathrm{p}}}\right)f_{X_{ak}}(x_{ak})\mathrm{d}x_{ai}\,\mathrm{d}x_{ak} \\ &= F_{X_{ai}}\left(\frac{\gamma}{\mathscr{P}_{\mathrm{m}}}\right)F_{X_{ak}}(\varepsilon) + 1 - F_{X_{ak}}(\varepsilon) - \int_{\varepsilon}^{\infty}\Gamma\left(m_{ai}, \frac{\alpha_{ai}\gamma x_{ak}/I_{\mathrm{p}}}{\Gamma(m_{ai})}\right)f_{X_{ak}}(x_{ak})\mathrm{d}x_{ak}. \end{aligned} \tag{26}$$

Using [23, Eq. (8.352.2)] to expand the incomplete gamma function as a finite sum, the integral in (26) is computed as

$$\begin{aligned} &\int_{\varepsilon}^{\infty}\Gamma\left(m_{ai}, \frac{\alpha_{ai}\gamma x_{ak}/I_{\mathrm{p}}}{\Gamma(m_{ai})}\right)f_{X_{ak}}(x_{ak})\mathrm{d}x_{ak} \\ &= \int_{\varepsilon}^{\infty}(m_{ai}-1)!\frac{1}{\Gamma(m_{ai})}\mathrm{e}^{-\alpha_{ai}\gamma x_{ak}/I_{\mathrm{p}}}\sum_{s_1=0}^{m_{ai}-1}\frac{(\alpha_{ai}\gamma x_{ak}/I_{\mathrm{p}})^{s_1}}{s_1!} \times \sum_{u_1=0}^{L-1}\binom{L-1}{u_1}(-1)^{u_1}\sum_{\substack{l_1, l_2, \ldots, l_{\mathrm{m}_{ak}} \geq 0 \\ l_1 + l_2 + \cdots + l_{\mathrm{m}_{ak}} = u_1}}\frac{u_1!}{l_1!l_2!\ldots l_{\mathrm{m}_{ak}}!} \\ &\times \prod_{\mathrm{w}_1=0}^{m_{ak}-1}\left(\frac{\alpha_{ak}^{\mathrm{w}_1}}{\mathrm{w}_1!}\right)^{l_{\mathrm{w}_1+1}}\frac{L\alpha_{ak}^{m_{ak}}}{\Gamma(m_{ak})}x_{ak}^{m_{ak} + \tilde{l}_{ak} - 1}\mathrm{e}^{-(u_1+1)\alpha_{ak}x_{ak}}\mathrm{d}x_{ak}, \end{aligned} \tag{27}$$

where $\tilde{l}_{ak} = \sum_{\mathrm{w}_1=0}^{m_{ak}-1}\mathrm{w}_1 l_{\mathrm{w}_1+1}$.

With the help of [23, Eq. 3.351.2] and using the fact that γ_{AR_i} and γ_{BR_i} take the same form, the CDFs of γ_{AR_i} and γ_{BR_i} are derived by (19) and (20), respectively, where $X_{bk} = \max_{k=1,2,\ldots,L}\left|h_{\mathrm{BP}_k}\right|^2$, $\tilde{l}_{bk} = \sum_{\mathrm{w}_2=0}^{m_{bk}-1}\mathrm{w}_2 l_{\mathrm{w}_2+1}$.

Having $F_{\gamma_{\mathrm{AR}_i}}(\gamma)$ and $F_{\gamma_{\mathrm{BR}_i}}(\gamma)$ at hands, we are now in a position to derive the CDF of $\gamma_{\mathrm{R}_i}(\gamma) = \min(\gamma_{R_i\mathrm{A}}, \gamma_{R_i\mathrm{B}})$. It is noted that $\gamma_{R_i\mathrm{A}}$ and $\gamma_{R_i\mathrm{B}}$ are correlated due to the common random variable $X_{i\mathrm{p}}$. Using the conditional probability, we can write the CDF of γ_{R_i} as follows:

$$F_{\gamma_{R_i}}(\gamma) = \int_0^{\infty}F_{\gamma_{R_i}|X_{i\mathrm{p}}}(\gamma)f_{X_{i\mathrm{p}}}(x_{i\mathrm{p}})\,\mathrm{d}x_{i\mathrm{p}} \tag{28}$$

$$\begin{aligned} F_{\gamma_{R_i}}(\gamma) = 1 &- \frac{\Gamma\left(m_{ia}, \frac{\alpha_{ia}\gamma}{\mathscr{P}_m}\right)}{\Gamma(m_{ia})}\frac{\Gamma\left(m_{ib}, \frac{\alpha_{ib}\gamma}{\mathscr{P}_m}\right)}{\Gamma(m_{ib})}\left(1 - \frac{\Gamma(m_{ip}, \alpha_{ip}\varepsilon)}{\Gamma(m_{ip})}\right)^L \\ &- \sum_{\mathrm{w}_a=0}^{m_{ia}-1}\frac{(\alpha_{ia}\gamma/I_{\mathrm{p}})^{\mathrm{w}_a}}{\mathrm{w}_a!}\sum_{\mathrm{w}_b=0}^{m_{ib}-1}\frac{(\alpha_{ib}\gamma/I_{\mathrm{p}})^{\mathrm{w}_b}}{\mathrm{w}_b!}\sum_{u_3=0}^{L-1}\binom{L-1}{u_3} \\ &\times (-1)^{u_3}\sum_{\substack{l_1, l_2, \ldots, l_{\mathrm{m}_{ip}} \geq 0 \\ l_1 + l_2 + \cdots + l_{\mathrm{m}_{ip}} = u_3}}\frac{u_3!}{l_1!l_2!\ldots l_{\mathrm{m}_{ip}}!}\prod_{\mathrm{w}_3=0}^{m_{ip}-1}\left(\frac{\alpha_{ip}^{\mathrm{w}_3}}{\mathrm{w}_3!}\right)^{l_{\mathrm{w}_3+1}}\frac{L\alpha_{ip}^{m_{ip}}}{\Gamma(m_{ip})} \\ &\times \frac{\Gamma\left(\mathrm{w}_a + \mathrm{w}_b + \tilde{l}_{ip} + m_{ip}, \left(\frac{\alpha_{ia} + \alpha_{ib}}{I_{\mathrm{p}}}\gamma + (u_3+1)\alpha_{ip}\right)\varepsilon\right)}{\left[(\alpha_{ia} + \alpha_{ib})\gamma/I_{\mathrm{p}} + (u_3+1)\alpha_{ip}\right]^{(\mathrm{w}_a + \mathrm{w}_b + \tilde{l}_{ip} + m_{ip})}}. \end{aligned} \tag{21}$$

where $F_{\gamma_{R_i}|X_{ip}}(\gamma)$ is given by

$$\begin{aligned} F_{\gamma_{R_i}|X_{ip}}(\gamma) &= 1 - \Pr\left(\gamma_{R_i\mathrm{A}} > \gamma|X_{ip}\right)\Pr\left(\gamma_{R_i\mathrm{B}} > \gamma|X_{ip}\right) \\ &= 1 - \left[1 - F_{\gamma_{R_i\mathrm{A}}|X_{ip}}(\gamma)\right]\left[1 - F_{\gamma_{R_i\mathrm{B}}|X_{ip}}(\gamma)\right]. \end{aligned} \quad (29)$$

From (6) and (7), we can easily find the following:

$$F_{\gamma_{\mathrm{R}_i\mathrm{A}}|X_{ip}}(\gamma) = \begin{cases} F_{X_{ia}}\left(\dfrac{\gamma}{\mathscr{P}_\mathrm{m}}\right) & \text{for } X_{ip} < \varepsilon \\ F_{X_{ia}}\left(\dfrac{\gamma}{I_\mathrm{p}}X_{ip}\right) & \text{for } X_{ip} > \varepsilon \end{cases} \quad (30)$$

and

$$F_{\gamma_{\mathrm{R}_i\mathrm{B}}|X_{ip}}(\gamma) = \begin{cases} F_{X_{ib}}\left(\dfrac{\gamma}{\mathscr{P}_\mathrm{m}}\right) & \text{for } X_{ip} < \varepsilon \\ F_{X_{ib}}\left(\dfrac{\gamma}{I_\mathrm{p}}X_{ip}\right) & \text{for } X_{ip} > \varepsilon \end{cases}. \quad (31)$$

On the basis of the above equations, (29) can be represented by the following form:

$$F_{\gamma_{R_i}}(\gamma) = \underbrace{\int_0^{\varepsilon} F_{\gamma_{R_i}|X_{ip}}(\gamma) f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip}}_{I_1} + \underbrace{\int_{\varepsilon}^{\infty} F_{\gamma_{R_i}|X_{ip}}(\gamma) f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip}}_{I_2}. \quad (32)$$

To obtain $F_{\gamma_{R_i}}(\gamma)$, we need to derive I_1 and I_2. We first consider I_1, which can be computed by

$$\begin{aligned} I_1 &= \int_0^{\varepsilon}\left[1 - \left[1 - F_{X_{ia}}\left(\frac{\gamma}{\mathscr{P}_\mathrm{m}}\right)\right]\left[1 - F_{X_{ib}}\left(\frac{\gamma}{\mathscr{P}_\mathrm{m}}\right)\right]\right] f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip} \\ &= \int_0^{\varepsilon} f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip} \\ &\quad - \frac{\Gamma(m_{ia}, \alpha_{ia}\gamma/\mathscr{P}_\mathrm{m})}{\Gamma(m_{ia})}\frac{\Gamma(m_{ib}, \alpha_{ib}\gamma/\mathscr{P}_\mathrm{m})}{\Gamma(m_{ib})}\left[1 - \frac{\Gamma(m_{ip}, \alpha_{ip}\varepsilon)}{\Gamma(m_{ip})}\right]^L. \end{aligned} \quad (33)$$

For I_2, we have

$$\begin{aligned} I_2 = \int_{\varepsilon}^{\infty}&\left(1 - \frac{\Gamma(m_{ia}, \alpha_{ia}\gamma x_{ip}/I_\mathrm{p})}{\Gamma(m_{ia})}\frac{\Gamma(m_{ib}, \alpha_{ib}\gamma x_{ip}/I_\mathrm{p})}{\Gamma(m_{ib})}\right) \\ &\times f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip} \end{aligned} \quad (34)$$

Using Corollary 1 and [23, Eq. (8.352.2)] to expand the incomplete gamma function as a finite sum, I_2 can be computed by

$$\begin{aligned} I_2 &= \int_{\varepsilon}^{\infty} f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip} \\ &\quad - \int_{\varepsilon}^{\infty}\sum_{\mathrm{w}_a=0}^{m_{ia}-1}\frac{(\alpha_{ia}\gamma/I_\mathrm{p})^{\mathrm{w}_a}}{\mathrm{w}_a!}\sum_{\mathrm{w}_b=0}^{m_{ib}-1}\frac{(\alpha_{ib}\gamma/I_\mathrm{p})^{\mathrm{w}_b}}{\mathrm{w}_b!} \\ &\quad \times \sum_{u_3=0}^{L-1}\binom{L-1}{u_3}(-1)^{u_3}\sum_{\substack{l_1, l_2, \ldots, l_{\mathrm{m}_{ip}} \geq 0 \\ l_1 + l_2 + \cdots + l_{\mathrm{m}_{ip}} = u_3}}\frac{u_3!}{l_1!l_2!\ldots l_{\mathrm{m}_{ip}}!} \\ &\quad \times \prod_{\mathrm{w}_3=0}^{m_{ip}-1}\left(\frac{\alpha_{ip}^{\mathrm{w}_3}}{\mathrm{w}_3!}\right)^{l_{\mathrm{w}_3+1}}\frac{L\alpha_{ip}^{m_{ip}}}{\Gamma(m_{ip})}x_{ip}^{\mathrm{w}_a+\mathrm{w}_b+m_{ip}+\tilde{l}_{ip}-1} \\ &\quad \times \mathrm{e}^{-((\alpha_{ia}+\alpha_{ib})\gamma/I_\mathrm{p}+(u_3+1)\alpha_{ip})x_{ip}}\mathrm{d}x_{ip}, \end{aligned} \quad (35)$$

where $\tilde{l}_{ip} = \sum_{\mathrm{w}_3=0}^{m_{ip}-1}\mathrm{w}_3 l_{\mathrm{w}_3+1}$.

Observing I_1 and I_2, we note that $\int_0^{\varepsilon} f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip} + \int_{\varepsilon}^{\infty} f_{X_{ip}}(x_{ip})\mathrm{d}x_{ip} = 1$. In addition, using the identity [23, Eq. (8.352.2)] for the second integral in (35), after tedious manipulations, we can obtain the closed-form expression for $F_{\gamma_{R_i}}(\gamma)$ as in (21), which also completes the proof. □

In the next sections, we will derive the asymptotic expression of OP at high and low SNR regimes to give insights. By considering two cases of $\widetilde{\mathscr{P}_\mathrm{m}} \ll \tilde{I}_\mathrm{p}$ and $\widetilde{\mathscr{P}_m} \gg \tilde{I}_\mathrm{p}$, we have

$$\begin{aligned} P_\mathsf{X} &= \min\left(\widetilde{\mathscr{P}_\mathrm{m}}, \frac{\tilde{I}_\mathrm{p}}{\max_{k=1,2,\ldots,L}|h_{\mathsf{X}U_k}|^2}\right) \\ &\backsimeq \begin{cases} \widetilde{\mathscr{P}_\mathrm{m}}, & \widetilde{\mathscr{P}_\mathrm{m}} \ll \tilde{I}_\mathrm{p} \\ \dfrac{I_\mathrm{p}}{\max_{k=1,2,\ldots,L}|h_{\mathsf{X}U_k}|^2}, & \widetilde{\mathscr{P}_\mathrm{m}} \gg \tilde{I}_\mathrm{p} \end{cases}, \end{aligned} \quad (36)$$

where $\mathsf{X} = \{\mathrm{A}, \mathrm{B}, \mathrm{R}_i\}$.

3.2 Asymptotic expression of OP at high SNR regime

For the case of $\widetilde{\mathscr{P}_\mathrm{m}} \ll \tilde{I}_\mathrm{p}$, we adopt $P_\mathrm{A} = P_\mathrm{B} = P_{R_i} = \widetilde{\mathscr{P}_\mathrm{m}}$ for analysis simplicity leading to

$$\begin{aligned} \gamma_{\mathrm{A}R_i} &= \mathscr{P}_\mathrm{m}|h_{\mathrm{A}R_i}|^2, \quad \gamma_{\mathrm{B}R_i} = \mathscr{P}_\mathrm{m}|h_{\mathrm{B}R_i}|^2, \\ \gamma_{R_i\mathrm{A}} &= \mathscr{P}_\mathrm{m}|h_{R_i\mathrm{A}}|^2, \quad \gamma_{R_i\mathrm{B}} = \mathscr{P}_\mathrm{m}|h_{R_i\mathrm{B}}|^2. \end{aligned} \quad (37)$$

We can easily compute the CDF of γ_{AR_i} at high SNR as follows:

$$F_{\gamma_{\mathrm{AR}_i}}^{\infty}(\gamma) = \Pr\left(\mathscr{P}_\mathrm{m}|h_{\mathrm{AR}_i}|^2 < \gamma\right) = 1 - \frac{\Gamma(m_{ai}, ((\alpha_{ai}\gamma)/\mathscr{P}_\mathrm{m}))}{\Gamma(m_{ai})}. \quad (38)$$

Similarly, we have the CDF of γ_{BR_i} given by

$$F_{\gamma_{\mathrm{BR}_i}}^{\infty}(\gamma) = F_{|h_{\mathrm{BR}_i}|^2}\left(\frac{\gamma}{\mathscr{P}_\mathrm{m}}\right) = 1 - \frac{\Gamma(m_{bi}, ((\alpha_{bi}\gamma)/\mathscr{P}_\mathrm{m}))}{\Gamma(m_{bi})}. \quad (39)$$

The CDF of γ_{R_i} at high SNR, $F_{\gamma_{R_i}}^{\infty}$ can be computed as

$$\begin{aligned} F_{\gamma_{R_i}}^{\infty}(\gamma) &\backsimeq \Pr\left[\min\left(\mathscr{P}_\mathrm{m}|h_{\mathrm{R}_i\mathrm{A}}|^2, \mathscr{P}_\mathrm{m}|h_{\mathrm{R}_iB}|^2\right) < \gamma\right] \\ &= \Pr\left[\min\left(|h_{\mathrm{R}_i\mathrm{A}}|^2, |h_{\mathrm{R}_i\mathrm{B}}|^2\right) < \frac{\gamma}{\mathscr{P}_\mathrm{m}}\right]. \end{aligned} \quad (40)$$

It is noted here that $|h_{\mathrm{R}_i\mathrm{A}}|^2$ and $|h_{\mathrm{R}_i\mathrm{B}}|^2$ are independent of each other. Hence, with the help of [24, Eq. (6-81)], we have

$$\begin{aligned} F_{\gamma_{R_i}}^{\infty}(\gamma) &= F_{|h_{\mathrm{R}_i\mathrm{A}}|^2}\left(\frac{\gamma}{\mathscr{P}_\mathrm{m}}\right) + F_{|h_{\mathrm{R}_i\mathrm{B}}|^2}\left(\frac{\gamma}{\mathscr{P}_\mathrm{m}}\right) \\ &\quad - F_{|h_{\mathrm{R}_i\mathrm{A}}|^2}\left(\frac{\gamma}{\mathscr{P}_\mathrm{m}}\right)F_{|h_{\mathrm{R}_i\mathrm{B}}|^2}\left(\frac{\gamma}{\mathscr{P}_\mathrm{m}}\right) \\ &= 1 - \frac{\Gamma(m_{ia}, ((\alpha_{ia}\gamma)/\mathscr{P}_m))}{\Gamma(m_{ia})}\frac{\Gamma(m_{ib}, ((\alpha_{ib}\gamma)/\mathscr{P}_\mathrm{m}))}{\Gamma(m_{ib})}. \end{aligned} \quad (41)$$

Substituting (38), (39) and (41) into (18), we obtain an asymptotic closed-form expression of the system OP at high SNR regime as shown in (42) at the top of the next page.

$$\mathrm{OP}^{\infty}=\sum_{n=0}^{N}\sum_{t_a=0}^{N-n}\sum_{t_b=0}^{N-n-t_a}\binom{N}{n}\binom{N-n}{t_a}\binom{N-n-t_a}{t_b}$$
$$\times\left[\frac{\Gamma(m_{ai},((\alpha_{ai}\gamma)/\mathscr{P}_{\mathrm{m}}))}{\Gamma(m_{ai})}\right]^{n+t_a}$$
$$\times\left[1-\frac{\Gamma(m_{ai},((\alpha_{ai}\gamma)/\mathscr{P}_{\mathrm{m}}))}{\Gamma(m_{ai})}\right]^{N-n-t_a}$$
$$\times\left[\frac{\Gamma(m_{bi},((\alpha_{bi}\gamma)/\mathscr{P}_{\mathrm{m}}))}{\Gamma(m_{bi})}\right]^{n+t_b}\left[1-\frac{\Gamma(m_{bi},((\alpha_{bi}\gamma)/\mathscr{P}_{\mathrm{m}}))}{\Gamma(m_{bi})}\right]^{N-n-t_b}$$
$$\times\left[1-\frac{\Gamma(m_{ia},((\alpha_{ia}\gamma)/\mathscr{P}_{\mathrm{m}}))}{\Gamma(m_{ia})}\frac{\Gamma(m_{ib},((\alpha_{ib}\gamma)/\mathscr{P}_{\mathrm{m}}))}{\Gamma(m_{ib})}\right]^{n}. \tag{42}$$

3.3 Asymptotic expression of OP at low SNR regime

On the basis of (36), the transmit signal powers of the secondary nodes in low SNR regime $\left(\widetilde{\mathscr{P}_m}\gg\widetilde{I}_{\mathrm{p}}\right)$ can be determined as follows:

$$P_{\mathrm{X}}=\frac{\widetilde{I}_{\mathrm{p}}}{\max_{k=1,2,\ldots,L}\left|h_{\mathrm{XU}_k}\right|^2}, \tag{43}$$

where $\mathrm{X}\in\{\mathrm{A},\mathrm{B},\mathrm{R}_i\}$.

Noting that if $\widetilde{\mathscr{P}_m}\gg\widetilde{I}_{\mathrm{p}}$ then $\varepsilon\to 0$. Furthermore, according to the definition of the incomplete gamma function as $\Gamma(\alpha,x)=\int_x^{\infty}\mathrm{e}^{-t}t^{\alpha-1}\,\mathrm{d}t$ [23, Eq. (8.350.2)], we can see that if $x\to 0$, then $\Gamma(\alpha,x)\to\Gamma(\alpha,0)=\Gamma(\alpha)$. For α being a positive integer, with the help of [23, Eq. (8.339.1)], i.e. $\Gamma(\alpha)=(\alpha-1)!$, we have

$$\Gamma\left(m_{ak}+\widetilde{l}_{ak}+s_1,\left((u_1+1)\alpha_{ak}+\alpha_{ai}\gamma/I_{\mathrm{p}}\right)\varepsilon\right)\overset{\varepsilon\to 0}{\simeq}\left(m_{ak}+\widetilde{l}_{ak}+s_1-1\right)! \tag{44}$$

and

$$F_{X_{ak}}(\varepsilon)\overset{\varepsilon\to 0}{\simeq}0. \tag{45}$$

From the CDF of $\gamma_{\mathrm{A}R_i}$ in (26), we can derive the asymptotic expression of the CDF for $\gamma_{\mathrm{A}R_i}$ at low SNR regime, $F^0_{\gamma_{\mathrm{AR}_i}}(\gamma)$, as shown by (46). Using the same approximation method as for (46), we also obtain the asymptotic CDF expressions of $\gamma_{\mathrm{B}R_i}$ and γ_{R_i}, i.e. $F^0_{\gamma_{\mathrm{BR}_i}}(\gamma)$ and $F^0_{\gamma_{R_i}}(\gamma)$, as shown in (47) and (48), respectively. Finally, substituting (46)–(48) into (18), we obtain a more simplified expression of the OP at low SNR regime

$$F^0_{\gamma_{\mathrm{AR}_i}}(\gamma)=1-\sum_{u_1=0}^{L-1}\binom{L-1}{u_1}(-1)^{u_1}\sum_{\substack{l_1,l_2,\ldots,l_{\mathrm{m}_{ak}}\ge 0\\ l_1+l_2+\cdots+l_{\mathrm{m}_{ak}}=u_1}}\frac{u_1!}{l_1!l_2!\ldots l_{\mathrm{m}_{ak}}!}$$
$$\times\prod_{\mathrm{w}_1=0}^{m_{ak}-1}\left(\frac{\alpha_{ak}^{\mathrm{w}_1}}{\mathrm{w}_1!}\right)^{l_{\mathrm{w}_1+1}}\frac{L\alpha_{ak}^{m_{ak}}}{\Gamma(m_{ak})}\sum_{s_1=0}^{m_{ai}-1}\frac{\left(\alpha_{ai}\gamma/I_{\mathrm{p}}\right)^{s_1}}{s_1!}$$
$$\times\frac{\left(m_{ak}+\widetilde{l}_{ak}+s_1-1\right)!}{\left[(u_1+1)\alpha_{ak}+\alpha_{ai}\gamma/I_{\mathrm{p}}\right]^{\left(m_{ak}+\widetilde{l}_{ak}+s_1\right)}}. \tag{46}$$

$$F^0_{\gamma_{\mathrm{BR}_i}}(\gamma)=1-\sum_{u_2=0}^{L-1}\binom{L-1}{u_2}(-1)^{u_2}\sum_{\substack{l_1,l_2,\ldots,l_{\mathrm{m}_{bk}}\ge 0\\ l_1+l_2+\cdots+l_{\mathrm{m}_{bk}}=u_2}}\frac{u_2!}{l_1!l_2!\ldots l_{\mathrm{m}_{bk}}!}$$
$$\times\prod_{\mathrm{w}_2=0}^{m_{bk}-1}\left(\frac{\alpha_{bk}^{\mathrm{w}_2}}{\mathrm{w}_2!}\right)^{l_{\mathrm{w}_2+1}}\frac{L\alpha_{bk}^{m_{bk}}}{\Gamma(m_{bk})}\sum_{s_2=0}^{m_{bi}-1}\frac{\left(\alpha_{bi}\gamma/I_{\mathrm{p}}\right)^{s_2}}{s_2!}$$
$$\times\frac{\left(m_{bk}+\widetilde{l}_{bk}+s_2-1\right)!}{\left[(u_2+1)\alpha_{bk}+\alpha_{bi}\gamma/I_{\mathrm{p}}\right]^{\left(m_{bk}+\widetilde{l}_{bk}+s_2\right)}}. \tag{47}$$

(see (48)) .

In a cognitive system, imperfect CSI (due to channel estimation error) incurs an erroneous transmit power adjustment at nodes in the secondary network. Therefore, increasing channel estimation error leads to an increase in the probability of the event that co-channel interference from the secondary network to primary receivers exceeds the peak power $\widetilde{I}_{\mathrm{p}}$. As a result, CSI imperfection degrades the performance of the primary network. In the secondary network, channel estimation error also degrades the performance of coherent detection (at nodes A, B and R_i) and relay selection. A detailed formulation of the performance degradation due to imperfect channel estimation can be considered as a future work of this paper.

4 Numerical results

This section provides numerous numerical results to verify the derivations of OP expressions and analyse the performance of the considered cognitive radio system. For the system model, it is assumed that all nodes are located on a two-dimensional plane and the distance between two primary source nodes is normalised to one. Without loss of generality, we set coordinates of nodes as follows: A(0,0), B(1,0), $\mathrm{U_k}(0.5,1)$ and $\mathrm{R_i}(0.5,0)\,\forall\mathrm{i},\mathrm{k}$. Taking into account channel path loss, we consider $\alpha_{\mathrm{XY}}=d_{\mathrm{XY}}^{-\eta}$, where d_{XY} is the physical distance between nodes X and Y and η is the path loss exponent. In the simulated system settings, we set $\eta=3$ and $\gamma_{\mathrm{th}}=7$. In figures of numerical results, curves of numerical OP values obtained by (i) an exact theoretical expression, (ii) an asymptotic expression in low SNRs, (iii) an asymptotic expression in high SNRs and (iv) simulations are made legend by ‘exact’, ‘asym-low’, ‘asym-high’ and ‘simulation’, respectively.

Fig. 2 shows theoretical and empirical OP values of the secondary network with the simulated system parameters as follows $m_{ia}=1$, $m_{ib}=3$, $m_{ap}=3$, $m_{ip}=2$, $m_{bp}=1$, $L=3$ and $\mathscr{P}_m$ = 10 dB. As observed, theoretical OP values and related empirical ones are in good agreement which verifies the derivations of OP expressions in this paper. As observed, when increasing I_{p} to a certain value, the secondary network's OP will approach to irreducible floor values corresponding to different numbers of relay nodes N. This is due to the fact that when I_{p} is large enough, transmit powers of secondary nodes are upper bounded by the power constraint $\mathscr{P}_{\mathrm{m}}$. In addition, adding more relay nodes in the secondary network will reduce the floor OP values. In other words, increasing the number of relays N produces more diversity gains to enhance the OP performance of the considered system.

Fig. 3 shows the OP performance of the secondary network under different numbers of receivers in the primary network. As can be seen, when I_{p} is smaller than 16 dB, increasing the number of primary receivers will degrade the performance of secondary networks. More specifically, adding more receivers in the primary

$$F^0_{\gamma_{R_i}}(\gamma)=1-\sum_{\mathrm{w}_a=0}^{m_{ia}-1}\frac{\left(\alpha_{ia}\gamma/I_{\mathrm{p}}\right)^{\mathrm{w}_a}}{\mathrm{w}_a!}\sum_{\mathrm{w}_b=0}^{m_{ib}-1}\frac{\left(\alpha_{ib}\gamma/I_{\mathrm{p}}\right)^{\mathrm{w}_b}}{\mathrm{w}_b!}\sum_{u_3=0}^{L-1}\binom{L-1}{u_3}(-1)^{u_3}\sum_{\substack{l_1,l_2,\ldots,l_{\mathrm{m}_{ip}}\ge 0\\ l_1+l_2+\cdots+l_{\mathrm{m}_{ip}}=u_3}}\frac{u_3!}{l_1!l_2!\ldots l_{\mathrm{m}_{ip}}!}$$
$$\times\prod_{\mathrm{w}_3=0}^{m_{ip}-1}\left(\frac{\alpha_{ip}^{\mathrm{w}_3}}{\mathrm{w}_3!}\right)^{l_{\mathrm{w}_3+1}}\frac{L\alpha_{ip}^{m_{ip}}}{\Gamma\left(m_{ip}\right)}\frac{\left(\mathrm{w}_a+\mathrm{w}_b+\widetilde{l}_{ip}+m_{ip}-1\right)!}{\left[(\alpha_{ia}+\alpha_{ib})\gamma/I_{\mathrm{p}}+(u_3+1)\alpha_{ip}\right]^{\left(\mathrm{w}_a+\mathrm{w}_b+\widetilde{l}_{ip}+m_{ip}\right)}}. \tag{48}$$

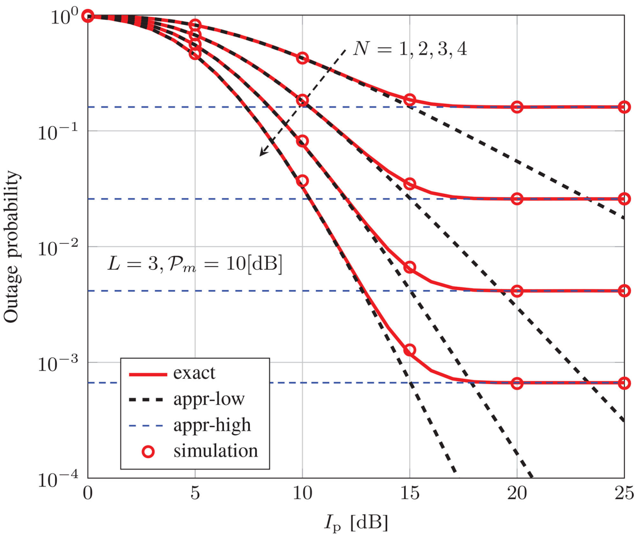

Fig. 2 *Analytical and empirical outage performance versus $I_{\rm p}$*

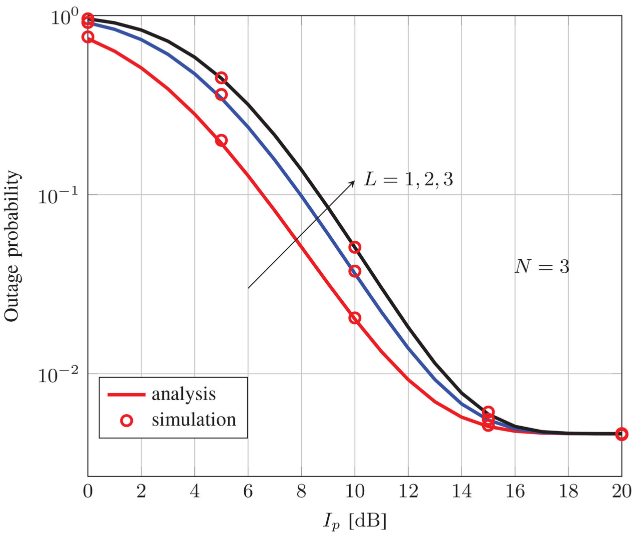

Fig. 3 *Effect of multiple primary receivers on the secondary network's performance*

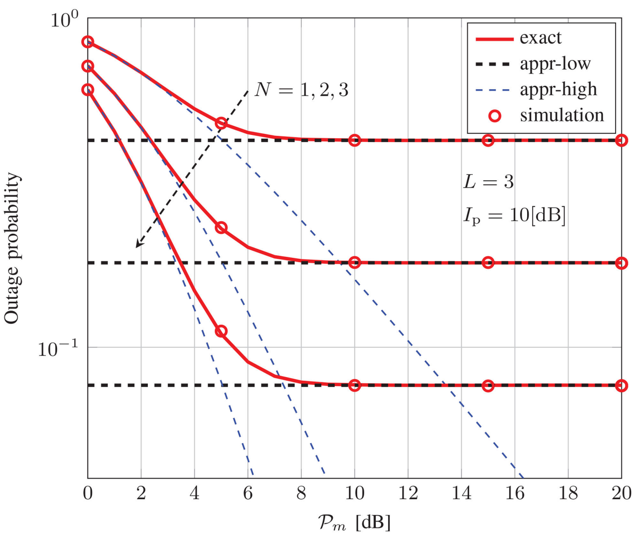

Fig. 4 *OP performance versus transmit power constraint at secondary nodes*

network will increase the probability of having a co-channel interference link (from a secondary node to a primary receiver) with a large channel gain. According to (3), large channel gains of interference links result in small transmit powers at secondary nodes. In other words, increasing the number of primary receivers likely induce an increase in OP of the secondary network. However, when $I_{\rm p}$ is greater than a certain value (e.g. 16 dB), the performances of secondary networks do not depend on the number of primary receivers. In fact, when $I_{\rm p}$ is large enough, the secondary network's performance mainly depends on the transmit power constraint of secondary nodes (i.e. $\mathscr{P}_m$).

To investigate the effect of transmit power constraint at secondary nodes on the system performance, Fig. 4 provides several curves of OP values versus $\mathscr{P}_{\rm m}$ under different numbers of relay nodes N. In this figure, the simulated system settings are as follows: $m_{ia} = 1$, $m_{ib} = 3$, $m_{ap} = 3$, $m_{ip} = 2$, $m_{bp} = 1$, $L = 3$ and $I_{\rm p} = 10$ dB. As observed, when transmit power constraint $\mathscr{P}_{\rm m}$ is large enough, the outage probabilities are approaching irreducible floors whose values depend on the tolerable interference power level $I_{\rm p}$ at primary receivers. As observed, the irreducible OP floor value can be reduced by adding more relay nodes (having more diversity gain) to the secondary network.

5 Conclusion

This paper studied the outage performance of a cognitive two-way DF relay system with multiple primary receivers and multiple two-way relays in secondary networks over Nakagami-*m* fading channels. The exact and asymptotic closed-form OP expressions were formulated and verified by empirical OP performance. The analytical and empirical OP values show that an increase in the number of primary receivers results in performance degradation of secondary network, which can be alleviated by adding more relay nodes to secondary networks.

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