Opportunistic Relaying for Cognitive Two-Way Network with Multiple Primary Receivers over Nakagami-m Fading

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***Abstract*—In this paper, we consider a cognitive two-way relay system with multiple relay nodes and multiple primary receivers over Nakagami-m fading channels. Relay nodes process signal following selective decode-and-forward (DF) relay technique. The opportunistic relay selection is used to choose the best relay. We derive the exact closed-form expression of the overall outage probability over Nakagami-m fading channels. These analytical results are exactly verified by Monte-Carlo simulation.**

***Index Terms*—Two-way relaying, cognitive radio, multiple receivers, decode-and-forward, Nakagami-m fading.**

I. Introduction

Cognitive radio technique has emerged as a promising solution in order to increase in system spectral efficiency [1]. In cognitive radio systems, secondary users (SUs) are allowed to operate on the radio frequency spectrum that have been licensed for primary users (PUs) as long as the communication of primary networks are protected [2]. To avoid any interference to primary networks, the transmit power of secondary transmitters is usually limited resulting the secondary network coverage relatively small. To increase the network coverage of secondary networks, relaying and cooperative communications are efficient solutions [2], [3].

To further improve the system spectral efficiency, two way relaying has been considered for cognitive radio networks [4]–[10]. In particular, Boris *el. al.* in [4] proposed two half-duplex relaying protocols where a bidirectional connection between two wireless terminals is established using one half-duplex AF or DF relay that is able to mitigate the loss in spectral efficiency due to the half-duplex operation of the relays. Duy *et. al.* in [5] derived an exact formula of outage probability for opportunity node selection DF two-way relay communication systems. Paper [6] solved the best relay node selection problem and simultaneously allocated optimal power among secondary nodes. In [7], the authors analyzed the performance of opportunity node selection two-way relay communication systems having a primary receiver and derived an exact expression of system outage probability and average bit error rate probability over Rayleigh fading channels. Considering primary transmitter effect on secondary system in two-way relay network, Zhang et. al. [8] derived the exact outage probability of the best relay node selection two-way secondary communication systems. Considering two-way relay systems having many primary transmitters and receivers, in [9] and [10], the authors analyzed the performance of the opportunity the best relay node selection AF (amplify-and-forward) and DF two-way relay communication systems on Rayleigh fading channel. However, it is assumed in [9], [10] that the number of relay nodes should be greater than the number of primary receivers making the results inapplicable for the case of arbitrary number of relay nodes and primary receivers.

In this paper, we study the outage probability of secondary two way relaying networks with multiple relays and multiple primary receivers . We also propose a new derivation approach, which is valid for all cases of the number of secondary relays and the number of primary receivers. Different with all of the above-mentioned papers, the channel model under consideration is Nakagami-m fading channels, which is well known as a versatile statistical distribution used to model a variety of fading environments covering Rayleigh fading channels as a special case.

The rest of this paper is organized as follows. In Section II, we introduce the system and channel model used in this paper. Section III presents the detailed derivations of the outage probability. Section IV will give the numerical results used to examine the effects of the system and channel parameters, followed by some conclusions in Section V.

II. System Model

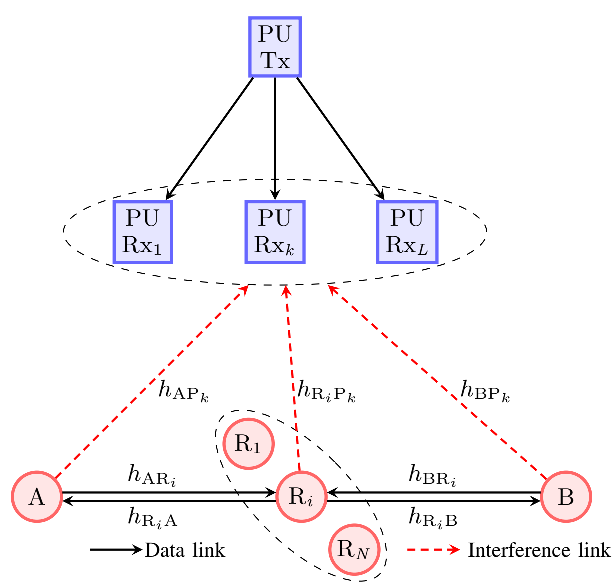

Fig. 1. A cognitive two-way relay network with multiple primary receivers.

In the considered cognitive two-way relaying network, two secondary sources are denoted as A and B, respectively. N

secondary relays are denoted as R_i with $i = 1, 2, \ldots, N$ coexisting with a primary transmitter, denoted as $\mathrm{PU} - \mathrm{Tx}$, and L primary receivers, denoted as $\mathrm{PU} - \mathrm{Rx}_k$ with $k = 1, 2, \ldots, L$. These mentioned nodes are illustrated in Fig. 1. In this model, we assume that there is no direct link between node A and node B due to the severe shadowing and channel pathloss. Each node is equipped with single antenna and is assumed to operate in a half-duplex mode.

Assuming that the channels amplitudes are flat-fading Nakagami-m distributed, the fading channel gain, i.e., $|h_{\mathsf{XY}}|^2$ with h_{XY} representing the channel coefficient of the $\mathsf{X} \to \mathsf{Y}$ link with $\mathsf{X} \in \{\mathrm{A}, \mathrm{B}, \mathrm{R}_i\}$ and $\mathsf{Y} \in \{\mathrm{A}, \mathrm{B}, \mathrm{R}_i, \mathrm{P}_k\}$, is a random variable having gamma distribution with parameters m_{XY} and Ω_{XY}. As a result, the cumulative distribution function (CDF) and probability density function (PDF) of $|h_{\mathsf{XY}}|^2$ is can be written respectively as

$$F_{|h_{\mathsf{XY}}|^2}(z) = 1 - \frac{\Gamma\left(m_{\mathsf{XY}}, \alpha_{\mathsf{XY}} z\right)}{\Gamma\left(m_{\mathsf{XY}}\right)}, \tag{1}$$

$$f_{|h_{\mathsf{XY}}|^2}\left(z\right) = \frac{\alpha_{\mathsf{XY}}^{m_{\mathsf{XY}}}}{\Gamma\left(m_{\mathsf{XY}}\right)} z^{m_{\mathsf{XY}}-1} e^{-\alpha_{\mathsf{XY}} z}. \tag{2}$$

Each transmission period between node A and node B is divided in three phases, where the transmit powers of all secondary transmitters are properly adjusted, i.e., below the peak interference constraint at primary receivers denoted as $\widetilde{I}_{\mathrm{p}}$, to protect the communication of the primary network. Considering the maximum power of secondary transmitters ($\widetilde{\mathcal{P}}_m$), the transmit power of node A, node B and relay R_i are respectively given as

$$P_{\mathrm{A}} = \min\left(\widetilde{\mathcal{P}}_m, \frac{\widetilde{I}_{\mathrm{p}}}{\max\limits_{k=1,2,\ldots,L} |h_{\mathrm{AP}_k}|^2}\right), \tag{3}$$

$$P_{\mathrm{B}} = \min\left(\widetilde{\mathcal{P}}_m, \frac{\widetilde{I}_{\mathrm{p}}}{\max\limits_{k=1,2,\ldots,L} |h_{\mathrm{BP}_k}|^2}\right), \tag{4}$$

$$P_{\mathrm{R}_i} = \min\left(\widetilde{\mathcal{P}}_m, \frac{\widetilde{I}_{\mathrm{p}}}{\max\limits_{k=1,2,\ldots,L} |h_{\mathrm{R}_i\mathrm{P}_k}|^2}\right). \tag{5}$$

In the first phase, node A transmits its data to all relay nodes. Then, node B broadcasts its data to relay nodes in the second phase. The instantaneous signal-to-noise ratios (SNR) received at the ith relay node in the first and second phase are respectively written as

$$\gamma_{\mathrm{AR}_i} = |h_{\mathrm{AR}_i}|^2 \min\left(\mathcal{P}_m, \frac{I_{\mathrm{p}}}{\max\limits_{k=1,2,\ldots,L} |h_{\mathrm{AP}_k}|^2}\right), \tag{6}$$

$$\gamma_{\mathrm{BR}_i} = |h_{\mathrm{BR}_i}|^2 \min\left(\mathcal{P}_m, \frac{I_{\mathrm{p}}}{\max\limits_{k=1,2,\ldots,L} |h_{\mathrm{BP}_k}|^2}\right). \tag{7}$$

where $\mathcal{P}_m = \widetilde{\mathcal{P}}_m / N_0$, $I_{\mathrm{p}} = \widetilde{I}_{\mathrm{p}} / N_0$ and the additive white Gaussian noise at secondary receivers have the common distribution $\mathcal{CN}\left(0, N_0\right)$ (circularly symmetric complex Gaussian variables with zero mean and variance N_0).

At the end of the second phase, all relays will decode the received signals from A and B using selective decode-and-forward. Denoting $\mathcal{R}_{\mathrm{A}}$ and $\mathcal{R}_{\mathrm{B}}$ as the set of successfully decoding relays from A and B, respectively, we can write the set of relays both successfully decoding relays from A and B as $\mathcal{R} = \mathcal{R}_{\mathrm{A}} \cap \mathcal{R}_{\mathrm{B}}$.

In the third phase, only the best relay among $\mathcal{R}$ will broadcast the encoded signal towards A and B. Denoting R_{i^*} as the selected relay, we have [8]:

$$i^* = \arg \max_{i=1,2,\ldots,n} \gamma_{\mathrm{R}_i}, \tag{8}$$

where n is the cardinality of $\mathcal{R}$, i.e., $n = |\mathcal{R}|$ and

$$\gamma_{\mathrm{R}_i} = \min\left(\gamma_{\mathrm{R}_i\mathrm{A}}, \gamma_{\mathrm{R}_i\mathrm{B}}\right). \tag{9}$$

In (9), $\gamma_{\mathrm{R}_i\mathrm{A}}$ and $\gamma_{\mathrm{R}_i\mathrm{B}}$ are of the form as follows:

$$\gamma_{\mathrm{R}_i\mathrm{A}} = |h_{\mathrm{R}_i\mathrm{A}}|^2 \min\left(\mathcal{P}_m, \frac{I_{\mathrm{p}}}{\max\limits_{k=1,2,\ldots,L} |h_{\mathrm{R}_i\mathrm{P}_k}|^2}\right), \tag{10}$$

$$\gamma_{\mathrm{R}_i\mathrm{B}} = |h_{\mathrm{R}_i\mathrm{B}}|^2 \min\left(\mathcal{P}_m, \frac{I_{\mathrm{p}}}{\max\limits_{k=1,2,\ldots,L} |h_{\mathrm{R}_i\mathrm{P}_k}|^2}\right). \tag{11}$$

Making use the fact that γ_{R_i} are independent each other, we can write $\gamma_{\mathrm{R}_{i^*}}$ as

$$\gamma_{\mathrm{R}_{i^*}} = \max_{i=1,\ldots,n} \gamma_{\mathrm{R}_i}. \tag{12}$$

III. Performance Analysis

In this section, we will derive the closed-form expression for the system outage probability over Nakagami-m fading channels. Recalling that the number of successful decoding nodes in the first phase is a discrete random number, the probability of the event $|\mathcal{R}_{\mathrm{A}}| = n_{\mathrm{A}}$ with $n_{\mathrm{A}} \in \{0, 1, 2, \ldots, N\}$ is as follows:

$$\begin{aligned} &\Pr(|\mathcal{R}_{\mathrm{A}}| = n_{\mathrm{A}}) \\ &= \binom{N}{n_{\mathrm{A}}} \Pr\left[\bigcap_{i\in\mathcal{R}_{\mathrm{A}}} \left(\gamma_{\mathrm{AR}_i} \geq \gamma_{\mathrm{th}}\right), \bigcap_{i\notin\mathcal{R}_{\mathrm{A}}} \left(\gamma_{\mathrm{AR}_i} < \gamma_{\mathrm{th}}\right)\right]. \end{aligned} \tag{13}$$

All related channels are assumed to be independent, then

$$\begin{aligned} &\Pr(|\mathcal{R}_{\mathrm{A}}| = n_{\mathrm{A}}) \\ &= \binom{N}{n_{\mathrm{A}}} \left[1 - F_{\gamma_{\mathrm{AR}_i}}\left(\gamma_{\mathrm{th}}\right)\right]^{n_{\mathrm{A}}} \left[F_{\gamma_{\mathrm{AR}_i}}\left(\gamma_{\mathrm{th}}\right)\right]^{N-n_{\mathrm{A}}}, \end{aligned} \tag{14}$$

where γ_{th} is the outage SNR threshold and $F_{\gamma_{\mathrm{AR}_i}}(\gamma)$ denotes the CDF of γ_{AR_i}.

Similar to the first phase, we have $\Pr(|\mathcal{R}_{\mathrm{B}}| = n_{\mathrm{B}})$ for the second phase as

$$\begin{aligned} \Pr(|\mathcal{R}_{\mathrm{B}}| =& n_{\mathrm{B}}) \\ &= \binom{N}{n_{\mathrm{B}}} \Pr\left[\bigcap_{i\in\mathcal{R}_{\mathrm{B}}} \left(\gamma_{\mathrm{BR}_i} \geq \gamma_{\mathrm{th}}\right), \bigcap_{i\notin\mathcal{R}_{\mathrm{B}}} \left(\gamma_{\mathrm{BR}_i} < \gamma_{\mathrm{th}}\right)\right] \\ &= \binom{N}{n_{\mathrm{B}}} \left[1 - F_{\gamma_{\mathrm{BR}_i}}\left(\gamma_{\mathrm{th}}\right)\right]^{n_{\mathrm{B}}} \left[F_{\gamma_{\mathrm{BR}_i}}\left(\gamma_{\mathrm{th}}\right)\right]^{N-n_{\mathrm{B}}}. \end{aligned} \tag{15}$$

At the end of the second phase, the selected relay is chosen from the intersection of two sets $\mathcal{R}_{\rm A}$ and $\mathcal{R}_{\rm B}$, i.e., $\mathcal{R}=\mathcal{R}_{\rm A}\cap\mathcal{R}_{\rm A}$, contains all of relays that are success in decoding in the first and the second phase. Then, we have $n\le n_{\rm A}$, $n\le n_{\rm B}$ and $n=|\mathcal{R}|$. For ease of presentation and with no loss of generality, it is assumed that

$$\mathcal{R}_{\rm A}=\{R_1,R_2,\ldots,R_n,R_{n+1},\ldots,R_{n+t_{\rm A}}\}, \quad (16)$$
$$\mathcal{R}_{\rm B}=\{R_1,R_2,\ldots,R_n,R_{n+t_{\rm A}+1},\ldots,R_{n+t_{\rm A}+t_{\rm B}}\}, \quad (17)$$
$$\mathcal{R}=\{R_1,R_2,\ldots,R_n\}, \quad (18)$$

where $0\le t_{\rm A}\le N-n$ and $0\le t_{\rm B}\le N-n-t_{\rm A}$.

For the given combination of $(n,t_{\rm A},t_{\rm B})$, we shall have $\binom{N}{n}\binom{N-n}{t_{\rm A}}\binom{N-n-t_{\rm A}}{t_{\rm B}}$ instances having n relay nodes decoding successfully both from node A and node B with $t_{\rm A}$ nodes only decoding from node A successfully and $t_{\rm B}$ nodes only decoding from node B successfully. Based on the theorem of total probability, we have

$$\text{OP}=\sum_{n=0}^{N}\Pr\left(|\mathcal{R}|=n\right)\Pr\left(\gamma_{{\rm R}_{i^*}}<\gamma_{\rm th}\right). \quad (19)$$

The following theorem will provide the closed-form expression of OP.

Theorem 1: Over Nakagami-m fading channels, the system outage probability is expressed under a closed-form expression as

$$\begin{aligned}\text{OP}=&\sum_{n=0}^{N}\sum_{t_{\rm A}=0}^{N-n}\sum_{t_{\rm B}=0}^{N-n-t_{\rm A}}\binom{N}{n}\binom{N-n}{t_{\rm A}}\binom{N-n-t_{\rm A}}{t_{\rm B}}\\&\times\left[1-F_{\gamma_{{\rm AR}_i}}(\gamma_{\rm th})\right]^{n+t_{\rm A}}\left[F_{\gamma_{{\rm AR}_i}}(\gamma_{\rm th})\right]^{N-n-t_{\rm A}}\\&\times\left[1-F_{\gamma_{{\rm BR}_i}}(\gamma_{\rm th})\right]^{n+t_{\rm B}}\left[F_{\gamma_{{\rm BR}_i}}(\gamma_{\rm th})\right]^{N-n-t_{\rm B}}\\&\times\left[F_{\gamma_{{\rm R}_i}}(\gamma_{\rm th})\right]^{n},\end{aligned} \quad (20)$$

where $F_{\gamma_{{\rm AR}_i}}(\gamma)$, $F_{\gamma_{{\rm BR}_i}}(\gamma)$, and $F_{\gamma_{{\rm R}_i}}(\gamma_{\rm th})$ are given as (21), (22), and (23), shown at the top of the next page.

Proof: To obtain OP, we need to calculate $F_{\gamma_{{\rm AR}_i}}(\gamma_{\rm th})$, $F_{\gamma_{{\rm BR}_i}}(\gamma_{\rm th})$, and $F_{\gamma_{{\rm R}_i}}(\gamma_{\rm th})$. We first consider $F_{\gamma_{{\rm AR}_i}}(\gamma)$, which can be written as

$$\begin{aligned}F_{\gamma_{{\rm AR}_i}}(\gamma)&=\Pr\left(\gamma_{{\rm AR}_i}<\gamma\right)\\&=\Pr\left[|h_{{\rm AR}_i}|^2\min\left(\mathcal{P}_m,\frac{I_{\rm p}}{\max\limits_{k=1,2,\ldots,L}|h_{{\rm AP}_k}|^2}\right)<\gamma\right],\end{aligned} \quad (24)$$

For notational simplicity, we introduce $X_{ai}=|h_{{\rm AR}_i}|^2$, $X_{ia}=|h_{{\rm R}_i{\rm A}}|^2$, $X_{ak}=\max\limits_{k=1,2,\ldots,L}|h_{{\rm AP}_k}|^2$, $X_{ip}=\max\limits_{k=1,2,\ldots,L}|h_{{\rm R}_i{\rm P}_k}|^2$, $\varepsilon=\frac{I_{\rm p}}{\mathcal{P}_m}$, the CDF of $\gamma_{{\rm AR}_i}$ can be rewritten as

$$\begin{aligned}F_{\gamma_{{\rm AR}_i}}(\gamma)=&\Pr\left(X_{ai}<\frac{\gamma}{\mathcal{P}_m},\ X_{ak}<\varepsilon\right)\\&+\Pr\left(X_{ai}<\frac{\gamma X_{ak}}{I_{\rm p}},\ X_{ak}>\varepsilon\right).\end{aligned} \quad (25)$$

It is noted that X_{ak} is the maximum of L i.i.d. gamma random variables. The following corollary will be useful in deriving the CDF and PDF of $\gamma_{{\rm AR}_i}$.

Corollary 1: Let Z be the maximum of L i.i.d. gamma random variables Z_i, $(i=1,2,\ldots,L)$ with parameters m_z and α_z. The CDF and PDF of the random variable Z are respectively given by

$$\begin{aligned}F_Z(z)=&\sum_{u=0}^{L}\binom{L}{u}(-1)^u\sum_{\substack{l_1,l_2,\ldots,l_{m_z}\ge0\\l_1+l_2+\cdots+l_{m_z}=u}}\frac{u!}{l_1!l_2!\ldots l_{m_z}!}\\&\times\prod_{{\rm w}=0}^{m_z-1}\left(\frac{\alpha_z^{\rm w}}{{\rm w}!}\right)^{l_{{\rm w}+1}}z^{\tilde{l}_z}e^{-\alpha_z uz},\end{aligned} \quad (26)$$

$$\begin{aligned}f_Z(z)=&\sum_{u=0}^{L-1}\binom{L-1}{u}(-1)^u\sum_{\substack{l_1,l_2,\ldots,l_{m_z}\ge0\\l_1+l_2+\cdots+l_{m_z}=u}}\frac{u!}{l_1!l_2!\ldots l_{m_z}!}\\&\times\prod_{{\rm w}=0}^{m_z-1}\left(\frac{\alpha_z^{\rm w}}{{\rm w}!}\right)^{l_{{\rm w}+1}}z^{m_z+\tilde{l}_z-1}e^{-(u+1)\alpha_z z},\end{aligned} \quad (27)$$

where $\tilde{l}_z=\sum\limits_{{\rm w}=0}^{m_z-1}{\rm w}l_{{\rm w}+1}$, $\binom{n}{k}\triangleq\frac{n!}{k!(n-k)!}$, and $0\le k\le n$.

Proof: See [11]. □

From (24) and utilizing Corollary 1, we have

$$\begin{aligned}F_{\gamma_{{\rm AR}_i}}=&F_{X_{ai}}\left(\frac{\gamma}{\mathcal{P}_m}\right)F_{X_{ak}}(\varepsilon) \quad (28)\\&+\int_{\varepsilon}^{\infty}\int_{0}^{\frac{\gamma}{I_{\rm p}}x_{ak}}f_{X_{ai}}\left(\frac{\gamma x_{ak}}{I_{\rm p}}\right)f_{X_{ak}}(x_{ak})\,dx_{ai}dx_{ak}\\=&F_{X_{ai}}\left(\frac{\gamma}{\mathcal{P}_m}\right)F_{X_{ak}}(\varepsilon)\ +1-F_{X_{ak}}(\varepsilon)\\&-\int_{\varepsilon}^{\infty}\Gamma\left(m_{ai},\frac{\alpha_{ai}\gamma x_{ak}/I_{\rm p}}{\Gamma(m_{ai})}\right)f_{X_{ak}}(x_{ak})\,dx_{ak}. \quad (29)\end{aligned}$$

Using [12, Eq. (8.352.2)] to expand the incomplete Gamma function as a finite sum, the integral in (29) is computed as

$$\begin{aligned}&\int_{\varepsilon}^{\infty}\Gamma\left(m_{ai},\frac{\alpha_{ai}\gamma x_{ak}/I_{\rm p}}{\Gamma(m_{ai})}\right)f_{X_{ak}}(x_{ak})\,dx_{ak}\\&=\int_{\varepsilon}^{\infty}(m_{ai}-1)!\frac{1}{\Gamma(m_{ai})}e^{-\alpha_{ai}\gamma x_{ak}/I_{\rm p}}\sum_{s_1=0}^{m_{ai}-1}\frac{(\alpha_{ai}\gamma x_{ak}/I_{\rm p})^{s_1}}{s_1!}\\&\times\sum_{u_1=0}^{L-1}\binom{L-1}{u_1}(-1)^{u_1}\sum_{\substack{l_1,l_2,\ldots,l_{m_{ak}}\ge0\\l_1+l_2+\cdots+l_{m_{ak}}=u_1}}\frac{u_1!}{l_1!l_2!\ldots l_{m_{ak}}!}\\&\times\prod_{{\rm w}_1=0}^{m_{ak}-1}\left(\frac{\alpha_{ak}^{{\rm w}_1}}{{\rm w}_1!}\right)^{l_{{\rm w}_1+1}}\frac{L\alpha_{ak}^{m_{ak}}}{\Gamma(m_{ak})}x_{ak}^{m_{ak}+\tilde{l}_{ak}-1}e^{-(u_1+1)\alpha_{ak}x_{ak}}dx_{ak}.\end{aligned} \quad (30)$$

where $\tilde{l}_{ak}=\sum\limits_{{\rm w}_1=0}^{m_{ak}-1}{\rm w}_1 l_{{\rm w}_1+1}$.

With the help of [12, eq. 3.351.2[11]] and making use the fact that $\gamma_{{\rm AR}_i}$ and $\gamma_{{\rm BR}_i}$ take the same form, the CDF of $\gamma_{{\rm AR}_i}$ and $\gamma_{{\rm BR}_i}$ are derived as (21) and (22), respectively, shown at the top of the next page, where $X_{bk}=\max\limits_{k=1,2,\ldots,L}|h_{{\rm BP}_k}|^2$,$\tilde{l}_{bk}=\sum\limits_{{\rm w}_2=0}^{m_{bk}-1}{\rm w}_2 l_{{\rm w}_2+1}$.

$$F_{\gamma_{\mathrm{AR}_i}}(\gamma) = F_{X_{ai}}\left(\frac{\gamma}{\mathcal{P}_m}\right) F_{X_{ak}}(\varepsilon) \ + 1 - F_{X_{ak}}(\varepsilon) - \sum_{u_1=0}^{L-1} \binom{L-1}{u_1} (-1)^{u_1} \sum_{\substack{l_1,l_2,\ldots,l_{m_{ak}} \geq 0 \\ l_1+l_2+\cdots+l_{m_{ak}} = u_1}} \frac{u_1!}{l_1! l_2! \ldots l_{m_{ak}}!}$$
$$\times \prod_{\mathrm{w}_1=0}^{m_{ak}-1} \left(\frac{\alpha_{ak}^{\mathrm{w}_1}}{\mathrm{w}_1!}\right)^{l_{\mathrm{w}_1+1}} \frac{L\alpha_{ak}^{m_{ak}}}{\Gamma(m_{ak})} \sum_{s_1=0}^{m_{ai}-1} \frac{(\alpha_{ai}\gamma/I_{\mathrm{p}})^{s_1}}{s_1!} \times \frac{\Gamma\left(m_{ak} + \tilde{l}_{ak} + s_1, \ ((u_1+1)\alpha_{ak} + \alpha_{ai}\gamma/I_{\mathrm{p}})\varepsilon\right)}{[(u_1+1)\alpha_{ak} + \alpha_{ai}\gamma/I_{\mathrm{p}}]^{\left(m_{ak}+\tilde{l}_{ak}+s_1\right)}}. \tag{21}$$

$$F_{\gamma_{\mathrm{BR}_i}}(\gamma) = F_{X_{bi}}\left(\frac{\gamma}{\mathcal{P}_m}\right) F_{X_{bk}}(\varepsilon) \ + 1 - F_{X_{bk}}(\varepsilon) - \sum_{u_2=0}^{L-1} \binom{L-1}{u_2} (-1)^{u_2} \sum_{\substack{l_1,l_2,\ldots,l_{m_{bk}} \geq 0 \\ l_1+l_2+\cdots+l_{m_{bk}} = u_2}} \frac{u_2!}{l_1! l_2! \ldots l_{m_{bk}}!}$$
$$\times \prod_{\mathrm{w}_2=0}^{m_{bk}-1} \left(\frac{\alpha_{bk}^{\mathrm{w}_2}}{\mathrm{w}_2!}\right)^{l_{\mathrm{w}_2+1}} \frac{L\alpha_{bk}^{m_{bk}}}{\Gamma(m_{bk})} \sum_{s_2=0}^{m_{bi}-1} \frac{(\alpha_{bi}\gamma/I_{\mathrm{p}})^{s_2}}{s_2!} \times \frac{\Gamma\left(m_{bk} + \tilde{l}_{bk} + s_2, \ ((u_2+1)\alpha_{bk} + \alpha_{bi}\gamma/I_{\mathrm{p}})\varepsilon\right)}{[(u_2+1)\alpha_{bk} + \alpha_{bi}\gamma/I_{\mathrm{p}}]^{\left(m_{bk}+\tilde{l}_{bk}+s_2\right)}}. \tag{22}$$

$$F_{\gamma_{\mathrm{R}_i}}(\gamma) = 1 - \frac{\Gamma\left(m_{ia}, \frac{\alpha_{ia}\gamma}{\mathcal{P}_m}\right)}{\Gamma(m_{ia})} \frac{\Gamma\left(m_{ib}, \frac{\alpha_{ib}\gamma}{\mathcal{P}_m}\right)}{\Gamma(m_{ib})} \left(1 - \frac{\Gamma(m_{ip}, \alpha_{ip}\varepsilon)}{\Gamma(m_{ip})}\right)^L - \sum_{\mathrm{w}_a=0}^{m_{ia}-1} \frac{(\alpha_{ia}\gamma/I_{\mathrm{p}})^{\mathrm{w}_a}}{\mathrm{w}_a!} \sum_{\mathrm{w}_b=0}^{m_{ib}-1} \frac{(\alpha_{ib}\gamma/I_{\mathrm{p}})^{\mathrm{w}_b}}{\mathrm{w}_b!} \sum_{u_3=0}^{L-1} \binom{L-1}{u_3}$$
$$\times (-1)^{u_3} \sum_{\substack{l_1,l_2,\ldots,l_{m_{ip}} \geq 0 \\ l_1+l_2+\cdots+l_{m_{ip}} = u_3}} \frac{u_3!}{l_1! l_2! \ldots l_{m_{ip}}!} \prod_{\mathrm{w}_3=0}^{m_{ip}-1} \left(\frac{\alpha_{ip}^{\mathrm{w}_3}}{\mathrm{w}_3!}\right)^{l_{\mathrm{w}_3+1}} \frac{L\alpha_{ip}^{m_{ip}}}{\Gamma(m_{ip})} \times \frac{\Gamma\left(\mathrm{w}_a + \mathrm{w}_b + \tilde{l}_{ip} + m_{ip}, \left(\frac{\alpha_{ia}+\alpha_{ib}}{I_{\mathrm{p}}}\gamma + (u_3+1)\alpha_{ip}\right)\varepsilon\right)}{[(\alpha_{ia}+\alpha_{ib})\gamma/I_{\mathrm{p}} + (u_3+1)\alpha_{ip}]^{\left(\mathrm{w}_a+\mathrm{w}_b+\tilde{l}_{ip}+m_{ip}\right)}}. \tag{23}$$

Having $F_{\gamma_{\mathrm{AR}_i}}(\gamma)$ and $F_{\gamma_{\mathrm{BR}_i}}(\gamma)$ at hands, we are now in a position to derive the CDF of $\gamma_{\mathrm{R}_i}(\gamma) = \min(\gamma_{\mathrm{R}_i\mathrm{A}}, \gamma_{\mathrm{R}_i\mathrm{B}})$. It is noted that $\gamma_{\mathrm{R}_i\mathrm{A}}$ and $\gamma_{\mathrm{R}_i\mathrm{B}}$ are correlated due to the common random variable X_{ip}. Using the conditional probability, we can write the CDF of γ_{R_i} as follows:

$$F_{\gamma_{\mathrm{R}_i}}(\gamma) = \int_0^\infty F_{\gamma_{\mathrm{R}_i}}(\gamma | X_{ip}) f_{X_{ip}}(x_{ip})\, dx_{ip}. \tag{31}$$

where $F_{\gamma_{\mathrm{R}_i}}(\gamma | X_{ip})$ is given by

$$\begin{aligned} F_{\gamma_{\mathrm{R}_i}}(\gamma | X_{ip}) &= 1 - \Pr(\gamma_{\mathrm{R}_i\mathrm{A}} > \gamma | X_{ip}) \Pr(\gamma_{\mathrm{R}_i\mathrm{B}} > \gamma | X_{ip}) \\ &= 1 - \left[1 - F_{\gamma_{\mathrm{R}_i\mathrm{A}}}(\gamma | X_{ip})\right]\left[1 - F_{\gamma_{\mathrm{R}_i\mathrm{B}}}(\gamma | X_{ip})\right]. \end{aligned} \tag{32}$$

From (10) and (11), we easily find out

$$F_{\gamma_{\mathrm{R}_i\mathrm{A}}}(\gamma | X_{ip}) = \begin{cases} F_{X_{ia}}\left(\frac{\gamma}{\mathcal{P}_m}\right), & \text{for } X_{ip} < \varepsilon \\ F_{X_{ia}}\left(\frac{\gamma}{I_{\mathrm{p}}} X_{ip}\right), & \text{for } X_{ip} > \varepsilon \end{cases} \tag{33}$$

and

$$F_{\gamma_{\mathrm{R}_i\mathrm{B}}}(\gamma | X_{ip}) = \begin{cases} F_{X_{ib}}\left(\frac{\gamma}{\mathcal{P}_m}\right), & \text{for } X_{ip} < \varepsilon \\ F_{X_{ib}}\left(\frac{\gamma}{I_{\mathrm{p}}} X_{ip}\right), & \text{for } X_{ip} > \varepsilon \end{cases} \tag{34}$$

leading to (32) being of the form as

$$F_{\gamma_{\mathrm{R}_i}}(\gamma) = \underbrace{\int_0^\varepsilon F_{\gamma_{\mathrm{R}_i}}(\gamma | X_{ip}) f_{X_{ip}}(x_{ip})\, dx_{ip}}_{I_1} + \underbrace{\int_\varepsilon^\infty F_{\gamma_{\mathrm{R}_i}}(\gamma | X_{ip}) f_{X_{ip}}(x_{ip})\, dx_{ip}}_{I_2}. \tag{35}$$

To obtain $F_{\gamma_{\mathrm{R}_i}}(\gamma)$, we need to derive I_1 and I_2. We first consider I_1, which is computed as

$$\begin{aligned} I_1 &= \int_0^\varepsilon \left[1 - \left[1 - F_{X_{ia}}\left(\frac{\gamma}{\mathcal{P}_m}\right)\right]\left[1 - F_{X_{ib}}\left(\frac{\gamma}{\mathcal{P}_m}\right)\right]\right] f_{X_{ip}}(x_{ip})\, dx_{ip} \\ &= \int_0^\varepsilon f_{X_{ip}}(x_{ip})\, dx_{ip} \\ &\quad - \frac{\Gamma(m_{ia}, \alpha_{ia}\gamma/\mathcal{P}_m)}{\Gamma(m_{ia})} \frac{\Gamma(m_{ib}, \alpha_{ib}\gamma/\mathcal{P}_m)}{\Gamma(m_{ib})} \left[1 - \frac{\Gamma(m_{ip}, \alpha_{ip}\varepsilon)}{\Gamma(m_{ip})}\right]^L. \end{aligned} \tag{36}$$

For I_2, we have

$$I_2 = \int_{\varepsilon}^{\infty} \left(1 - \frac{\Gamma\left(m_{ia}, \alpha_{ia}\gamma x_{ip}/I_{\rm p}\right)}{\Gamma\left(m_{ia}\right)} \frac{\Gamma\left(m_{ib}, \alpha_{ib}\gamma x_{ip}/I_{\rm p}\right)}{\Gamma\left(m_{ib}\right)}\right) \times f_{X_{ip}}\left(x_{ip}\right) dx_{ip}$$
$$= \int_{\varepsilon}^{\infty} f_{X_{ip}}\left(x_{ip}\right) dx_{ip} - \int_{\varepsilon}^{\infty} \sum_{\mathrm{w}_a=0}^{m_{ia}-1} \frac{\left(\alpha_{ia}\gamma/I_{\rm p}\right)^{\mathrm{w}_a}}{\mathrm{w}_a!} \sum_{\mathrm{w}_b=0}^{m_{ib}-1} \frac{\left(\alpha_{ib}\gamma/I_{\rm p}\right)^{\mathrm{w}_b}}{\mathrm{w}_b!} \times \sum_{u_3=0}^{L-1} \binom{L-1}{u_3} (-1)^{u_3} \sum_{\substack{l_1,l_2,\ldots,l_{m_{ip}} \geq 0 \\ l_1+l_2+\cdots+l_{m_{ip}}=u_3}} \frac{u_3!}{l_1! l_2! \ldots l_{m_{ip}}!} \times \prod_{\mathrm{w}_3=0}^{m_{ip}-1} \left(\frac{\alpha_{ip}^{\mathrm{w}_3}}{\mathrm{w}_3!}\right)^{l_{\mathrm{w}_3+1}} \frac{L\alpha_{ip}^{m_{ip}}}{\Gamma\left(m_{ip}\right)} x_{ip}^{\mathrm{w}_a+\mathrm{w}_b+m_{ip}+\tilde{l}_{ip}-1} \times e^{-((\alpha_{ia}+\alpha_{ib})\gamma/I_{\rm p}+(u_3+1)\alpha_{ip})x_{ip}} dx_{ip}, \quad (37)$$

where $\tilde{l}_{ip} = \sum_{\mathrm{w}_3=0}^{m_{ip}-1} \mathrm{w}_3 l_{\mathrm{w}_3+1}$.

Observing I_1 and I_2, we note that $\int_0^{\varepsilon} f_{X_{ip}}\left(x_{ip}\right) dx_{ip} + \int_{\varepsilon}^{\infty} f_{X_{ip}}\left(x_{ip}\right) dx_{ip} = 1$. In addition, using the identity [12, Eq. (8.352.2)] for the second integral in (37), after tedious manipulations, we obtain the closed form expression for $F_{\gamma_{\mathrm{R}_i}}(\gamma)$ as in (23), which also completes the proof. □

IV. Numerical Results

The purpose of this section is to verify the proposed derivation approach and to study the performance of the system under consideration. For the network model, it is assumed that all nodes are located on a 2D plane and the distance between the two source terminals are normalized by one. Without loss of generality, we set coordinates of all nodes as follows: A(0,0), B(1,0), $\mathrm{PU}-\mathrm{Rx}_k$(0.5,1), and $\mathrm{R}_i(0.5, 0)$ ∀i. Taking into account the channel path loss, we adopt $\alpha_{\mathsf{XY}} = d_{\mathsf{XY}}^{-\eta}$, where d_{XY} is the physical distance between node X and Y and η is the path loss exponent. For illustrative purpose, we set $\eta = 3$ and $\gamma_{\rm th} = 1$.

Fig. 2 presents the secondary system outage probability in three different cases with $\mathcal{P}_m$ = 10 dB, i.e., Case 1: $N = L = 1$, $m_{ia} = 1, m_{ib} = 2$, $m_{ip} = 1, m_{ap} = 3, m_{bp} = 1$; Case 2: $N = L = 2$, $m_{ia} = 2$, $m_{ib} = 2$, $m_{ip} = 2$, $m_{ap} = 2, m_{bp} = 1$; and Case 3: $N = L = 3$, $m_{ia} = 1, m_{ib} = 3$, $m_{ip} = 1, m_{ap} = 2$, $m_{bp} = 3$. It can be seen that the analysis results are in excellent agreement with the simulation results confirm the correctness of the derivation approach. Among three cases, Case 2 outperforms Case 3, which, in turns, outperforms Case 1, showing that the system outage probability depends not only the number of secondary relays but also the number of primary receivers.

In Fig. 3, we study the effect of number of relays on the system performance by plotting the system outage probability as a function of $I_{\rm p}$. We fix the channel and network parameters, i.e., $L = 3$, $m_{ia} = 1, m_{ib} = 2$, $m_{ip} = 1, m_{ap} = 3, m_{bp} = 1$, $\mathcal{P}_m$=10 dB while varying the number of relays from 1 to 3, i.e., $N = 1 \to 3$. It can be seen that increasing the number relays will improve the system outage probability significantly. In addition, there exists the irreducible outage probability at high regime of $I_{\rm p}$ due to the constraint of $\mathcal{P}_m$. As one would expect, the irreducible outage probability becomes smaller since the number relays increases.

In Fig. 4, the effect of $\mathcal{P}_m$ is investigated by increasing $\mathcal{P}_m$ from 5 dB to 10 dB. We consider two cases of network models including $N = L = 2$ and $N = L = 3$. As expected, we can see that increasing $\mathcal{P}_m$ will make the system outage probability floor smaller. Stated another way, the system outage probability just depend on the $\mathcal{P}_m$ at high regimes of $I_{\rm p}$. This observation is also repeated in Fig. 5, where we plot the system outage probability versus $I_{\rm p}$ by

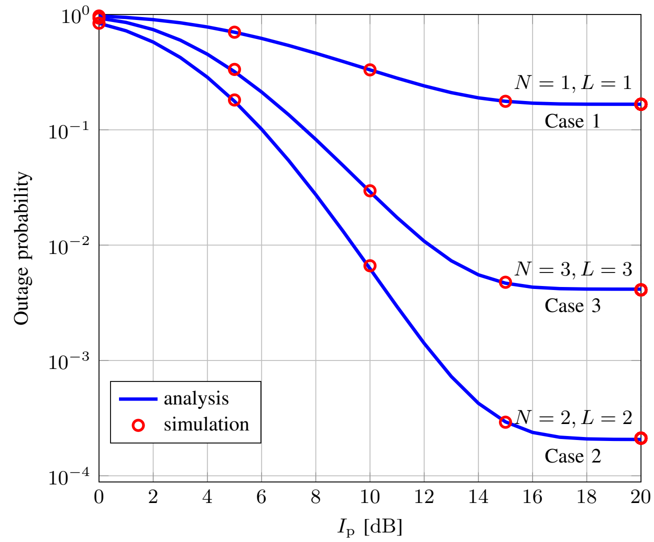

Fig. 2. System outage probability versus $I_{\rm p}$.

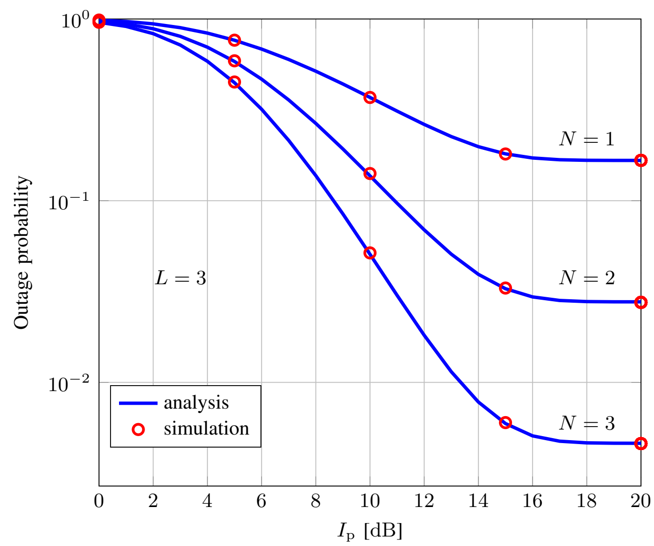

Fig. 3. Effect of the number of relay nodes on the system outage performance.

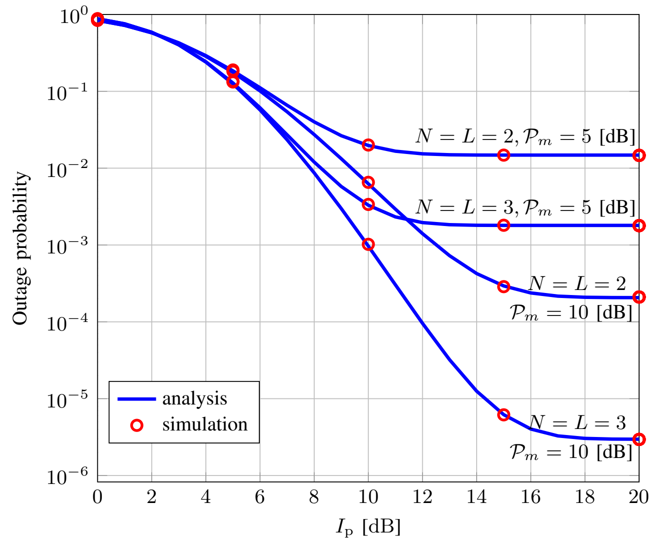

Fig. 4. Effect of $\mathcal{P}_m$ on the system outage probability.

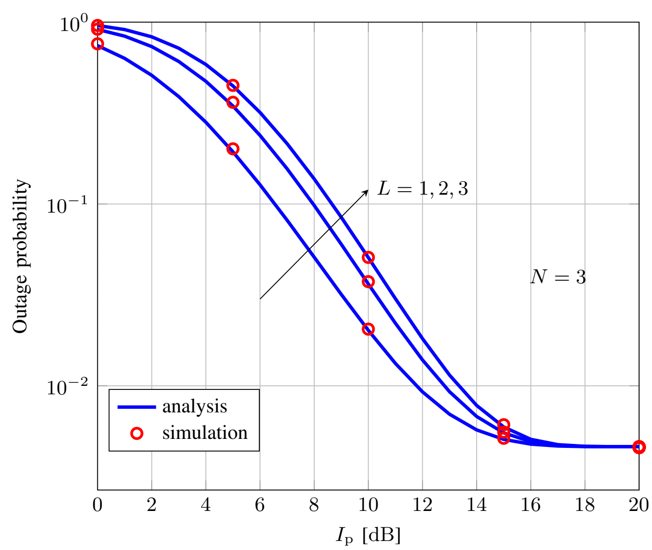

Fig. 5. Effect of the number of primary receiver on the system outage probability.

varying L and fixing N. Fig. 5 shows that for a fixed number of L, diminishing gain is obtained as the number of relays increases. Also, we can see that the system outage probability does not depend on the number of relays at high SNRs.

V. Conclusion

In this paper, we have studied the outage performance of cognitive two-way DF relay network with multiple primary receivers and multiple relay nodes over Nakagami-m channels is investigated. The obtained results consider the results in [10] as a special case. Numerical results showed that the system outage probability of secondary networks depends not only on the number of relays but also on the number of primary receivers and the maximum transmit power at secondary transmitters.

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