# Improved LDPC Iterative Decoding Algorithm Based on the Reliable Extrinsic Information and Its Histogram

Nguyen Anh Tuan[1(✉)] and Pham Xuan Nghia[2]

[1] University of Information and Communication Technology, Thai Nguyen, Viet Nam
natuan@ictu.edu.vn

[2] Le Quy Don Technical University, Ha noi, Viet Nam

**Abstract.** In this paper we proposed a new method to prevent propagating errors due to passing the error Extrinsic information between nodes during the iterative decoding of low density parity check codes based on reliable Extrinsic information. Moreover, we also proposed a new method to analyze the convergence of the iterative LDPC decoding by using the histogram of Extrinsic information.

**Keywords:** LDPC decoding · Convergence of decoding · Reliable extrinsic information

## 1 Introduction

The convergence of iterative LDPC decoding processes are analyzed by the Density Evolution (DE) algorithm was proposed by Richardson et al. [1] or the Extrinsic Transfer Exit Chart devised by ten Brink [2]. Those above method help us in predicting the convergence of the LDPC codes and base on it we will decide the number of iterations used for decoding the LDPC codes. In this paper we introduce a novel method to predict the convergence based on analyzing the histogram of the Extrinsic information and also based on this histogram we will propose a new decoding method depended on the reliable Extrinsic information which are transferred between nodes during the LDPC decoding process.

## 2 A Novel Method to Predict the Convergence of the Iterative LDPC Decoding Process

The probabilistic LDPC decoding process is provided in [3] as following:

Based on the received soft values $y_j$ at the output of the channel, the intrinsic probability of the $jth$ bit being a binary 1 or binary 0 can be calculated as:

$$P(y|s_1) = \frac{1}{\sqrt{\pi N_0}} e^{\frac{-(y+\sqrt{E_b})^2}{N_0}}; P(y|s_0) = \frac{1}{\sqrt{\pi N_0}} e^{\frac{-(y-\sqrt{E_b})^2}{N_0}} \tag{1}$$


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where, y and $N_0$ denotes the received soft channel output value and the power of channel noise, respectively.

The $P^1_{i,j}$ values the probability equals to 1of the neighbouring non-zero entries of the Equivalent Parity Check Matrix $\boldsymbol{H}_e$ are initialized by the $p(y|s_1)$ in Eq. (1).

The Extrinsic information $LR_{i,j}$ values corresponding to each non-zero entry in a given row of the $\boldsymbol{H}_e$ are updated as the below equation:

$$LR_{i,j} = \frac{1 + \prod_{l \in \{Ci\}, l \neq j} (1 - 2P^1_{i,j})}{1 - \prod_{l \in \{Ci\}, l \neq j} (1 - 2P^1_{i,j})} \tag{2}$$

where M, N are the number of rows and columns of the $\boldsymbol{H}_e$.

The probability ratio values corresponding to each non-zero entry in a given column of the $\boldsymbol{H}_e$ are updated:

$$PR_{i,j} = \frac{1 - P^1_j}{P^1_j} \prod_{k \in \{R_j\}, k \neq j} LR_{k,j} \tag{3}$$

The overall a posteriori probability ratio of the jth coded $PR(x_j)$ is updated as following:

$$PR(x_j) = \frac{1 - P^1_j}{P^1_j} \prod_{i \in \{R_j\}} LR_{i,j} with j = 1...N \tag{4}$$

The ${P^1}_{i,j}$ value corresponding to each non-zero entry of the $\boldsymbol{H}_e$ is updated according to $1/(1 + PR_{i,j})$, where $PR_{i,j}$ represents the updated values.

Based on the $PR(x_j)$ values updated in step 5, a tentative hard decision is made and this tentatively decoded codeword is multiplied with $\boldsymbol{H}^T$.

If the resultant syndrome vector is an all-zero vector, we declare a legitimate codeword has been found and the iterative decoding process is terminated.

By contrast, if the syndrome vector is not an all-zero vector and the maximum number of LDPC iterations is reached, we will declare a decoding failure and output the tentatively decoded codeword.

If the maximum affordable complexity has not been exhausted, go back to step 3.

Assuming that probabilities of the input bit having “1” and “0” values are equal each others. This means that $p(s_1) = p(s_0) = 1/2$. The error condition probability to receive transmitted $s_1$ and $s_2$ is given in the following equations:

$$P(e|s_1) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{+\infty} e^{\frac{-(y-\sqrt{E_b})^2}{N_0}} dy = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{5}$$

$$P(e|s_0) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{+\infty} e^{\frac{-(y+\sqrt{E_b})^2}{N_0}} dy = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{6}$$

where: $erfc(x) = \frac{2}{\sqrt{\pi}} \int\limits_{x}^{+\infty} e^{x^2} dx$ is the error compensating function. The error probability to fail to receive a transmitted bit is calculated as following:

$$P_b = \frac{1}{2} \mathrm{er}fc\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{7}$$

From Eqs. (2), (5), (6) and (7) we can see that a single error received bit can be distributed to many other coded bit via the exchanging extrinsic information between nodes of the Tanner graph [4]. This distribution is very fast when the Eb/N0 is small and this error distribution causes the error avalanche. When the Eb/N0 value is high enough the error propagation is reduced, but this will delay the convergence of the LDPC decoding process and causes the error floor. To prevent this issue we will analyze the distribution of extrinsic information $LLR_{i,j}$ values (Log Likelihood Ratio) passed between nodes during the iterative LDPC decoding with the different the number of decoding iterations and Eb/N0 values in the next section.

To lead to the novel method predicting the convergence of the iterative LDPC decoding process we will analyze the distribution of $LLR_{i,j}$ values via the number of decoding iterations. We will simulate the histogram of $LLR_{i,j}$ values with the different parameters (The size of LDPC code word: 60, 120; code rate: $\frac{1}{2}$ Number of code words: 1000; Eb/N0 ratio: 4 dB; Modulation: BPSK). The LDPC is used in this simulation having the parity check matrix structure and using the decoding method proposed.

The distribution of the extrinsic information values after 2, 4 and 15 decoding iterations are plotted in the Figs. 1, 2 and 3. The transmission channel is the AWGN channel, the modulation is BPSK and the Eb/N0 = 4 dB. Observing the Figs. 1, 2 and 3, the distribution of the extrinsic information $LLR_{i,j}$ is changed via different numbers of decoding iterations. Those $LLR_{i,j}$ values are expanded toward two sides of the horizontal axes when the number of decoding iteration

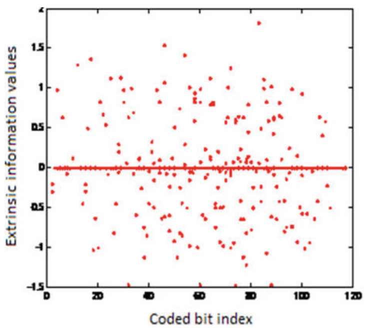


**Fig. 1.** The distribution of extrinsic information $LLR_{i,j}$ at Eb/N0 = 4 dB, Itermax = 2.

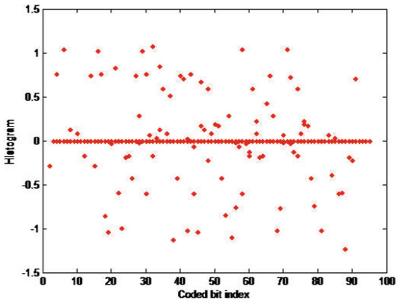


**Fig. 2.** The distribution of extrinsic information $LLR_{i,j}$ at Eb/N0 = 4 dB, Itermax = 4.

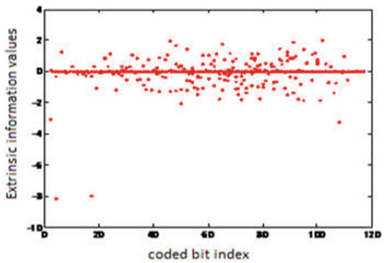


**Fig. 3.** The distribution of extrinsic information $LLR_{i,j}$ at Eb/N0 = 4 dB, Itermax = 15.

changes from 2 to 4, but most of them are converged around the horizontal axis at the 15th iteration. We can identify as following:

– At the number of decoding iterations equals to 2, most $LLR_{i,j}$ values concentrate near to the horizontal axis and when increasing the number of decoding iterations those values will be expanded to two sides of the horizontal axis as observed in the Fig. 2. However, when increasing the number of decoding iterations to 15 those above values will be converged back around the horizontal axis. This means that with the number of decoding iterations higher than 15 the values of the extrinsic information will be not so much increased. In the other word, there is no more valuable gain when increasing the number of decoding iterations over 15.
– There are quite a lot $LLR_{i,j}$ values equal to zero at different decoding iterations. This means that existing a lot of nodes not involved to the extrinsic information transferring process. This is caused because of the He having low density. The He having low density will prevent the error propagating during the LDPC decoding iteration. However, this also creates the error floor issue in decoding LDPC codes.

- By observing the distribution of extrinsic information values it is also provide for us a new method to analyze the convergent of the LDPC decoding having the same utility in comparison with the EXIT chart (Extrinsic Information Transfer) [5] or Density Evolution [6] methods. In our simulation, the $LLR_{i,j}$ values will be reduce toward the horizontal axis after the 15th iteration at Eb/N0 = 4 dB. This means that the LDPC decoding is almost converged after 15 decoding iterations. We will stop the decoding process after the 15th iteration at Eb/N0= 4 dB instead of continuing to iterate more the LDPC decoding. This help to reduce a lot the complexity of the decoding process.

By observing the distribution of the extrinsic information values $LLR_{i,j}$ at the different Eb/N0 ratios we also can improve the BER performance of LDPC codes by using the reliable $LLR_{i,j}$ values as presented in the following section.

## 3 A Method to Improving the Performance of LDPC Codes by Using the Reliable Extrinsic Information Values During the Iterative Decoding

Figures 4 and 5 are the distribution of the information values versus different Eb/N0 values at the number of decoding iterations equals to 2. As observing in the Figs. 4 and 5 we can notice that:

- At the low Eb/N0 values, the error transferring probability increases from the first to the second decoding iterations and then decreases from the second to the 15th iterations. Therefore, at the low Eb/N0, if we increase the number of decoding iterations to more than 2 times the BER will be increase, accordingly. The error increases because the iterative decoding process propagates errors via passing the error extrinsic information from one node to other related nodes.
- When the Eb/N0 values increase the extrinsic information values $LLR_{i,j}$ are also increase and their distribution will be expanded to two sides of the horizontal axis as seen in the above figures. At the high enough Eb/N0, the $LLR_{i,j}$ values are more reliable.

With the number of decoding iterations is bigger than 2 such as 15 times, the distribution of extrinsic information values $LLR_{i,j}$ are expanded to two side of zero axis. There are not so many abnormal values. Both the error and correct extrinsic information are propagated after each decoding iteration hence we could not identify the reliable extrinsic information values.

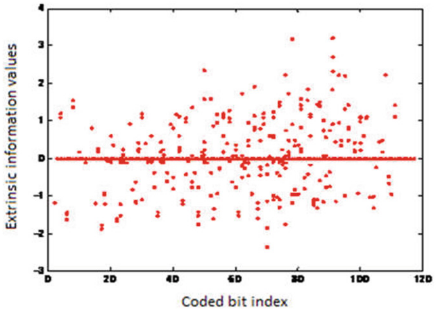


**Fig. 4.** The distribution of extrinsic information $LLR_{i,j}$ values at Eb/N0 = 2 dB after the 2nd iteration.

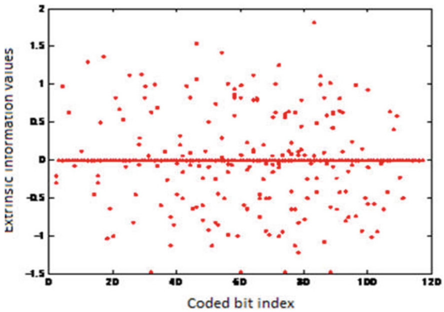


**Fig. 5.** The distribution of extrinsic information $LLR_{i,j}$ values at Eb/N0 = 4 dB after the 2nd iteration.

With the number of decoding iterations equals to 2, the $LLR_{i,j}$ values are distributed very close to the horizontal axis. Most of them are smaller than ±1.5. There are some values are bigger than ±1.5. We can say at the number of iterations equals to 2 the error extrinsic information is prevented from propagating to different nodes.

We need to consider to choice the right threshold at the as small as possible number of decoding iterations to prevent the error propagating issue in advance and also to reduce the total complexity of the iterative decoding. The decoding process in Fig. 6 is explained as following:

– At the first step: The soft bit yi from the demodulator are passed to the input of the decoder.
– The initial $P^1{}_j$ values are set to these soft bit values.
– Calculating the extrinsic information ratio $LLR_{i,j}$ values and check with the given threshold.
– If those values are satisfied the condition: $|LL\mathrm{R}_{i,j}| \leq \pm 1.5$ reset these values to zero and updating the values $PR_{i,j}$ with the Eq. (3).
– If it is not, updating the values $PR_{i,j}$ with the Eq. (3).

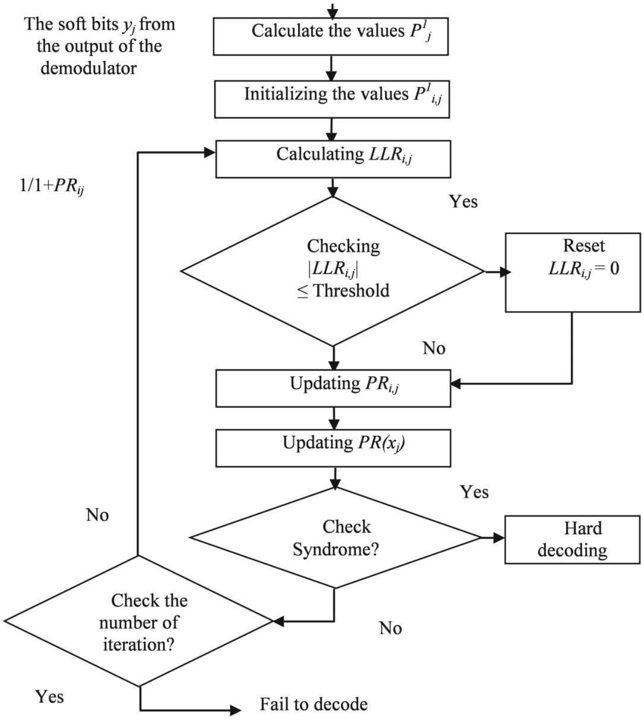


**Fig. 6.** The iterative decoding process based on reliable extrinsic information.

– Then updating the values $PR(x_j)$ with the Eq. (4).
– Checking the soft syndrome, if it is satisfied then pass the $PR(x_j)$ values to hard bit decoder and get the hard bits at the output. If it is not satisfied feeding back the value $1/(1 + PRi, j)$ to establishing the probabilities $P^1{}_{i,j}$ and continue with the next decoding processes. The simulation results of the novel decoding method are shown in the next section of this paper.

## 4 Simulation Result

The simulation parameters are listed in the Sect. 3. The LDPC is used in this simulation having the parity check matrix structure and using the BPA-EMS (Belief Propagation Algorithm based on Equivalent Parity Check Matrix and Minimum Weight of Syndrome) and BPA (Belief Propagation Algorithm) decoding method.

Figures 7 and 8 are the simulation BER performance versus Eb/N0 of LDPC codes using the BPA-EMS and BPA decoding method and our proposed method which uses the BPA-EMS and BPA decoding method based on the reliable extrinsic information to decode LDPCs after 10 decoding iterations.

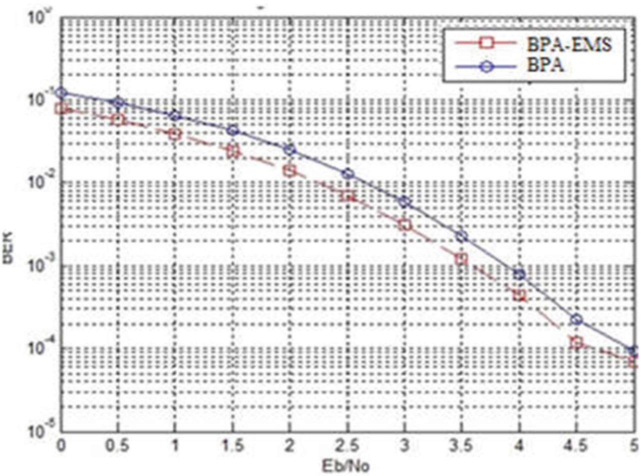


**Fig. 7.** The BER performance of LDPCs using the BPA-EMS and BPA decoding methods, 10 iterations, modulation BPSK in AWGN channel.

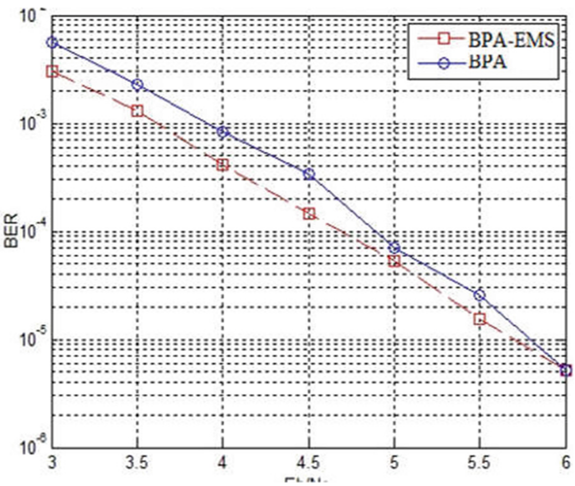


**Fig. 8.** The BER performance of LDPCs using the BPA-EMS and BPA decoding methods based on the reliable extrinsic information, number of iterations equals to 10 in AWGN channel.

As we can see in Figs. 7 and 8, LDPCs using the BPA-EMS and BPA decoding methods require the $Eb/N0 \geq 0.5\,\text{dB}$ to achieve the $BER = 10^{-4}$, while if LDPCs using our proposed decoding method require only 4.5 dB.

Figures 9 and 10 are the simulation BER performances versus $Eb/N0$ of LDPCs using the BPA-EMS and BPA decoding methods and our proposed after 15 decoding iterations. To archive the same $BER = 10^{-6}$ LDPCs using the BPA-

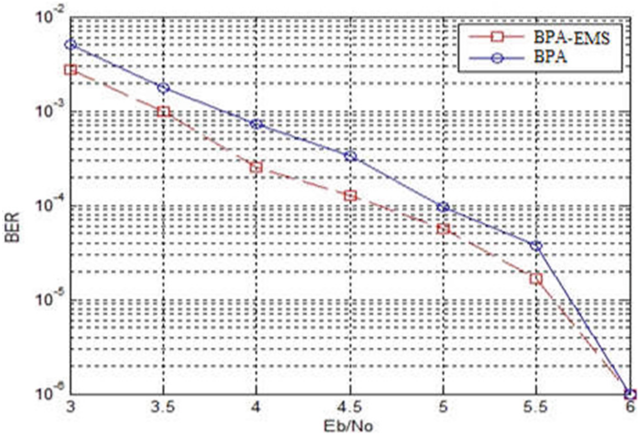


**Fig. 9.** The BER performance of LDPCs using the BPA-EMS and BPA decoding methods, the number decoding iterations is 15, modulation BPSK in AWGN channel.

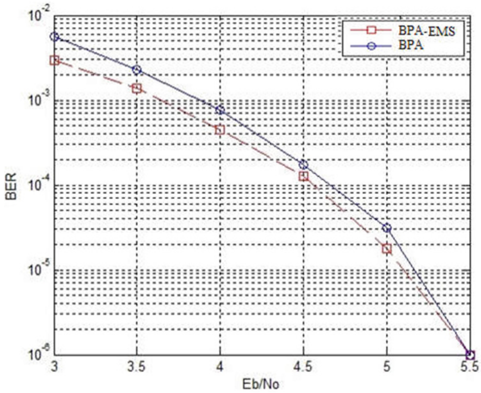


**Fig. 10.** The BER performance of LDPCs using the BPA-EMS and BPA decoding methods based on the reliable extrinsic information, the number of decoding iterations is 15.

EMS and BPA decoding methods require up to $Eb/N0 = 6\,\text{dB}$, while LDPCs using our proposed method only need $Eb/N0 = 5.5\,\text{dB}$.

## 5 Conclusion

In this paper we proposed our novel contributions which are a new method to analyze the convergence behavior of LDPC decoding process and an improved decoding method based on reliable extrinsic information to limit the error propagation during the iterative decoding of LDPCs. By using two methods proposed in this paper, the BER versus $Eb/N0$ performance of LDPCs gains 0.5 dB and the complexity of the LDPC decoding process is also reduced a lot due to predicting the optimal number of decoding iterations. In the coming research we will concentrate to optimize these two methods to achieve better performances of LDPCs.

## References

1. Chung, S.Y., Richardson, T.J., Urbanke, R.L.: Analysis of sum-product decoding of low densityparity check codes using a Gaussian approximation. IEEE Trans. Inf. Theory **47**, 599–618 (2001)
2. ten Brink, S.: Convergence behavior of iteratively decoded parallel concatenated codes. IEEE Trans. Commun. **49**, 1727–1737 (2001)
3. Hanzo, L., Liew, T.H., Yeap, B.L., Ng, S.X.: Turbo Coding, Turbo Equalisantion and Space Time Coding for Transmission Over Fading Channels. Wiley and IEEE (2002)
4. Tanner, R.M.: A recursive approach to low complexity codes. IEEE Trans. Inf. Theory **27**(5), 533–547 (1981)
5. Kollu, J., Hamid, J.: On the EXIT chart analysis of low density parity check codes. In: IEEE Global Telecommunications Conference, vol. 3 (2005)
6. Chen, J., Fossorier, M.P.C.: Density evolution for two improved BP-based decoding algorithms of LDPC codes. Commun. Lett. **6**, 208–210 (2002). IEEE