Exact Outage Probability of Energy Harvesting Incremental Relaying Networks with MRC Receiver

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***Abstract*—In this paper, we consider an incremental cooperative relaying network (IR-CRN), where a relay node harvests the radio energy from the source signal using the time-switching mechanism. Specif cally, we propose a scheme named as IR-CRN which enhances spatial diversity gain by utilizing the incremental relaying protocol. The destination node of the IR-CRN scheme employs maximal ratio combining (MRC) technique to combine the signals from the source node and the relay node. For performance evaluation, we derive exact expressions of outage probability of the proposed system over Rayleigh fading channels with half-duplex based that energy harvesting relaying. Our results show that the system performances of the proposed scheme are much better than of the direct transmission (DT) scheme. In addition, the optimal time splitting and the position of the cooperative relay are also investigated. Our derivations are validated by Monte Carlo simulations.**

***Index Terms*—Dual-hop systems, half-duplex relaying, energy harvesting, maximal ratio combining, outage probability.**

I. Introduction

Recently, energy harvesting (EH) for wireless communications has become a promising technology to overcome the crucial battery problem of wireless devices and the battery-powers [1], [2]. Wireless devices or mobile terminals are equipped with the EH modules [2], which scavenge energy from their surrounding natural environment (solar, electromagnetic, radiation, vibration, etc.) and convert them to suitable electrical power, in order to supply operation. Recently, the application of the wireless energy harvesting on cooperative relaying networks has recently studied by many researchers in [3]–[6]. In the EH technique, the relay node scavenges the energy from the RF signals and consumes the harvested energy for supporting the transmission between the source and the destination node. In [4], the outage performance of a typical cooperative communication system was studied. In [4], the authors studied energy harvesting in an OSTBC based amplify-and-forward MIMO relay systems. However, the authors in [3], [4] assumed that the relay has its own internal energy source and does not need external charging. The problem of joint transfer of information and energy for multi-user systems has been investigated in [5]. The outage performance for amplify-and-forward (AF) relaying networks under EH constraints was derived in [6]. In [7], [8], the energy harvesting protocols based on the power splitting and time switching receivers in cooperative networks were studied. However, these works only considered the cooperative transmission in the absence of the direct link. For a decode-and-forward (DF) relaying network [9], power-allocation strategies and outage performance under the EH constraints were considered.

In cooperative communications, incremental relaying has been shown to provide signif cant performance benef ts in terms of spectral eff ciency and network performance. It has attracted considerable attention in recent years, for example, see [10]–[14]. In particular, the authors in [10] proposed the incremental relaying protocol with selection DF relaying to increase network capacity and performance. Considering AF relaying, the paper [11] studied cognitive underlay incremental relaying networks with Nth best partial relay selection. Over independent non-identical Rayleigh fading channels, Ikki *et. al.* in [12] presented a completely analytical approach in obtaining closed-form expressions for the error rate and SNR outage probability of incremental relaying cooperative diversity networks. As extension of [12], paper [13] and [14] presented a complete performance analysis of the Nth best-relay selection scheme and partial relay selection, respectively, for any SNR value over Rayleigh fading channels. In most of the work mentioned above that deals with dual relaying transmission, it is normally assumed that the direct link between the source and the destination does not exist due to showing and pathloss. However, in many practical scenarios, the relaying link is only requested if the direct link is not good enough for decoding. Recently, in [15], Son *et. al.* proposed incremental relaying with energy harvesting and derived the exact outage probability over Rayleigh fading channels. However, the obtained exact system outage probability is still in the form of a single integral making it computationally intensive.

In this paper, we for the f rst time to derive the exact outage probability for energy harvesting incremental relaying networks with MRC Receiver over Rayleigh fading channels using Taylor series expansion. The system outage probability is shown to be expressible in terms of inf nite series suitable for numerical evaluation. Numerical results are also provided to verify the analysis approach and to investigate effects of channel and system parameters on the system performance.

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We also show that for the same channel settings, the proposed system outperforms the direct transmission with 10 dB gain at outage probability of 10^{-1}.

The rest of the paper is organized as follows. The system model is described in Section II. In Section III, the outage probability performance is analyzed. The simulation results are presented in Section IV and Section V concludes the paper.

II. System Model

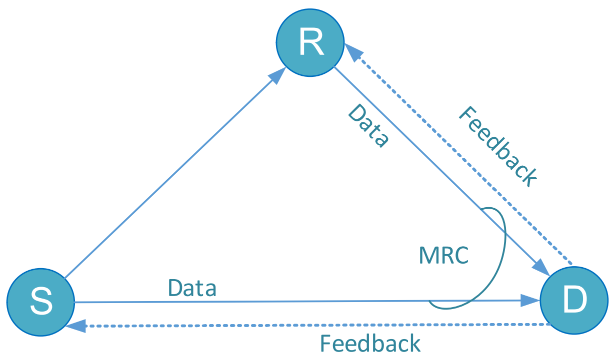

Fig. 1. The IR-CRN scheme with MRC receiver.

We consider the DF cooperative communication network consisting of one source node (S), one destination (D) and one relay node (R) as shown in Fig. 1. The source broadcasts the information which is heard by both the relay and the destination. We assume that each node is equipped with one antenna and the relay has the limited power supply. It must harvest energy from the source RF signals as in [6] for its operation.

We denote h_{XY} and d_{XY} as Rayleigh-fading coefficient and distance between two nodes X and Y, respectively, where $\mathrm{X}, \mathrm{Y} \in \{\mathrm{S}, \mathrm{R}, \mathrm{D}\}$. As stated in [11], the channel gain $\gamma_{\mathrm{XY}} \left(\gamma_{\mathrm{XY}} = |h_{\mathrm{XY}}|^2\right)$ follows an exponential distributions with parameter $\lambda_{\mathrm{XY}} = \mathbb{E}\{\gamma_{\mathrm{XY}}\} = d_{\mathrm{XY}}^{\beta}$, where β is the path-loss exponent and $\mathbb{E}\{.\}$ is an expectation operator. We assume that the fading channel coefficient h_{XY} independent and identically distribution during the transmission time. We further assume that the harvested energy at the relay is stored in a super-capacitor and then fully consumed to forward the source signal toward the destination.

We first model the received signal at the destination from the source as

$$y_{\mathrm{SD}} = \sqrt{P_{\mathrm{S}}} h_{\mathrm{SD}} x_{\mathrm{S}} + n_{\mathrm{SD}}, \tag{1}$$

where x is the normalized signal of the source, i.e., $\mathrm{E}\left\{|x_{\mathrm{S}}|^2\right\} = 1$, n_{SD} is the additive white Gaussian noise (AWGN) with zero-mean and variance N_0 and P_{S} is the average transmit power at S. The SNR of the S $\to$ D link is given as

$$\gamma_{\mathrm{SD}} = \Psi |h_{\mathrm{SD}}|^2, \tag{2}$$

where $\Psi = P_{\mathrm{S}}/N_0$ is the average transmit signal-to-noise ratio (SNR).

Different from the conventional cooperative networks [1], the relay always forward the received signal from the source toward the destination even when the SNR of the direct link is good enough for successful decoding. The proposed system first checks the signal quality of the direct link between the source and the destination. If the direct link is able to support the switching rate, i.e., $\log_2(1+\gamma_{\mathrm{SD}}) \geq \mathcal{R}_{\mathrm{s}}$, the source will send a new signal in the following time slot while the relays will not perform any transmission, i.e. still keep idle, after receiving a feedback message from the destination indicating the successful reception. Otherwise, the destination will request the help from the wirelessly harvested energy based relay, named as the incremental relaying phase. Similar to [7], we consider the time-switching architecture for the incremental relaying phase with each data frame separated into three consecutive time slots for energy transmission, information transmission and information reception, respectively. For the given total block duration T and the time splitting ratio α, the corresponding duration for each time slot are αT, $(1-\alpha)/2T$, and $(1-\alpha)/2T$. To enhance the system performance, the value of α should be designed appropriately.

For the incremental relaying phase, we have the harvested energy at the relay in the first time slot of αT given by [7, Eq. (2)]

$$E_{\mathrm{R}} = \eta P_{\mathrm{S}} |h_{\mathrm{SR}}|^2 \alpha T, \tag{3}$$

where η as the energy conversion efficiency with $0 < \eta \leq 1$.

From (3), the transmitted power at the relay during $(1-\alpha)T/2$ is written as

$$\begin{aligned} P_{\mathrm{R}} &= \frac{E_{\mathrm{R}}}{(1-\alpha)\, T/2} \\ &= \kappa P_{\mathrm{S}} |h_{\mathrm{SR}}|^2, \end{aligned} \tag{4}$$

where κ is defined as $\kappa = 2\eta\alpha(1-\alpha)$.

Next, the received signal at the relay during in the second time slot of $(1-\alpha)T/2$ is obtained as

$$y_{\mathrm{SR}} = \sqrt{P_{\mathrm{S}}} h_{\mathrm{SR}} x_{\mathrm{S}} + n_{\mathrm{SR}}, \tag{5}$$

resulting in the instantaneous SNR at the relay as

$$\gamma_{\mathrm{SR}} = \Psi |h_{\mathrm{SR}}|^2. \tag{6}$$

Since fixed decode-and-forward relaying is used at the relay, the received signal at the destination from the relay during the third time slot of $(1-\alpha)T/2$ can be expressed as

$$y_{\mathrm{RD}} = \sqrt{P_{\mathrm{R}}} h_{\mathrm{RD}} x_{\mathrm{R}} + n_{\mathrm{RD}}, \tag{7}$$

where x_{R} denotes the re-encoded version of x_{S}.

Making use the fact that the dual-hop relaying is dominated by the weakest hop, we can write the equivalent SNR of the $\mathrm{S} \to \mathrm{R} \to \mathrm{D}$ as [10, Eq. (1)]

$$\begin{aligned} \gamma_{\mathrm{R}} &= \min\left(\gamma_{\mathrm{SR}}, \gamma_{\mathrm{RD}}\right) \\ &= \min\left(\Psi |h_{\mathrm{SR}}|^2, \kappa\Psi |h_{\mathrm{SR}}|^2 |h_{\mathrm{RD}}|^2\right), \end{aligned} \tag{8}$$

where $\gamma_{\rm RD}$ is defined as

$$\gamma_{\rm RD} = \frac{P_{\rm R}}{N_0}|h_{\rm RD}|^2 = \kappa\Psi|h_{\rm SR}|^2|h_{\rm RD}|^2. \tag{9}$$

At the end of the incremental relaying phase, the destination will combine two signals received directly from the source and indirectly via relay using maximal ratio combining. Mathematically, we have the output SNR of the MRC combiner as [15, Eq.(1)]:

$$\gamma_{\rm MRC} = \gamma_{\rm R} + \gamma_{\rm SD}. \tag{10}$$

III. Outage Probability Analysis

In this section, we provide new closed-form expressions of the outage probability (OP) of the half-duplex based EH relaying scheme. According to the total probability law, the outage probability of the system is defined as

$$\begin{aligned}\mathrm{OP}^{\rm MRC} =& \Pr\left[\log_2(1+\gamma_{\rm SD})<\mathcal{R}_{\rm s}, \frac{1-\alpha}{2}\log_2(1+\gamma_{\rm MRC}) < \mathcal{R}_{\rm o}\right] \\ &+ \Pr\left[\log_2(1+\gamma_{\rm SD}) \geq \mathcal{R}_{\rm s}, \log_2(1+\gamma_{\rm SD}) < \mathcal{R}_{\rm o}\right] \\ =& \Pr\left[\gamma_{\rm SD}<2^{\mathcal{R}_{\rm s}}-1, \gamma_{\rm MRC} < 2^{2\mathcal{R}_{\rm o}/(1-\alpha)}-1\right] \\ &+\Pr\left(2^{\mathcal{R}_{\rm s}}-1 \leq \gamma_{\rm SD} \leq 2^{\mathcal{R}_{\rm o}}-1\right),\end{aligned} \tag{11}$$

where the pre-factor $\frac{1-\alpha}{2}$ is accounted for communication between the source node and the destination node.

Theorem 1: The exact closed form expression for the system outage probability over Rayleigh fading channels for $\mathcal{R}_{\rm s} \leq \mathcal{R}_{\rm o}$ and $\mathcal{R}_{\rm s} > \mathcal{R}_{\rm o}$ are given respectively as

$$\mathrm{OP}^{\rm MRC} = \begin{cases} \mathbb{I}_1 + \mathbb{I}_2, & \mathcal{R}_{\rm s} \leq \mathcal{R}_{\rm o} \\ \mathbb{I}_1, & \mathcal{R}_{\rm s} > \mathcal{R}_{\rm o} \end{cases}, \tag{12}$$

where $\mathbb{I}_1$ and $\mathbb{I}_2$ are given by (13) and (14), respectively, shown at the top of the next page with $\mathrm{Ei}(x) = -\int_{-x}^{\infty} \frac{\exp(-t)}{t}dt$ being the exponential integral function [14, Eq. (8.211.1)] and $\Gamma(n,x) = \int_x^{\infty} \exp(-t)\, t^{n-1}dt$ being the incomplete Gamma function [16, Eq.(8.350.2)]. And $\mathbb{I}_2$ is of the form

$$\mathbb{I}_2 = \exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm s}}{\Psi}\right) - \exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm o}}{\Psi}\right), \tag{14}$$

where $\rho_{\rm s} = 2^{\mathcal{R}_{\rm s}} - 1$ and $\rho_{\rm th} = 2^{2\mathcal{R}_{\rm o}/(1-\alpha)} - 1$.

Proof: We start the proof with $\mathbb{I}_1$ given by

$$\begin{aligned}\mathbb{I}_1 &= \Pr\left[\gamma_{\rm SD}<2^{\mathcal{R}_{\rm s}}-1, \gamma_{\rm MRC} < 2^{2\mathcal{R}_{\rm o}/(1-\alpha)}-1\right] \\ &= \int_0^{\rho_{\rm s}} f_{\gamma_{\rm SD}}(x)\, F_{\gamma_{\rm R}}(\rho_{\rm th}-x)\, dx,\end{aligned} \tag{15}$$

where $f_{\gamma_{\rm SD}}(.)$ and $F_{\gamma_{\rm R}}(.)$ denotes the PDF and CDF of $\gamma_{\rm SD}$ and $\gamma_{\rm R}$, respectively. Noting that $\rho_{\rm th} > \rho_{\rm s}$ and making a change of variable, i.e., $t = \rho_{\rm th} - x$, we have

$$\mathbb{I}_1 = \int_{\rho_{\rm th}-\rho_{\rm s}}^{\rho_{\rm th}} f_{\gamma_{\rm SD}}(\rho_{\rm th}-t)\, F_{\gamma_{\rm R}}(t)\, dt. \tag{16}$$

In (16), $F_{\gamma_{\rm R}}(x)$ can be rewritten as follows:

$$\begin{aligned}F_{\gamma_{\rm R}}(x) &= \Pr\left[\min\left(\Psi|h_{\rm SR}|^2, \kappa\Psi|h_{\rm SR}|^2|h_{\rm RD}|^2\right) \leq x\right] \\ &= 1 - \Pr\left[\min\left(\Psi|h_{\rm SR}|^2, \kappa\Psi|h_{\rm SR}|^2|h_{\rm RD}|^2\right) > x\right] \\ &= 1 - \int_{x/\Psi}^{\infty} f_{|h_{\rm SR}|^2}(y)\left[1 - F_{|h_{\rm RD}|^2}\left(\frac{x}{\kappa\Psi y}\right)\right] dy.\end{aligned} \tag{17}$$

Substituting $f_{|h_{\rm SR}|^2}(y)$ and $F_{|h_{\rm RD}|^2}(y)$, i.e.,

$$f_{|h_{\rm SR}|^2}(y) = \lambda_{\rm SR}\exp(-\lambda_{\rm SR}x), \tag{18}$$

and

$$F_{|h_{\rm RD}|^2}(y) = 1 - \exp(-\lambda_{\rm RD}y), \tag{19}$$

into (17), we rewrite (17) as

$$F_{\gamma_{\rm R}}(x) = 1 - \int_{x/\Psi}^{\infty} \lambda_{\rm SR}\exp(-\lambda_{\rm SR}y)\exp\left(-\frac{\lambda_{\rm RD}x}{k\Psi y}\right) dy. \tag{20}$$

Utilizing the Taylor expansion for $\exp\left(-\frac{\lambda_{\rm RD}x}{k\Psi y}\right)$ gives

$$\exp\left(-\frac{\lambda_{\rm RD}x}{\kappa\Psi y}\right) = 1 - \frac{\lambda_{\rm RD}x}{\kappa\Psi y} + \sum_{n=2}^{+\infty}\frac{(-1)^{n-2}}{n!}\left(\frac{\lambda_{\rm RD}x}{\kappa\Psi}\right)^n\frac{1}{y^n}. \tag{21}$$

Plugging (21) into (20) and after some manipulation, we have

$$\begin{aligned}F_{\gamma_{\rm R}}(x) =& 1 - \exp\left(-\frac{\lambda_{\rm SR}x}{\Psi}\right) - \frac{\lambda_{\rm SR}\lambda_{\rm RD}x}{\kappa\Psi}\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}x}{\Psi}\right) \\ &- \lambda_{\rm SR}\sum_{n=2}^{+\infty}\frac{(-1)^{n-2}}{n!}\left(\frac{\lambda_{\rm RD}x}{\kappa\Psi}\right)^n \underbrace{\int_{x/\Psi}^{\infty}\frac{\exp(-\lambda_{\rm SR}y)}{y^n}dy}_{\mathbb{T}_1}.\end{aligned} \tag{22}$$

By applying [16, Eq.(3.353.1)] for $\mathbb{T}_1$, we have

$$\begin{aligned}F_{\gamma_{\rm R}}(x) =& 1 - \exp\left(-\frac{\lambda_{\rm SR}x}{\Psi}\right) - \frac{\lambda_{\rm SR}\lambda_{\rm RD}x}{\kappa\Psi}\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}x}{\Psi}\right) \\ &- \sum_{n=2}^{+\infty}\sum_{k=1}^{n-1}\frac{(-1)^{2n-k-3}(k-1)!}{n!\,(n-1)!} \\ &\times\left(\frac{\lambda_{\rm SR}}{\Psi}\right)^{n-k}\left(\frac{\lambda_{\rm RD}}{\kappa}\right)^n x^{n-k}\exp\left(-\frac{\lambda_{\rm SR}x}{\Psi}\right) \\ &+ \sum_{n=2}^{+\infty}\frac{(-1)^{2n-3}(\lambda_{\rm SR})^n}{n!\,(n-1)!}\left(\frac{\lambda_{\rm RD}x}{\kappa\Psi}\right)^n\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}x}{\Psi}\right).\end{aligned} \tag{23}$$

Substituting (23) and $f_{\gamma_{\rm SD}}(x) = \frac{\lambda_{\rm SD}}{\Psi}\exp\left(-\frac{\lambda_{\rm SD}x}{\Psi}\right)$ into (16) and after some manipulation gives $\mathbb{I}_1$ as (24), shown at the top of the next page.

Considering $\mathbb{J}_1$ and using partial integral method, we arrive at (25).

$$\begin{aligned}\mathbb{I}_1 =&1-\exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm s}}{\Psi}\right) - \frac{\lambda_{\rm SD}}{\lambda_{\rm SD}-\lambda_{\rm SR}}\left[\exp\left(-\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)-\exp\left(\frac{-\lambda_{\rm SD}\rho_{\rm s}-\lambda_{\rm SR}\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right]\\
&-\frac{\lambda_{\rm SR}\lambda_{\rm RD}}{\Psi\kappa}\left[\ \mathrm{Ei}\left(-\tfrac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)\left[\rho_{\rm th}-\tfrac{\Psi}{\lambda_{\rm SD}}\right]-\mathrm{Ei}\left(-\tfrac{\lambda_{\rm SR}(\rho_{\rm th}-\rho_{\rm s})}{\Psi}\right)\exp\left(-\tfrac{\lambda_{\rm SD}\rho_{\rm s}}{\Psi}\right)\left[\rho_{\rm th}-\rho_{\rm s}-\tfrac{\Psi}{\lambda_{\rm SD}}\right]\right.\\
&\left.+\frac{\Psi}{\lambda_{\rm SD}}\exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm th}}{\Psi}\right)\left[\mathrm{Ei}\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\rho_{\rm th}}{\Psi}\right)-\mathrm{Ei}\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right]\right]\\
&-\sum_{n=2}^{+\infty}\sum_{k=1}^{n-1}\sum_{i=0}^{n-k}\frac{(-1)^i\,(n-k)!}{(n-k-i)!\left(\frac{\lambda_{\rm SD}-\lambda_{\rm SR}}{\Psi}\right)^{i+1}}\frac{(-1)^{2n-k-3}\,(k-1)!}{n!\,(n-1)!}\frac{\lambda_{\rm SD}}{\Psi}\left(\frac{\lambda_{\rm SR}}{\Psi}\right)^{n-k}\left(\frac{\lambda_{\rm RD}}{\kappa}\right)^{n}\\
&\times\left[\rho_{\rm th}^{n-k-i}\exp\left(-\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)-\left(\rho_{\rm th}-\rho_{\rm s}\right)^{n-k-i}\exp\left(\frac{-\lambda_{\rm SD}\rho_{\rm s}-\lambda_{\rm SR}\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right]\\
&+\sum_{n=2}^{+\infty}\sum_{m=0}^{+\infty}\frac{1}{m!\,(n+m+1)}\left(\frac{\lambda_{\rm SD}}{\Psi}\right)^{m+1}\frac{(-1)^{2n-3}}{n!\,(n-1)!}\left(\frac{\lambda_{\rm SR}\lambda_{\rm RD}}{\Psi\kappa}\right)^{n}\exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm th}}{\Psi}\right)\\
&\times\left[\rho_{\rm th}^{n+m+1}\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)-\left(\rho_{\rm th}-\rho_{\rm s}\right)^{n+m+1}\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right.\\
&\left.+\left(\frac{\lambda_{\rm SR}}{\Psi}\right)^{-n-m-1}\left[\Gamma\left(n+m+1,\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)-\Gamma\left(n+m+1,\frac{\lambda_{\rm SR}\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right]\right].\end{aligned}\tag{13}$$

$$\begin{aligned}\mathbb{I}_1 =& 1-\exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm s}}{\Psi}\right) - \frac{\lambda_{\rm SD}}{\lambda_{\rm SD}-\lambda_{\rm SR}}\left[\exp\left(-\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)-\exp\left(\frac{-\lambda_{\rm SD}\rho_{\rm s}-\lambda_{\rm SR}\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right]\\
&-\frac{\lambda_{\rm SD}\lambda_{\rm SR}\lambda_{\rm RD}}{\Psi^2\kappa}\exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm th}}{\Psi}\right)\underbrace{\int_{\rho_{\rm th}-\rho_{\rm s}}^{\rho_{\rm th}} t\exp\left(\frac{\lambda_{\rm SD}t}{\Psi}\right)\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}t}{\Psi}\right)dt}_{\mathbb{J}_1}\\
&-\sum_{n=2}^{+\infty}\sum_{k=1}^{n-1}\frac{(-1)^{2n-k-3}\,(k-1)!}{n!\,(n-1)!}\frac{\lambda_{\rm SD}}{\Psi}\left(\frac{\lambda_{\rm SR}}{\Psi}\right)^{n-k}\left(\frac{\lambda_{\rm RD}}{\kappa}\right)^{n}\exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm th}}{\Psi}\right)\underbrace{\int_{\rho_{\rm th}-\rho_{\rm s}}^{\rho_{\rm th}} t^{n-k}\exp\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)t}{\Psi}\right)dt}_{\mathbb{J}_2}\\
&+\sum_{n=2}^{+\infty}\frac{(-1)^{2n-3}}{n!\,(n-1)!}\frac{\lambda_{\rm SD}}{\Psi}\left(\frac{\lambda_{\rm SR}\lambda_{\rm RD}}{\Psi\kappa}\right)^{n}\exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm th}}{\Psi}\right)\underbrace{\int_{\rho_{\rm th}-\rho_{\rm s}}^{\rho_{\rm th}}\exp\left(\frac{\lambda_{\rm SD}t}{\Psi}\right)t^n\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}t}{\Psi}\right)dt}_{\mathbb{J}_3}.\end{aligned}\tag{24}$$

$$\begin{aligned}\mathbb{J}_1 =&\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)\exp\left(\frac{\lambda_{\rm SD}\rho_{\rm th}}{\Psi}\right)\left[\frac{\Psi\rho_{\rm th}}{\lambda_{\rm SD}}-\left(\frac{\Psi}{\lambda_{\rm SD}}\right)^2\right]\\
&-\mathrm{Ei}\left(-\frac{\lambda_{\rm SR}\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\exp\left(\frac{\lambda_{\rm SD}\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\left[\frac{\Psi\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\lambda_{\rm SD}}-\left(\frac{\Psi}{\lambda_{\rm SD}}\right)^2\right]\\
&-\frac{1}{\frac{\lambda_{\rm SD}}{\Psi}\left(\frac{\lambda_{\rm SD}-\lambda_{\rm SR}}{\Psi}\right)}\left[\exp\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\rho_{\rm th}}{\Psi}\right)-\exp\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right]\\
&+\frac{1}{\left(\frac{\lambda_{\rm SD}}{\Psi}\right)^2}\left[\mathrm{Ei}\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\rho_{\rm th}}{\Psi}\right)-\mathrm{Ei}\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right].\end{aligned}\tag{25}$$

For $\mathbb{J}_2$, with the help of [16, Eq.(2.321.2)], we obtain

$$\begin{aligned}\mathbb{J}_2 =& \sum_{i=0}^{n-k}\frac{(-1)^i\,(n-k)!}{(n-k-i)!\left(\frac{\lambda_{\rm SD}-\lambda_{\rm SR}}{\Psi}\right)^{i+1}}\\
&\times\left[\rho_{\rm th}^{n-k-i}\exp\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\rho_{\rm th}}{\Psi}\right)\right.\\
&\left.-\left(\rho_{\rm th}-\rho_{\rm s}\right)^{n-k-i}\exp\left(\frac{\left(\lambda_{\rm SD}-\lambda_{\rm SR}\right)\left(\rho_{\rm th}-\rho_{\rm s}\right)}{\Psi}\right)\right].\end{aligned}\tag{26}$$

For $\mathbb{J}_3$, expanding $\exp\left(\frac{\lambda_{\rm SD}t}{\Psi}\right)$ in the power series, i.e.,

$$\exp\left(\frac{\lambda_{\rm SD}t}{\Psi}\right)=\sum_{n=0}^{+\infty}\frac{1}{n!}\left(\frac{\lambda_{\rm SD}}{\Psi}\right)^n t^n.\tag{27}$$

and using [16, Eq.(3.353.1)], we have

$$\begin{aligned}\mathbb{J}_3 &= \sum_{m=0}^{+\infty} \frac{1}{m!}\left(\frac{\lambda_{\rm SD}}{\Psi}\right)^m \int_{\rho_{\rm th}-\rho_{\rm s}}^{\gamma_{\rm th}} t^{n+m}{\rm Ei}\left(-\frac{\lambda_{\rm SR}t}{\Psi}\right)dt \\ &= \sum_{m=0}^{+\infty} \frac{1}{m!\,(n+m+1)}\left(\frac{\lambda_{\rm SD}}{\Psi}\right)^m \rho_{\rm th}^{n+m+1}{\rm Ei}\left(-\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right) \\ &\quad\times\left[-(\rho_{\rm th}-\rho_{\rm s})^{n+m+1}{\rm Ei}\left(-\frac{\lambda_{\rm SR}(\rho_{\rm th}-\rho_{\rm s})}{\Psi}\right)\right. \\ &\quad+\left(\frac{\lambda_{\rm SR}}{\Psi}\right)^{-n-m-1}\left[\Gamma\left(n+m+1,\frac{\lambda_{\rm SR}\rho_{\rm th}}{\Psi}\right)\right. \\ &\quad\left.\left.-\Gamma\left(n+m+1,\frac{\lambda_{\rm SR}(\rho_{\rm th}-\rho_{\rm s})}{\Psi}\right)\right]\right]. \end{aligned} \tag{28}$$

Finally, putting everything together, i.e., (25), (26) and (28), we obtained the desired form for $\mathbb{I}_1$ as (13).

For $\mathbb{I}_2$, over Rayleigh fading channel, we have

$$\begin{aligned}\mathbb{I}_2 &= \Pr\left(2^{\mathcal{R}_s}-1 \le \gamma_{\rm SD} \le 2^{\mathcal{R}_o}-1\right) \\ &= \exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm s}}{\Psi}\right) - \exp\left(-\frac{\lambda_{\rm SD}\rho_{\rm o}}{\Psi}\right), \end{aligned} \tag{29}$$

where $\rho_{\rm o} = 2^{\mathcal{R}_o} - 1$.

Having $\mathbb{I}_1$ and $\mathbb{I}_2$ at hands, we can obtain the desired results of OP as in (12), which also complete the proof. ■

IV. Simulation Results

In this section, Monte-Carlo simulations are provided to verify the analytical expressions, which were presented in the previous section. The performance of the IR-CRN scheme is analyzed and evaluated in terms of the OP analysis. The performance of the IR-CRN scheme is also compared with that of the DT scheme with different target rates. We consider a two-dimensional network, where the source, the relay and the destination are located at (0,0), (0.6,0) and (1,0), respectively. In all of the simulations, we set the energy harvesting eff ciency as 0.8, i.e., $\eta = 0.8$, the pass loss exponent as 3, i.e., $\beta = 3$, and $\mathcal{R}_s = 2$ bps/Hz.

We f rst investigate the convergence of the approximation for the outage probability of the IR-CRN scheme. For the target rate $\mathcal{R}_o = 3$ bps/Hz, the approximations are presented in the Fig. 2 with N_{terms} denoting the number of f rst terms truncated in Taylor series in the expressions $\mathbb{J}_3$. As we can see from Fig. 2, the approximation for OP converges very fast. As we can see in Fig. 2, 17 terms are required to achieve high accuracy.

Figure 3 plots the system OP of the scheme as a function of the average SNR in dB. The theoretical results are in exact agreement with the simulation ones conf rming the correctness of our analysis. As shown in Fig. 3, all schemes achieve better performances when the average SNR increases. It is due to the fact that when the average SNR increases, the harvested energy at the relay increases making an improvement on the dual-hop relaying SNR. Fig. 3 also shows that for the same channel settings, the IR-CNR outperforms the direct transmission owing to the effect of using diversity combiner at receiver. For example, at the outage probability of 10^{-1}, the IR-CNR protocol obtains 10 dB gain as compared with the DT protocol.

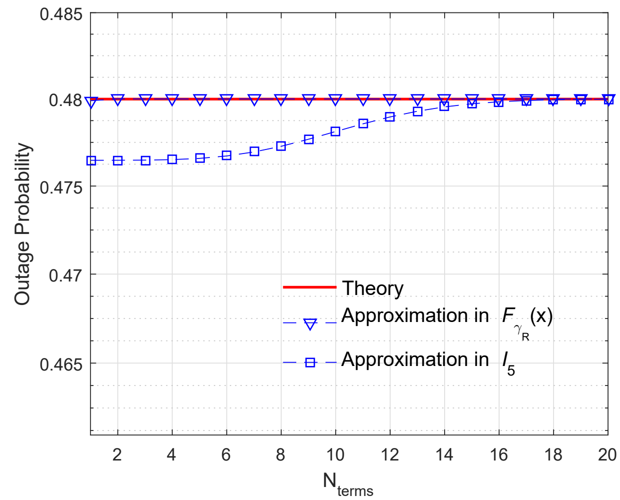

Fig. 2. The convergence of two approximations for outage probability of the IR-CRN scheme with $\mathcal{R}_{\rm th} = 2$bps/Hz and $\Psi = 10$ dB.

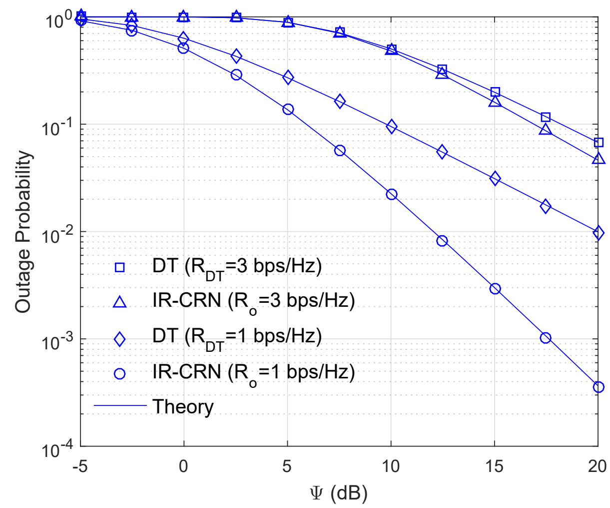

Fig. 3. Outage probability of the IR-CRN and DT protocols versus average SNR Ψ in dB with $\mathcal{R}_{\rm th} = 1$ bps/Hz and $\mathcal{R}_{\rm th} = 2$ bps/Hz.

In Fig. 4, we investigate the effect of relay positions on the system performance by varying the x-coordinate of the relay from 0.1 to 0.9 when $\Psi = 10$ dB. As expected, the system outage probability can be improved by locating the relay closely to the source. It can be explained by making use the fact that the harvested energy at relay is also a function of path loss.

Figure 5 deals with the impact of the time splitting α on the OP of the IR-CRN scheme. We vary from 0.1 to 0.7. As readily observed, the optimal α is a complicated function of channel and system parameters. It is slightly small and approximately 0.15 for the IR-CRN protocol with $\mathcal{R}_o = 1$ bps/Hz.

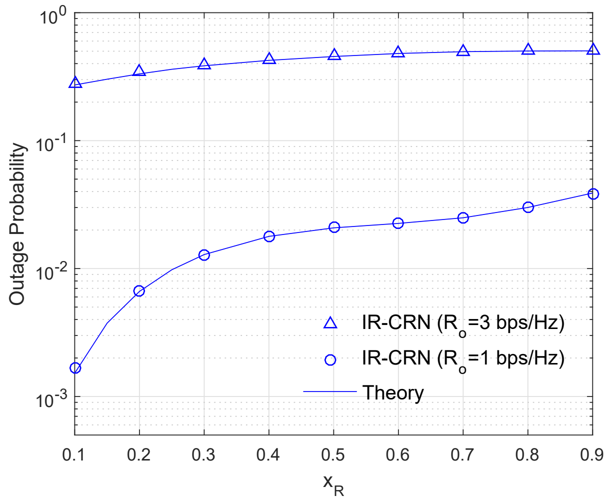

Fig. 4. Outage probability of the IR-CRN protocol versus positions of the relay (x_{R}) with $\alpha = 0.1$ and $\Psi = 10\,\mathrm{dB}$.

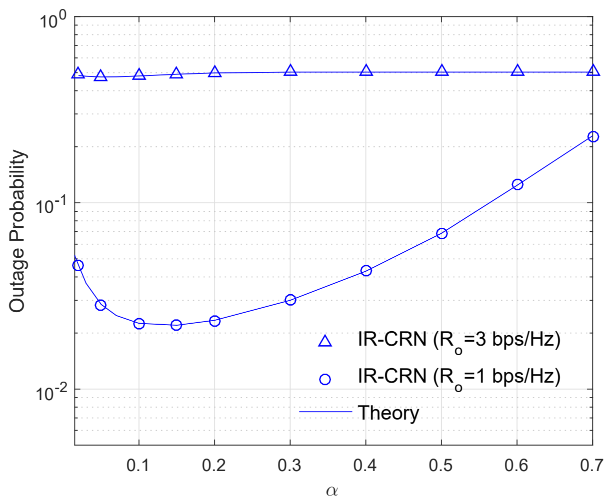

Fig. 5. Outage probability of the IR-CRN protocol versus $\alpha = 0.1$ with $\Psi = 10\,\mathrm{dB}$.

V. Conclusion

In this paper, we proposed an incremental cooperative relaying network (IR-CRN) with an energy harvesting relay and MRC-based destination using the time-switching mechanism. We derived the system outage probability over fading Rayleigh channels. The numerical results regarding the outage probability showed that the proposed system signifcantly outperforms the direct transmission for the same channels settings. As a result, it could be a promising protocol for wireless sensor networks to extend the network coverage as well as to improve the network performance.

Appendix

This appendix is to provide the outage probability of direct transmission over Rayleigh fading channels. In direct transmission, the source transmits signals directly to the destination without the assistance of the relay. As a result, the OP of the DT protocol can be computed as

$$\mathrm{OP}^{\mathrm{DT}} = 1 - \exp\left(-\frac{\lambda_{\mathrm{SD}}\rho_{\mathrm{o}}^{\mathrm{DT}}}{\Psi}\right), \tag{30}$$

where $\rho_{\mathrm{o}}^{\mathrm{DT}} = 2^{\mathcal{R}_{\mathrm{o}}} - 1$.

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References

[1] J. N. Laneman, D. N. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Effcient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[2] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: Architecture design and rate-energy tradeoff," *IEEE Trans. Commun.*, vol. 61, no. 11, pp. 4754–4767, Nov. 2013.

[3] K. Ishibashi, H. Ochiai, and V. Takrokh, "Energy harvesting cooperative communications," in *Proc. of the 23rd. IEEE Inter. Per. Indoor and Mobile Radio Commun.*, Sep. 2012, pp. 1819–1823.

[4] B. K. Chalise, Y. D. Zhang, and M. G. Amin, "Energy harvesting in an OSTBC based amplify-and-forward MIMO relay system," in *Proc. of IEEE Inter. Conf. Speech and Signal Proces.*, Mar. 2012, pp. 3201–3204.

[5] A. M. Fouladgar and O. Simeone, "On the transfer of information and energy in multi-user systems," *IEEE Commun. Lett.*, vol. 16, no. 11, pp. 1733–1736, Nov. 2012.

[6] I. Krikidis, S. Timotheou, and S. Sasaki, "RF energy transfer for cooperative networks: Data relaying or energy harvesting?" *IEEE Commun. Lett.*, vol. 16, no. 11, pp. 1772–1775, Nov. 2012.

[7] A. Nasir, X. Zhou, S. Durrani, and R. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3622–3636, Jul. 2013.

[8] ——, "Throughput and ergodic capacity of wireless energy harvesting based DF relaying network," in *Proc. of IEEE Inter. Conf. Commun.*, Jun. 2014, pp. 4066–4071.

[9] Z. Ding, S. M. Perlaza, I. Esnaola, and H. V. Poor, "Power allocation strategies in energy harvesting wireless cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 846–860, Feb. 2014.

[10] V. N. Q. Bao and H. Y. Kong, "Performance analysis of incremental selection decode-and-forward relaying over Rayleigh fading channels," in *Proc. of IEEE Inter. Conf. Commun. Workshops*. IEEE, Dec. 2009, pp. 1–5.

[11] T. T. Duy and H. Y. Kong, "Performance analysis of incremental amplify-and-forward relaying protocols with N-th best partial relay selection under interference constraint," *Wirel. Pers. Commun.*, vol. 71, no. 4, pp. 2741–2757, Dec. 2013.

[12] S. S. Ikki and M. H. Ahmed, "Performance analysis of decode-and-forward incremental relaying cooperative-diversity networks over Rayleigh fading channels," in *Proc. of IEEE 69th Vehicular Tech. Conf.*, Apr. 2009, pp. 1–6.

[13] ——, "On the performance of cooperative-diversity networks with the N-th best relay selection scheme," *IEEE Trans. Commun.*, vol. 58, no. 11, pp. 3062–3069, Sep. 2010.

[14] V. N. Q. Bao and H. Y. Kong, "Incremental relaying with partial relay selection," *IEICE Trans. Commun.*, vol. 93, no. 5, pp. 1317–1321, May. 2010.

[15] P. N. Son, H. Y. Kong, and A. Anpalagan, "Exact outage analysis of a decode-and-forward cooperative communication network with N-th best energy harvesting relay selection," *Ann. Telecom.*, vol. 71, no. 5-6, pp. 251–263, Jun. 2016.

[16] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*. Academic Press, 2007.