Channel Quantization Based Physical-Layer Network Coding for MIMO Two-Way Relay Networks

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***Abstract*—In this paper, a new physical-layer network coding (PNC) scheme is proposed for multiple-input multiple-output two-way relay networks where all network nodes, including two terminal nodes and the relay node, are equipped with multiple antennas. Spatial division multiplexing (SDM) technique is used at the two source nodes to increase transmission rate while the physical-layer network coding based on channel quantization (CQ-PNC) is used at the relay node to achieve diversity gain. The simulation results show that the proposed scheme outperforms significantly both of the previous channel quantization based PNC (CQ-PNC) and SDM-PNC schemes.**

***Index Terms*—two-way relay wireless network, spatial division multiplexing, channel quantization, physical-layer network coding.**

I. INTRODUCTION

In recent years, various solutions for improvement of both quality of service and transmission rate of wireless communication networks have been considered in the literature. A typical one which has gained a lot of attention is the so-called network coding (NC). NC uses linear coding at the relay node to reduce the number of transmission phases as compared with the traditional bidirectional relaying [1]. In order to increase spectral efficiency, a special type of NC, which is referred to as the physical-layer network coding (PNC), was proposed [2]. Following the introduction of PNC, a large number of researches aiming at improving the system performance of PNC have been reported [3]–[6], [8]–[11]. Among the improved PNC schemes the channel quantization based PNC (CQ-PNC) proposed by Zhang *et. al.* [3]–[6] has attracted more attention as it can achieve significant performance improvement while requiring low computational complexity. In the CQ-PNC scheme, the two terminal nodes with single antenna communicate with each other via the help of a relay having two antennas. In order to perform PNC, the channel matrix is firstly decomposed using the QR decomposition. The relay then uses the channel quantization method to map a weighted linear combination of two transmitted symbols from the terminal nodes into a PNC form. When quantization is performed for either the real or imaginary part, the CQ-PNC system attains significant performance improvement as compared with the traditional network coding based on QR decomposition algorithm (QR-NC) [3], [4]. If quantization is taken for both the real and imaginary part, the CQ-PNC system can achieve full diversity order of 2 [5], [6].

Meanwhile, spatial division multiplexing (SDM) is known as a typical technique to achieve multiplexing gain for multiple-input multiple-output (MIMO) systems [7]. Thus, if the full MIMO configuration is used for a two-way relay network, combination of SDM and PNC will be a promising approach to improve both spectral efficiency and system performance. In [8], the authors proposed an SDM-PNC scheme where each terminal node has two antennas, while the relay node has four. The relay node uses linear processing to map the sum and the difference of a pair of transmitted symbols into a PNC form. As a result, the data transmission rate of the system is double as compared with the previous systems. In [9], a two-way relay scheme is proposed by Hui *et. al.*, where all nodes employs multiple antennas. The terminal nodes use zero-forcing (ZF) precoder/decoder and SDM while the relay node uses PNC together with max-min antenna selection and maximum likelihood (ML) detection. The system achieves diversity order $N_R - N_T + 1$ at the cost of increased computational complexity, where N_R and N_T are the number of receive and transmit antennas, respectively. In [10], a similar scheme is proposed for SDM-PNC system. Network coding form is performed based on computing the Euclidean distance between symbols at the relay node in this system. However, the computational complexity of the system is still as large as of ML detection.

In this paper, we propose a new PNC scheme for MIMO two-way relay network (MIMO-TWRN) where both terminal nodes have two antennas while the relay node is equipped with K antennas. In our proposed scheme, SDM is used at the two terminal nodes during the multiple access control (MAC) phase and CQ-PNC is performed at the relay node during the broadcast (BC) phase. The scheme is referred to as the SDM-CQ-PNC one. Although the SDM-CQ-PNC scheme is based on the idea of the CQ-PNC scheme [5], [6] and the SDM-PNC scheme [8] it differs from these schemes in the following points. Firstly, the SDM-CQ-PNC scheme uses SDM but the CQ-PNC [5], [6] does not. Secondly, the SDM-CQ-PNC scheme uses channel quantization to map a Gaussian integer weighted linear combination of 2 symbols into a PNC form while SDM-PNC scheme [8] uses linear detection based the log-likelihood estimation and selective combination. Moreover, the biggest difficulty of using spatial division multiplexing is to remove the co-channel interference between the signal streams. In the traditional network coding based on QR decomposition algorithm (QR-NC), the estimated value of the lower layer is used to cancel interference of the upper layer. However in this paper, only the residual interference, but not the full interference of the lower layer is used instead. By doing so, the error propagation problem of the proposed scheme is significantly reduced as compared with the traditional QR-NC.

The rest of this paper is organized as follows. Section II presents the proposed scheme and computational steps to obtain the PNC symbols. Simulation results are presented in Section III and, finally, conclusions are drawn in Section IV.

Throughout this paper, we will use the following mathematical notation. Bold letters denote vectors or matrices with a lower-case letter representing a vector and an upper-case letter A matrix. $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $|\cdot|$, and $\|.\|_F^2$ denote the matrix transpose, conjugate transpose, absolute value, and the Frobenious norm, respectively. $\mathcal{N}_c\left(0,\, \sigma_n^2\right)$ denotes a complex Gaussian distribution with zero mean and covariance σ_n^2 ; $\mathbf{A}^{K\times K}$ denotes a $K \times K$ matrix and $\mathbf{I}_K$ a $K \times K$ unit matrix.

II. System Model

A. Proposed Model of SDM-CQ-PNC

We consider a general model of a two-way relay system as illustrated in Fig. 1, where each terminal node $\mathrm{N}_q,\ (q = 1, 2)$ is equipped with two antennas. The two terminal nodes exchange data with each other through a relay node R which is equipped with $K > 2$ antennas.

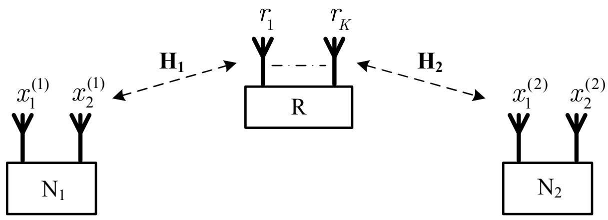

Fig. 1. System model of SDM-CQ-PNC.

It is assumed that there is no direct link between the terminal nodes and that the half-duplex mode is used at all nodes. Moreover, channel state information (CSI) is assumed to be known at the receiving node but not at the transmitting one. The channel between each pair of nodes is assumed to undergo independent uncorrelated Rayleigh fading. The channel gain is invariant during a packet transmission period but varies independently from one packet to another.

Let $\mathbf{x}^{(q)} = \left[x_1^{(q)}, x_2^{(q)}\right]^{\mathrm{T}}$ be the transmit vector of the q-th terminal node N_q. The channel matrices $\mathbf{H}_1$ between N_1 and R and $\mathbf{H}_2$ between N_2 and R are defined as follows:

$$\mathbf{H}_1 = \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} \\ h_{21}^{(1)} & h_{22}^{(1)} \\ \vdots & \vdots \\ h_{K1}^{(1)} & h_{K2}^{(1)} \end{bmatrix}, \mathbf{H}_2 = \begin{bmatrix} h_{11}^{(2)} & h_{12}^{(2)} \\ h_{21}^{(2)} & h_{22}^{(2)} \\ \vdots & \vdots \\ h_{K1}^{(2)} & h_{K2}^{(2)} \end{bmatrix},$$

where $h_{kv}^{(q)}$ denotes the channel gain between the k-th antenna $(k = 1, 2, ..., K)$ of the relay node R to the v-th antenna $(v = 1, 2)$ of the terminal node N_q. Each element $h_{kv}^{(q)}$ is modeled by a complex Gaussian variable with zero-mean and unit-variance, *i.e.*, $h_{kv}^{(q)} \sim \mathcal{N}_c\,(0, 1)$.

The two terminal nodes N_q communicate with each other over two phases, namely, the MAC and the BC one. Since the operation of the PNC system during the former is equivalent to that of a point-to-point system, we will focus only on the latter. During the MAC phase, the two terminal nodes N_q simultaneously transmit their symbols to the relay node. It is assumed that the transmitted symbols from the two terminal nodes arrive at the relay node simultaneously, *i.e.*, there is no propagation delay between the two terminal nodes N_q to the relay node R. The received signal vector at the relay node is given by:

$$\begin{aligned} \mathbf{y} &= \frac{1}{\sqrt{2}}\mathbf{H}_1\mathbf{x}^{(1)} + \frac{1}{\sqrt{2}}\mathbf{H}_2\mathbf{x}^{(2)} + \mathbf{n} \\ &= \frac{1}{\sqrt{2}}\mathbf{H}\mathbf{x} + \mathbf{n}. \end{aligned} \tag{1}$$

Here $\mathbf{y} = [y_1, y_2, ..., y_K]^{\mathrm{T}}$, where y_k is the received signal at the k-th antenna branch of the relay node; $\mathbf{x} = \left[x_1^{(1)}, x_2^{(1)}, x_1^{(2)}, x_2^{(2)}\right]^{\mathrm{T}}$; the fraction $\frac{1}{\sqrt{2}}$ is the power normalization factor; $\mathbf{n} = [n_1, \ldots, n_K]^{\mathrm{T}}$ is the noise vector, where each element n_k denotes the additive white Gaussian noise (AWGN) at the k-th received antenna of the relay node, $n_k \sim \mathcal{N}_c\left(0, \sigma_n^2\right)$. The equivalent channel $\mathbf{H}$ in (1) can be expressed as follows:

$$\mathbf{H} = [\mathbf{H}_1\, \mathbf{H}_2] = \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} & h_{11}^{(2)} & h_{12}^{(2)} \\ h_{21}^{(1)} & h_{22}^{(1)} & h_{21}^{(2)} & h_{22}^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ h_{K1}^{(1)} & h_{K2}^{(1)} & h_{K1}^{(2)} & h_{K2}^{(2)} \end{bmatrix}. \quad (2)$$

From the received vector $\mathbf{y}$ the relay node R needs to estimate a pair of PNC symbols $x_1^{(1)} \oplus x_2^{(2)}$ and $x_2^{(1)} \oplus x_1^{(2)}$, where $x_1 \oplus x_2 \triangleq \mathrm{Re}(x_1)\mathrm{Re}(x_2) + j\mathrm{Im}(x_1)\mathrm{Im}(x_2)$ and $\mathrm{Re}(x)$, $\mathrm{Im}(x)$ denote the real and imaginary part of x, respectively. In the following, we will denote the estimate of the PNC symbol $x_1 \oplus x_2$ by $\widehat{x_1 \oplus x_2}$.

In order to get the estimate of the first PNC symbol $\widehat{x_1^{(1)} \oplus x_2^{(2)}}$ we swap the first with the third column of $\mathbf{H}$ in (2) to get a new equivalent channel matrix

$$\tilde{\mathbf{H}}_1 = \begin{bmatrix} h_{11}^{(2)} & h_{12}^{(1)} & h_{11}^{(1)} & h_{12}^{(2)} \\ h_{21}^{(2)} & h_{22}^{(1)} & h_{21}^{(1)} & h_{22}^{(2)} \\ \vdots & \vdots & \vdots & \vdots \\ h_{K1}^{(2)} & h_{K2}^{(1)} & h_{K1}^{(1)} & h_{K2}^{(2)} \end{bmatrix}. \quad (3)$$

This equivalent matrix $\tilde{\mathbf{H}}_1$ can be decomposed as follows: $\left(\mathbf{Q}^{(1)}, \mathbf{R}^{(1)}\right) = \mathrm{qr}\left(\frac{1}{\sqrt{2}}\tilde{\mathbf{H}}_1\right)$, where $\mathrm{qr}(\cdot)$ denotes the QR decomposition operation; $\mathbf{Q}^{(1)}$ is a unitary matrix with $\mathbf{Q}^{(1)}\mathbf{Q}^{(1)\mathrm{H}} = \mathbf{I}_K$, $\mathbf{R}^{(1)}$ is a $K \times 4$ upper triangular matrix. Multiplying both sides of (3) by $\mathbf{Q}^{(1)\mathrm{H}}$ and defining $\mathbf{w}^{(1)} = \mathbf{Q}^{(1)\mathrm{H}}\mathbf{y}$, $\mathbf{u}^{(1)} = \mathbf{Q}^{(1)\mathrm{H}}\mathbf{n}$ while noting that $\mathbf{R}^{(1)} = \frac{1}{\sqrt{2}}\mathbf{Q}^{(1)\mathrm{H}}\tilde{\mathbf{H}}_1$, we obtain the following equivalent system equation:

$$\mathbf{w}^{(1)} = \mathbf{R}^{(1)}\left[x_1^{(2)}, x_2^{(1)}, x_1^{(1)}, x_2^{(2)}\right]^{\mathrm{T}} + \mathbf{u}^{(1)}. \quad (4)$$

Similarly, the estimate of the second PNC symbol $\widehat{x_2^{(1)} \oplus x_1^{(2)}}$ can be obtained by swapping the second with the first column and the third with fourth column of $\tilde{\mathbf{H}}_1$ in the equation (3). Because of the similarity in the process to obtain the estimates of the two PNC symbols, in the following we will limit our presentation only for the first symbol $\widehat{x_1^{(1)} \oplus x_2^{(2)}}$.

B. Physical-Layer Network Coding Based on Channel Quantization

1) The simple SDM-CQ-PNC: Without loss of generality, let us consider the case $K = 4$. From (4), the received signals are given by:

$$w_3^{(1)} = r_{33}^{(1)}x_1^{(1)} + r_{34}^{(1)}x_2^{(2)} + u_3^{(1)} \quad (5)$$

$$w_4^{(1)} = r_{44}^{(1)}x_2^{(2)} + u_4^{(1)}. \quad (6)$$

The simple SDM-CQ-PNC consists of the following four steps:

Step 1: Estimate the soft estimate of $x_2^{(2)}$ from equation (6). This can be done as follows:

$$\hat{x}_2^{(2)} = \tanh\left(\frac{r_{44}^{(1)}w_4^{(1)}}{\sigma_n^2}\right). \quad (7)$$

Step 2: Channel quantization. Divide both sides of (5) by the quantization step $r_{33}^{(1)}$ we have

$$u_0 = \frac{w_3^{(1)}}{r_{33}^{(1)}} = x_1^{(1)} + Lx_2^{(2)} + lx_2^{(2)} + \frac{u_3^{(1)}}{r_{33}^{(1)}}, \quad (8)$$

where L and l are given by:

$$L = L_r + jL_i = \mathrm{round}\left(\frac{r_{34}^{(1)}}{r_{33}^{(1)}}\right) \quad (9)$$

$$l = l_r + jl_i = \frac{r_{34}^{(1)}}{r_{33}^{(1)}} - L, \quad (10)$$

$\mathrm{round}(\cdot)$ denotes the rounding operator to a nearest complex integer, $j^2 = -1$ and L_r, l_r, L_i, l_i are the real and imaginary parts of L and l, respectively.

Step 3: Canceling the residual interference $lx_2^{(2)}$ in (8). In the conventional QR-NC, the estimate $\hat{x}_2^{(2)}$ of $x_2^{(2)}$ obtained from equation (6) is used to cancel $x_2^{(2)}$ from equation (5) to estimate $\hat{x}_1^{(1)}$. If $\hat{x}_2^{(2)}$ is perfectly estimated, it will not affect the estimate of $x_1^{(1)}$, denoted by $\hat{x}_1^{(1)}$. However, it is very difficult to achieve this assumption, and this may lead to incorrect estimation of $\hat{x}_1^{(1)}$. In order to reduce this estimation error, instead of $x_2^{(2)}$ only the residual interference of $x_2^{(2)}$, *i.e.* $lx_2^{(2)}$, is canceled from equation (8). As a result, the residual noise due to estimation error of $\hat{x}_2^{(2)}$ is reduced and the error performance of the proposed scheme is improved over the conventional QR-NC.

After removing the residual interference from equation (8) we have

$$u_1 = x_1^{(1)} + Lx_2^{(2)} + l\delta_{x_2} + \frac{u_3^{(1)}}{r_{33}^{(1)}}, \quad (11)$$

where $\delta_{x_2} = x_2^{(2)} - \hat{x}_2^{(2)}$ is the estimation error due to imperfect estimation of $\hat{x}_2^{(2)}$. Assume that QPSK modulation is used for transmission. Then the transmitted symbol from the m-th antenna of the terminal node N_i can be expressed as $x_m^{(q)} = 2s_m^{(q)} - (1+j)$, where $m = 1, 2$ and $s_m^{(q)} \in \{0, 1, j, 1+j\}$ is the unmodulated signal of $x_m^{(q)}$. Using this expression, the transmitted symbol $x_m^{(q)}$ in equation (11) can be transformed back to its unmodulated signal as follows:

$$\begin{aligned}\tilde{u}_1 &= \frac{1}{2}\left[u_1 + (1+j) + L(1+j)\right]\\ &= \frac{1}{2}\left[(x_1^{(1)} + 1 + j) + L(x_2^{(2)} + 1 + j) + l\delta_{x_2} + \frac{u_3^{(1)}}{r_{33}^{(1)}}\right]\\ &= s_1^{(1)} + Ls_2^{(2)} + \frac{l}{2}\delta_{x_2} + \frac{u_3^{(1)}}{2r_{33}^{(1)}}. \end{aligned} \tag{12}$$

Then the hard decision of $s_1^{(1)} + Ls_2^{(2)}$ can be obtained as follows:

$$\begin{aligned}\hat{u}_1 &= \mathrm{round}\left(\tilde{u}_1\right) = s_1^{(1)} + Ls_2^{(2)} + e\\ &= s_{1,2} + e,\end{aligned} \tag{13}$$

where $e = \mathrm{round}\left(\frac{l}{2}\delta_{x_2} + \frac{u_3^{(1)}}{2r_{33}^{(1)}}\right)$ is the estimation error due to rounding operation and $s_{1,2} \triangleq s_1^{(1)} + Ls_2^{(2)}$ is the desired part that we need to detect.

Step 4: PNC mapping. The network coding form depends on the value of L_r and L_i and can be given for the following 5 cases.

Case 1: L_r is odd and L_i is even. Note that

$$\begin{aligned}s_{1,2} \bmod 2 &= \left[\left(s_{1,r}^{(1)} + L_r s_{2,r}^{(2)} - L_i s_{2,i}^{(2)}\right) \bmod 2\right]\\ &\quad + j\left[\left(s_{1,i}^{(1)} + L_r s_{2,i}^{(2)} + L_i s_{2,r}^{(2)}\right) \bmod 2\right]\\ &= \left[\left(s_{1,r}^{(1)} + s_{2,r}^{(2)}\right) \bmod 2\right]\\ &\quad + j\left[\left(s_{1,i}^{(1)} + s_{2,i}^{(2)}\right) \bmod 2\right]\\ &= s_1^{(1)} \oplus s_2^{(2)},\end{aligned} \tag{14}$$

where $s_{m,r}^{(q)}$ and $s_{m,i}^{(q)}$ are the real and the imaginary part of $s_m^{(q)}$, respectively. Based on the observation that $x_1^{(1)} \oplus x_2^{(2)} = 2\left(s_1^{(1)} \oplus s_2^{(2)}\right) - (1+j)$, the estimated PNC symbol can be obtained by using the following decision rule:

$$\widehat{x_1^{(1)} \oplus x_2^{(2)}} = 2\left(\hat{u}_1 \bmod 2\right) - (1+j). \tag{15}$$

Case 2: L_r is even and L_i is odd. Similar to the first case, note that

$$\begin{aligned}s_{1,2} \bmod 2 &= \left[\left(s_{1,r}^{(1)} + L_r s_{2,r}^{(2)} - L_i s_{2,i}^{(2)}\right) \bmod 2\right]\\ &\quad + j\left[\left(s_{1,i}^{(1)} + L_r s_{2,i}^{(2)} + L_i s_{2,r}^{(2)}\right) \bmod 2\right]\\ &= \left[\left(s_{1,r}^{(1)} + s_{2,i}^{(2)}\right) \bmod 2\right]\\ &\quad + j\left[\left(s_{1,i}^{(1)} + s_{2,r}^{(2)}\right) \bmod 2\right]\\ &= s_1^{(1)} \oplus \left(js_2^{(2)}\right).\end{aligned} \tag{16}$$

Also, based on the observation that $x_1^{(1)} \oplus jx_2^{(2)} = 2\left(s_1^{(1)} \oplus js_2^{(2)}\right) - (1+j)$, the estimated PNC symbol is achieved by using the following decision rule:

$$\widehat{x_1^{(1)} \oplus jx_2^{(2)}} = 2\left(\hat{u}_1 \bmod 2\right) - (1+j). \tag{17}$$

Case 3: both L_r and L_i are even or odd and $|L_r| > 1$ or $|L_i| > 1$. In this case, it is easy to see that $|L| \geq 2$ and the received signal power of $x_2^{(2)}$ is much larger than that of the $x_1^{(1)}$, so the decision rules are chosen as follows:

$$\hat{x}_2^{(2)} = \mathrm{sign}\left(\frac{u_1}{L}\right) \tag{18}$$

$$\hat{x}_1^{(1)} = \mathrm{sign}\left(u_0 - (L+l)\,\hat{x}_2^{(2)}\right). \tag{19}$$

The estimated PNC symbol is then simply given by $\widehat{x_1^{(1)} \oplus x_2^{(2)}} = \hat{x}_1^{(1)} \oplus \hat{x}_2^{(2)}$.

Case 4: $|L_r| = |L_i| = 1$. In this case, unfortunately $x_1^{(1)} + (L_r + jL_i)\,x_2^{(2)}$ cannot be mapped into a point in the QPSK constellation. Alternatively, it can be mapped into a 5-QAM constellation point $\left(-j3/\sqrt{55} \pm 16/\sqrt{165} - j3\sqrt{55} \pm 8/\sqrt{165} + j\sqrt{5/11}\right)$. The detailed mapping rule can be found in [5], [6], [11].

Case 5: $L_r = L_i = 0$. The estimate $\hat{x}_1^{(1)}$ can be obtained using hard decision $\hat{x}_1^{(1)} = \mathrm{sign}\left(u_1\right)$. Then soft estimate $\tilde{x}_2^{(2)}$ can be given by $\tilde{x}_2^{(2)} = \frac{r_{33}^{(1)}}{r_{34}^{(1)}}\left(u_0 - \hat{x}_1^{(1)}\right)$. The estimate $\hat{x}_2^{(2)}$ can be then obtained by using the Maximal-Ratio Combining (MRC) of $\tilde{x}_2^{(2)}$ with $w_4^{(1)}$ in (6) as follows

$$\hat{x}_2^{(2)} = \mathrm{sign}\left(\frac{\left|r_{34}^{(1)}\right|^2 \tilde{x}_2^{(2)} + r_{44}^{(1)} w_4^{(1)}}{\left|r_{34}^{(1)}\right|^2 + \left|r_{44}^{(1)}\right|^2}\right). \tag{20}$$

If $K = 3$, there is no $w_4^{(1)}$ in equation (6), the hard decision of $\hat{x}_1^{(1)}$ and $\hat{x}_2^{(2)}$ are then given by:

$$\hat{x}_1^{(1)} = \mathrm{sign}\left(u_0\right) \tag{21}$$

$$\hat{x}_2^{(2)} = \mathrm{sign}\left(\frac{r_{33}^{(1)}}{r_{34}^{(1)}}\left(u_0 - \hat{x}_1^{(1)}\right)\right). \tag{22}$$

Finally, the estimated PNC symbol can be simply given by: $\widehat{x_1^{(1)} \oplus x_2^{(2)}} = \hat{x}_1^{(1)} \oplus \hat{x}_2^{(2)}$.

2) The Adaptive SDM-CQ-PNC: When the quantization error is large, for example, if $l_r \approx 0.5$, $l_i \approx 0.5$ and $x_2^{(2)} = 1 - j$ then the value of the residual interference $\left(l_r + jl_i\right) x_2^{(2)}$ tends to 1 and the error becomes very large. In this case, a smaller quantization step $\frac{r_{33}^{(1)}}{1+j}$ needs to be used. The combination of the two quantization steps $r_{33}^{(1)}$ and $\frac{r_{33}^{(1)}}{1+j}$ is called the adaptive quantization and such PNC scheme is referred to as the Adaptive SDM-CQ-PNC. On the other hand, from equation (11), it is clear that the residual interference $l\delta_{x_2}$ not only depends on the quantization error ($|l_r| + |l_i|$) but also the estimated error $\hat{x}_2^{(2)}$. Based on the two thresholds in [6], the Adaptive SDM-CQ-PNC has two cases:

If $r_{44}^{(1)} > \sqrt{\max\left\{0, 0.28\left(r_{33}^{(1)}\right)^2 - \sigma_n^2\right\}}$ or $|l_r|+|l_i| < \frac{2}{3}$, the residual interference is small enough, so the quantization step $r_{33}^{(1)}$ can be used. The following steps are similar with those in Sect. II.2.1.

If $r_{44}^{(1)} \leq \sqrt{\max\left\{0, 0.28r\left(_{33}^{(1)}\right)^2 - \sigma_n^2\right\}}$ and $1 > |l_r| + |l_i| \geq \frac{2}{3}$, the residual interference is large, so the smaller quantization step $\frac{r_{33}^{(1)}}{1+j}$ can be used. Equation (11) now becomes

$$u_1' = (1+j)x_1^{(1)} + L'x_2^{(2)} + l'\delta_{x_2} + (1+j)\frac{u_3^{(1)}}{r_{33}^{(1)}} \tag{23}$$

where:

$$\begin{aligned} &L' = L_r' + jL_i' \,,\; l' = l_r' + jl_i' \\ &L_r' = \mathrm{round}\left(L_r + l_r - L_i - l_i\right) \,, \\ &L_i' = \mathrm{round}\left(L_r + l_r + L_i + l_i\right) \,, \\ &l' = (1+j)\frac{r_{34}^{(1)}}{r_{33}^{(1)}} - L'. \end{aligned}$$

It is clear that if $\frac{2}{3} \leq |l_r| + |l_i| < 1$ then one of L_r' , L_i' must be odd and the other must be even [6]. Thus, there are the following three cases:

Case 1: if $L' \in \{\pm 1 \pm 2j, \pm 2 \pm j\}$, then $\hat{x}_1^{(1)}$ and $\hat{x}_2^{(2)}$ are estimated separately first, then they are combined to get the estimated PNC symbol $\widehat{x_1^{(1)} \oplus x_2^{(2)}} = \hat{x}_1^{(1)} \oplus \hat{x}_2^{(2)}$.

Case 2: if $|L'|^2 > 5$, the received signal power of $x_2^{(2)}$ is much larger than that of $x_1^{(1)}$. As a result, $\hat{x}_2^{(2)}$ is estimated first and then $\hat{x}_1^{(1)}$. Finally, they are combined to get the PNC symbol $\widehat{x_1^{(1)} \oplus x_2^{(2)}} = \hat{x}_1^{(1)} \oplus \hat{x}_2^{(2)}$

- Case 3: if $|L_r'| = 1$, $L_i' = 0$ or $L_r' = 0$, $|L_i'| = 1$ the mapping rule similar to that given in Case 4 of Sect. II.2.1 is used to get the estimated PNC symbol.

III. Performance Evaluation

This section presents the performance evaluation of the proposed SDM-CQ-PNC system using computer simulations. The simulation model includes two terminal nodes N_q equipped with two antennas and a relay node R with three or four antennas. The terminal nodes use SDM for transmission while the relay node uses the proposed channel quantization approach to map a Gaussian integer weighted linear combination of 2 symbols into a PNC symbol. In the simulations, QPSK is used for modulation. As for reference, we also compare the simulated results of the proposed scheme with those of the previous schemes. The system performance in terms of symbol error rate (SER) and effective throughput obtained at the relay node is used for evaluation.

When the relay node has three antennas, equation (8) shows that both SDM-PNC in [8] and the conventional QR-NC cannot estimate the PNC symbols using linear estimation because the number of the transmit antennas is larger than the number of the receive antennas. However, since the proposed SDM-CQ-PNC scheme is based on the combination of a weighted sum of the two terminal nodes' symbols, it can estimate a pair of the PNC symbols using linear estimation. Moreover, it is shown from Fig. 2 that the SER performance of the SDM-CQ-PNC with adaptive quantization is very close to that of the ML detection while the processing complexity is clearly lower than that of the ML detection. Fig. 2 also shows that the SER performance of the proposed SDM-CQ-PNC with adaptive quantization is significantly improved compared with no adaptive quantization. For example, about 2 dB SNR gain can be achieved at the SER of 10^{-2}.

Fig. 3 illustrates the SER performance of the SDM-CQ-PNC for the case the relay node has four antennas. It is clearly seen that both the SDM-PNC in [8] and the QR-NC only achieve the diversity order 1 while the maximum diversity order 2 can be obtained for the proposed SDM-CQ-PNC. As a result, at the SER of 10^{-3} the SDM-CQ-PNC outperforms the QR-NC by about 10 dB and the SDM-PNC by about 9.5 dB in SNR. This performance improvement becomes larger as SNR increases. It is also shown that the SDM-CQ-PNC with adaptive quantization can achieve the same performance

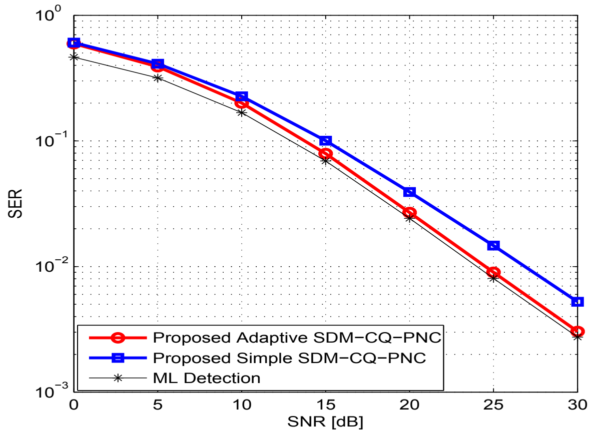

Fig. 2. Performance the SDM-CQ-PNC for $K = 3$.

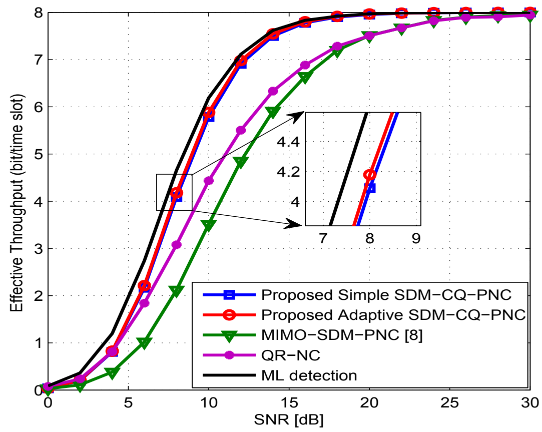

Fig. 4. Effective throughput of SDM-CQ-PNC with $M = 4$.

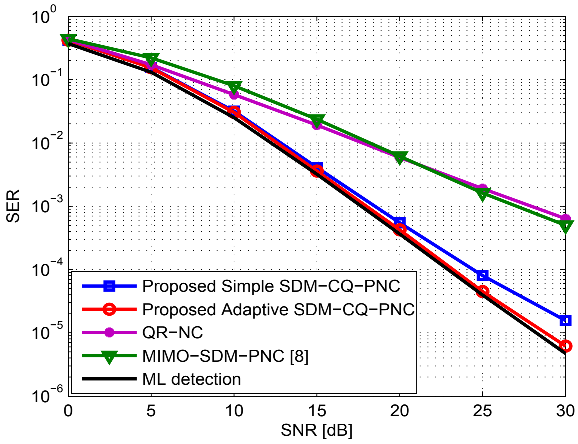

Fig. 3. Performance of the SDM-CQ-PNC for $K = 4$.

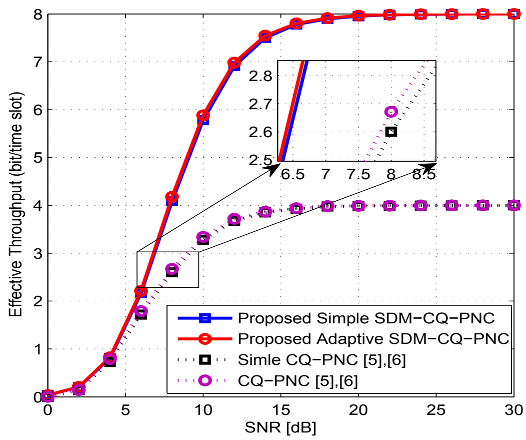

Fig. 5. Comparative effective throughput of different schemes.

of the ML detection while the SDM-CQ-PNC without adaptive quantization exhibits small performance loss.

Figs. 4 and 5 compare the effective throughputs of the proposed SDM-CQ-PNC with its counterparts. The effective throughput is calculated from the frame error rate (FER) as follows: $\eta = (1 - \mathrm{FER}) R \log_2 M$, where R is the symbol rate, and $M = 4$ is the modulation order. The simulated FER is obtained under the assumption that each frame has 20 symbols. The simulation results show that the proposed SDM-CQ-PNC scheme outperforms the existing schemes significantly. Specifically, in Fig. 4 at the effective throughput of 6 bits/time slot, the proposed SDM-CQ-PNC scheme achieves an SNR gain of 3 dB compared with the conventional QR-NC and that of 4 dB compared with the MIMO-SDM-PNC [8]. The effective throughput of the proposed SDM-CQ-PNC is approximately the same with that of the ML detection.

Fig. 5 further compares effective throughput results of different schemes. First, it is seen that the proposed system allows each terminal node to transmit 2 parallel streams using SDM, so it can achieve the maximum multiplexing gain of 2. Whereas the CQ-PNC in [5], [6] can only achieve the multiplexing gain of 1. This multiplexing gain is linearly increased with the number of the antennas at each terminal node. Specifically, for $\mathrm{SNR} > 10$ dB, the effective throughput of the proposed SDM-CQ-PNC scheme is nearly double that of the CQ-PNC in [5], [6].

IV. Conclusions

In this paper, a new SDM-CQ-PNC scheme for MIMO-TWRN has been proposed. In this model, each terminal node is equipped two antennas to implement spatial division multiplexing while the relay node has K antennas. The relay node uses the channel quantization approach to map a Gaussian integer weighted linear combination of two symbols into a PNC form. The SER performance of the proposed scheme can achieve near-maximum likelihood performance with lower computational complexity. In comparison to the SDM-PNC scheme [8], the proposed SDM-CQ-PNC scheme achieves an SNR gain of about 9.5dB. Moreover, the proposed scheme also has double effective throughput when compared with the conventional CQ-PNC scheme [5], [6].

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