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Modeling and Feedback Linearization Control of a Nonholonomic Wheeled Mobile Robot with Longitudinal, Lateral Slips

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Abstract— **This paper presents a systematic method to build the kinematic model and dynamic model of a nonholonomic wheeled mobile robot (WMR) with longitudinal and lateral slip, followed by the design of a control law using the input-output feedback linearization method to drive the mobile robot to track a given trajectory while longitudinal, and lateral slip exist. The asymptotical stability of the system is verified by solving second-order differential linear equations to find solutions for history time. Matlab-Simulink simulation results show the correctness and performances of the control law.**

I. Introduction

The wheel mobile robot has been researched and applied at many places in the world in recent years. It would be an area attracting attention of many researchers from everywhere in the world. The reason why the wheeled mobile robot (WMR) is applied widely is that it could be able to work in an unlimited area, and especially able to implement tasks intelligently without any human action. Besides, it can replace people on dangerous tasks such as look for explosive materials, transport of goods in harmful environments, etc.

A lot of researching effort for wheeled mobile robot focused on problems of motion control. In [1], [2], [3], [4] the controllers were designed taking account of nonholonomic kinematic model and dynamic model. In that situation, the absolute condition in which there is only pure rolling is always ensured.

However, in many practical applications, the condition in which there are not any slips between wheels and floor is not always satisfied. That condition depends on many factors, namely a centrifugal force acting on WMR when it moves in a circular path, an external force acting on WMR when it collides with another object, the frictional force between floor and walls, etc. Consequently, if we want motion control problem to be solved, then in kinematic, dynamic models of WMR, we have to take account of slips. In [5], the authors develop a generalized kinematic model, including various slips such as lateral slip, longitudinal slips. In [6], a central force in the lateral direction of WMR is suggested to regulate the position of WMR when there is lateral slip.

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For longitudinal slip, in [7], the coefficient of friction was modelled as a function of slip ratio. In [8], only longitudinal slip has been taken into account when an autonomous robot for agriculture was shown. Next, in [9], the authors have built a path-following problem when there are both longitudinal and lateral slips. After that, a motion controller was designed, which has taken into account longitudinal slip. For lateral slip, this controller bases on a lateral friction model.

The contribution of this paper comprises:

- Modeling a kinematic and dynamic model of a WMR which have incorporated both lateral and longitudinal slips.
- Designing a control law using the input-output feedback linearization method for the WMR to track a desired trajectory in such a way that tracking errors converge on zeros asymptotically.

This paper is organized as follows. Section 2 derives both kinematic and dynamic models which consider both longitudinal and lateral slip. Section 3 discusses how to design a control by using the input-output feedback linearization method to drive the WMR track to a desired trajectory in such a way that tracking errors converge on zeros asymptotically. Simulation results, performed by Matlab-Simulink are shown in Section 4. Finally, Section 5 presents our conclusions.

II. Modelling

A. Kinematic model

Let us consider a nonholonomic WMR as Fig. 1. V, ω is the linear and angular velocity of the WMR's platform, respectively. When there are not any slips between the wheels and the floor, V and ω are computed, respectively, as follows:

$$V = \frac{r\dot{\phi}_R + r\dot{\phi}_L}{2} \tag{1}$$

$$\omega = \frac{r\dot{\phi}_R - r\dot{\phi}_L}{2b} \tag{2}$$

where $\dot{\phi}_R, \dot{\phi}_L$ are the angular velocities of the right and left wheel, respectively; r is the radius of the wheels; b is a half of the distance between the two wheels. Consequently, the kinematic equations of the WMR can be expressed by:

$$\dot{x}_M = V\cos\theta , \tag{3}$$

$$\dot{y}_M = V\sin\theta , \tag{4}$$

$$\omega = \dot{\theta} , \tag{5}$$

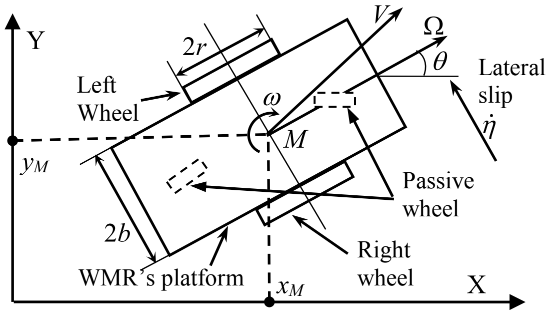


Figure 1. Nonholonomic wheeled mobile robot.

where x_M, y_M are coordinates of the midpoint, M, of the line segment joining the right and left wheel (see Fig. 1); θ is the direction of the WMR. Therefore, the nonholonomic constraint equations of the WMR are expressed by:

$$r\dot{\phi}_R = \dot{x}_M \cos\theta + \dot{y}_M \sin\theta + b\omega, \tag{6}$$

$$r\dot{\phi}_L = \dot{x}_M \cos\theta + \dot{y}_M \sin\theta - b\omega, \tag{7}$$

$$-\dot{x}_M \sin\theta + \dot{y}_M \cos\theta = 0, \tag{8}$$

Alternatively, when longitudinal and lateral slips between the wheels and the floor exist, let us express the coordinates of lateral slip, longitudinal slip of the right and left wheel as η, γ_R, γ_L, respectively. In this case, (1), (2) are rewritten as follows:

$$V = \sqrt{\Omega^2 + \dot{\eta}^2} \tag{9}$$

$$\omega = \frac{r\dot{\phi}_R - r\dot{\phi}_L}{2b} + \frac{\dot{\gamma}_R - \dot{\gamma}_L}{2b} \tag{10}$$

where Ω is the linear velocity along the longitudinal direction of the WMR's platform and is presented by:

$$\Omega = \frac{r\dot{\phi}_R + r\dot{\phi}_L}{2} + \frac{\dot{\gamma}_R + \dot{\gamma}_L}{2} \tag{11}$$

In this context, the kinematic model of the WMR is expressed as follows:

$$\dot{x}_M = \Omega\cos\theta - \dot{\eta}\sin\theta \tag{12}$$

$$\dot{y}_M = \Omega\sin\theta + \dot{\eta}\cos\theta \tag{13}$$

$$\omega = \dot{\theta} \tag{14}$$

Supposing that x_M, y_M, θ, $\dot{\phi}_R, \dot{\phi}_L$ are always measured exactly, according to the nonholonomic constraint equations of the WMR, the velocities of the slips can be represented as follows:

$$\dot{\gamma}_R = -r\dot{\phi}_R + \dot{x}_M \cos\theta + \dot{y}_M \sin\theta + b\omega \tag{15}$$

$$\dot{\gamma}_L = -r\dot{\phi}_L + \dot{x}_M \cos\theta + \dot{y}_M \sin\theta - b\omega \tag{16}$$

$$\dot{\eta} = -\dot{x}_M \sin\theta + \dot{y}_M \cos\theta \tag{17}$$

B. Dynamic Model

Without loss of generality, Let M be the center point of mass of the WMR's platform, and m_M is the mass of this platform, and I_M is the inertial moment of this platform about the vertical axis through the point M.

The kinetic energy of this platform is computed as follows:

$$K_M = \frac{1}{2} m_M \left(\dot{x}_M^2 + \dot{y}_M^2\right) + \frac{1}{2} I_M \omega^2 \tag{18}$$

The kinetic energy of the right and left wheel is computed, respectively, as follows:

$$K_L = \frac{1}{2} m_W \left(r^2 \dot{\phi}_L^2 + \dot{\gamma}_L^2 + \dot{\eta}^2\right) + \frac{1}{2} I_W \dot{\phi}_L^2 + \frac{1}{2} I_D \omega^2 \tag{29}$$

$$K_R = \frac{1}{2} m_W \left(r^2 \dot{\phi}_R^2 + \dot{\gamma}_R^2 + \dot{\eta}^2\right) + \frac{1}{2} I_W \dot{\phi}_R^2 + \frac{1}{2} I_D \omega^2 \tag{20}$$

where I_W or I_D respectively is an inertial moment of each wheel about its rotational axis and diameter axis.

The total kinetic energy of the whole system is:

$$\begin{aligned} K &= K_M + K_L + K_R \\ &= \frac{1}{2} m_W \left[r^2 \left(\dot{\phi}_L^2 + \dot{\phi}_R^2\right) + \dot{\gamma}_R^2 + \dot{\gamma}_L^2 \right] \\ &\quad + m_W \dot{\eta}^2 + \frac{1}{2} I_W \left(\dot{\phi}_L^2 + \dot{\phi}_R^2\right) \\ &\quad + \left(I_D + \frac{1}{2} I_M\right)\omega^2 + \frac{1}{2} m_M \left(\dot{x}_M^2 + \dot{y}_M^2\right) \end{aligned} \tag{21}$$

The potential energy of whole system always equals zero, so its Lagrange function is $L = K$.

Let $\mathbf{q} = \left[x_M, y_M, \theta, \eta, \gamma_R, \gamma_L, \phi_R, \phi_L\right]^{\mathrm{T}}$ be the Lagrange coordinate vector, the kinematic constraint equation can be written as follows:

$$\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0} \tag{22}$$

where $\mathbf{A}(\mathbf{q})$ is the matrix associated with the kinematic constraints. Comparing (15), (16), (17) with (22), $\mathbf{A}(\mathbf{q})$ can be described as follows:

$$\mathbf{A}(\mathbf{q}) = \begin{bmatrix} \cos\theta & \sin\theta & b & 0 & -1 & 0 & -r & 0 \\ \cos\theta & \sin\theta & -b & 0 & 0 & -1 & 0 & -r \\ -\sin\theta & \cos\theta & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

The Lagrange equation can be written in the following form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{u} + \mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda} \tag{24}$$

where $\boldsymbol{\lambda} = \left[\lambda_1, \lambda_2, \lambda_3\right]^{\mathrm{T}}$ is the vector of Lagrange multipliers, $\mathbf{u}$ is the generalized forced vector. Solving this Lagrange equation, the dynamic equation of the whole system can be represented by:

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{N}_1\boldsymbol{\tau} + \mathbf{N}_2 F_{lat} + \mathbf{N}_3 \mathbf{F}_{lon} + \mathbf{A}^{\mathrm{T}}\boldsymbol{\lambda}, \tag{25}$$

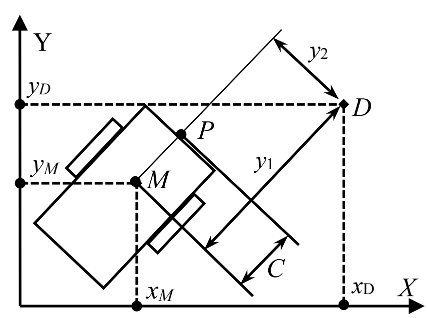


Figure 2. The output vector $\mathbf{y}$

$$\mathbf{M}=\begin{bmatrix} m_M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_M & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_M+2I_D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2m_W & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_W & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_W & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_W r^2+I_W & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_W r^2+I_W \end{bmatrix} \tag{26}$$

where $\mathbf{N}_1=\begin{bmatrix} 0\,0\,0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,0\,0\,0\,1 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{N}_2=\begin{bmatrix} 0\,0\,0\,1\,0\,0\,0\,0 \end{bmatrix}^{\mathrm{T}}$, $\mathbf{N}_3=\begin{bmatrix} 0\,0\,0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0 \end{bmatrix}^{\mathrm{T}}$ are input matrices, $\boldsymbol{\tau}=\left[\tau_R,\tau_L\right]^{\mathrm{T}}$ is the torque vector including torques at the rotational axis of the right and left wheels, F_{lat} is a unknown force making lateral slip, $\mathbf{F}_{lon}=\left[F_R,F_L\right]^{\mathrm{T}}$ is a vector which includes unknown forces, F_R and F_L, making longitudinal slip at the right and left wheel, respectively, and $\mathbf{M}$ is an 8×8 diagonal positive definite inertia matrix and is represented as (26).

It is easy to achieve the following equation:

$$\dot{\mathbf{q}}=\mathbf{S}_1(\mathbf{q})\mathbf{v}+\mathbf{S}_2(\mathbf{q})\dot{\eta}+\mathbf{S}_3(\mathbf{q})\dot{\boldsymbol{\gamma}}, \tag{27}$$

where $\mathbf{v}=\left[\dot{\phi}_R,\dot{\phi}_L\right]^{\mathrm{T}}$, $\mathbf{S}_1(\mathbf{q})$, $\mathbf{S}_2(\mathbf{q})$, and $\mathbf{S}_3(\mathbf{q})$ are matrices described as follows:

$$\mathbf{S}_1(\mathbf{q})=\begin{bmatrix} \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \\ \frac{r}{2b} & -\frac{r}{2b} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix};\mathbf{S}_2(\mathbf{q})=\begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{S}_3(\mathbf{q})=\begin{bmatrix} \frac{1}{2}\cos\theta & \frac{1}{2}\cos\theta \\ \frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Next, taking the time derivative of equation (27) gives:

$$\begin{aligned}\ddot{\mathbf{q}}&=\dot{\mathbf{S}}_1(\mathbf{q})\mathbf{v}+\mathbf{S}_1(\mathbf{q})\dot{\mathbf{v}}+\dot{\mathbf{S}}_2(\mathbf{q})\dot{\eta}\\&+\mathbf{S}_2(\mathbf{q})\ddot{\eta}+\dot{\mathbf{S}}_3(q)\dot{\boldsymbol{\gamma}}+\mathbf{S}_3(q)\ddot{\boldsymbol{\gamma}}\end{aligned}, \tag{28}$$

Moreover, it is always true to write:

$$\begin{aligned}&\mathbf{S}_1^{\mathrm{T}}(\mathbf{q})\mathbf{A}^{T}(\mathbf{q})=\mathbf{0},\quad \mathbf{S}_1^{\mathrm{T}}(\mathbf{q})\mathbf{N}_1=\mathbf{I},\\&\mathbf{S}_1^{\mathrm{T}}(\mathbf{q})\mathbf{N}_2=\mathbf{0},\quad \mathbf{S}_1^{\mathrm{T}}(\mathbf{q})\mathbf{N}_3=\mathbf{0},\end{aligned} \tag{29}$$

Hence, pre-multiplying the both sides of the equation (25) with $\mathbf{S}_1^{\mathrm{T}}(\mathbf{q})$ yields:

$$\begin{aligned}&\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_1(\mathbf{q})\right]\dot{\mathbf{v}}+\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}_1(\mathbf{q})\right]\mathbf{v}\\&+\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}_2(\mathbf{q})\right]\dot{\eta}+\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_2(\mathbf{q})\right]\ddot{\eta}\\&+\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_3(\mathbf{q})\right]\ddot{\boldsymbol{\gamma}}+\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}_3(\mathbf{q})\right]\dot{\boldsymbol{\gamma}}=\boldsymbol{\tau}\end{aligned}, \tag{30}$$

Also, it is obvious that

$$\begin{aligned}&\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}_1(\mathbf{q})=\mathbf{0},\\&\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_2(\mathbf{q})=\mathbf{0},\quad \mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}_3(\mathbf{q})=\mathbf{0}.\end{aligned}, \tag{31}$$

Substituting (31) into (30) results in:

$$\begin{aligned}&\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_1(\mathbf{q})\right]\dot{\mathbf{v}}+\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}_2(\mathbf{q})\right]\dot{\eta}\\&+\left[\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_3(\mathbf{q})\right]\ddot{\boldsymbol{\gamma}}=\boldsymbol{\tau}\end{aligned}, \tag{32}$$

Rewriting equation (32) yields:

$$\mathbf{m}\dot{\mathbf{v}}+\mathbf{b}\omega\dot{\eta}+\mathbf{z}\ddot{\boldsymbol{\gamma}}=\boldsymbol{\tau}, \tag{33}$$

where $\mathbf{m}=\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_1(\mathbf{q})=\begin{bmatrix}\bar{m}_{11} & \bar{m}_{12} \\ \bar{m}_{21} & \bar{m}_{22}\end{bmatrix}$,

$$\begin{aligned}&\bar{m}_{11}=\bar{m}_{22}\\&=m_M\left(\frac{r}{2}\right)^2+\left(I_M+2I_D\right)\left(\frac{r}{2b}\right)^2+\left(m_W r^2+I_W\right),\end{aligned}$$

$$\bar{m}_{12}=\bar{m}_{21}=m_M\left(\frac{r}{2}\right)^2-\left(I_M+2I_D\right)\left(\frac{r}{2b}\right)^2,$$

$$\mathbf{b}\omega=\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\dot{\mathbf{S}}_2(\mathbf{q})=-m_M\frac{r}{2}\begin{bmatrix}1 & 1\end{bmatrix}^{\mathrm{T}}\omega$$

$$\mathbf{z}=\mathbf{S}_1(\mathbf{q})^{\mathrm{T}}\mathbf{M}\mathbf{S}_3(\mathbf{q})=\frac{m_M r}{4}\begin{bmatrix}1 & 1 \\ 1 & 1\end{bmatrix}$$

III. Designing Control Law

A. Problem statement

Let $D(x_D,y_D)$ be a target which is moving with a invariable linear velocity, V_D, along an orientation θ_D which is either variable or constant. Thus, the motion equation of the point D can be written as follows:

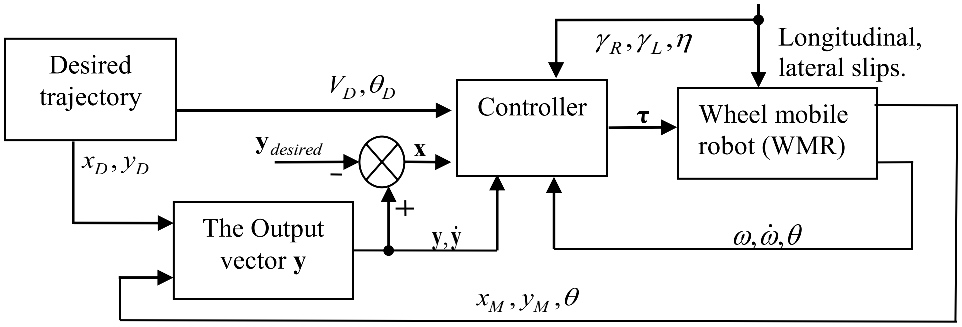


Figure 3. Scheme of control system for a nonholonomic wheeled mobile robot when there are slips.

$$\begin{aligned} \dot{x}_D &= V_D \cos\theta_D \\ \dot{y}_D &= V_D \sin\theta_D \end{aligned} \tag{34}$$

The requirement of this control problem is to control the WMR so that point P has to track point D with tracking errors converge on zero asymptotically (Fig. 2).

B. Describing the input- output relationship

In order to describe the input-output relationship of this system, an output vector of the whole system is presented in the following form:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_D - x_M \\ y_D - y_M \end{bmatrix}. \tag{35}$$

Taking the derivative with respect to time of (35) yields:

$$\dot{\mathbf{y}} = \begin{bmatrix} y_2.\omega - \Omega + V_D \cos(\theta - \theta_D) \\ -y_1\omega - \dot{\eta} - V_D \sin(\theta - \theta_D) \end{bmatrix}. \tag{36}$$

Taking the time derivative of equation (36) again results in:

$$\ddot{\mathbf{y}} = -\begin{bmatrix} \dot{\vartheta} \\ y_1 \dot{\mu} \end{bmatrix} + \mathbf{f}, \tag{37}$$

where $\mathbf{f}$ is a non-linear component which will be determined lately, $\mu = \frac{r}{2b}(\dot{\phi}_R - \dot{\phi}_L)$, $\vartheta = \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L)$, $\chi = \frac{\dot{\gamma}_R + \dot{\gamma}_L}{2}$, $\kappa = \frac{\dot{\gamma}_R - \dot{\gamma}_L}{2b}$. Hence, it yields: $\omega = \mu + \kappa$, and $\Omega = \vartheta + \chi$.

In (37), $\mathbf{f}$ is a vector depending on the trajectory of target D. If D moves on a line with linear velocity V_D and orientation θ_D, then $\mathbf{f}$ is replaced by $\mathbf{f}_L$ as follows:

$$\mathbf{f}_L = \begin{bmatrix} \dot{y}_2.\omega + y_2.\dot{\omega} - \dot{\chi} - V_D\omega\sin(\theta - \theta_D) \\ -y_1\dot{\kappa} - \dot{y}_1\omega - \ddot{\eta} - V_D\omega\cos(\theta - \theta_D) \end{bmatrix}. \tag{38}$$

Conversely, if $D(x_D, y_D)$ moves with an invariable linear velocity V_D along a circular path as follows:

$$(x_D - x_O)^2 + (y_D - y_O)^2 = R^2, \tag{39}$$

then $\mathbf{f}$ is replaced by $\mathbf{f}_C$ as follows:

$$\mathbf{f}_C = \begin{bmatrix} \dot{y}_2.\omega + y_2.\dot{\omega} - \dot{\chi} - V_D\left(\omega - \dot{\theta}_D\right)\sin(\theta - \theta_D) \\ -y_1\dot{\kappa} - \dot{y}_1\omega - \ddot{\eta} - V_D\left(\omega - \dot{\theta}_D\right)\cos(\theta - \theta_D) \end{bmatrix}, \tag{40}$$

In other words, it can be rewritten (37) as follows:

$$\ddot{\mathbf{y}} = -\mathbf{h}\dot{\mathbf{v}} + \mathbf{f}, \tag{41}$$

where

$$\mathbf{h} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ y_1\frac{r}{2b} & -\left(y_1\frac{r}{2b}\right) \end{bmatrix}, \tag{42}$$

If $y_1 > 0$, then $\mathbf{h}$ is always an invertible matrix.

With torque vector $\boldsymbol{\tau}$ considered as the input vector, combining (33) with (41), we achieve:

$$\ddot{\mathbf{y}} = -\mathbf{h}\mathbf{m}^{-1}\left(\boldsymbol{\tau} - \mathbf{b}\omega\dot{\eta} - \mathbf{z}\ddot{\boldsymbol{\gamma}}\right) + \mathbf{f}, \tag{43}$$

C. Design the input- output feedback linearization control law

We define the tracking error vector as $\mathbf{x} = [x_1, x_2]^{\mathrm{T}} = \mathbf{y} - \mathbf{y}_{desired}$ where $\mathbf{y}_{desired}$ is the desired vector of vector $\mathbf{y}$.

To choose a control law using the input-output feedback linearization method, we propose a scheme of the whole system as Fig. 3, and the torque vector is computed as follows:

$$\boldsymbol{\tau} = \mathbf{b}\omega\dot{\eta} + \mathbf{z}\ddot{\boldsymbol{\gamma}} + \mathbf{m}\mathbf{h}^{-1}\left(\mathbf{f} - \ddot{\mathbf{y}}_{desired} + \mathbf{K}_D\dot{\mathbf{x}} + \mathbf{K}_P\mathbf{x}\right), \tag{44}$$

where $\mathbf{K}_P$, $\mathbf{K}_D$ are diagonal, constant, positive definite matrices and are chosen arbitrarily as follows:

$$\mathbf{K}_D = \begin{bmatrix} K_{D1} & 0 \\ 0 & K_{D2} \end{bmatrix}, \quad \mathbf{K}_P = \begin{bmatrix} K_{P1} & 0 \\ 0 & K_{P2} \end{bmatrix}., \tag{45}$$

Substituting (44) into (43) results in:

$$\ddot{\mathbf{x}} + \mathbf{K}_D\dot{\mathbf{x}} + \mathbf{K}_P\mathbf{x} = \mathbf{0}. \tag{46}$$

We can write (46) more details as follows:

$$\begin{cases} \ddot{x}_1 + K_{D1}\dot{x}_1 + K_{P1}x_1 = 0, \\ \ddot{x}_2 + K_{D2}\dot{x}_2 + K_{P2}x_2 = 0, \end{cases} \tag{47}$$

The characteristics equations of (47) in frequency domain are written by:

$$\begin{cases} s^2 + K_{D1}s + K_{P1} = (s - a_{11})(s - a_{12}) = 0 \\ s^2 + K_{D2}s + K_{P2} = (s - a_{21})(s - a_{22}) = 0 \end{cases} \tag{48}$$

where s is the Laplace operator, a_{11}, a_{12}, a_{21}, a_{22} are solutions of (48) and described as follows:

$$a_{11,12} = \frac{-K_{D1} \pm \sqrt{K_{D1}^2 - 4K_{P1}}}{2},$$

$$a_{21,22} = \frac{-K_{D2} \pm \sqrt{K_{D2}^2 - 4K_{P2}}}{2},$$

Next, we solve (47) for time history t:

For $x_1(t)$, either $x_1(t) = z_{11}e^{a_{11}t} + z_{12}e^{a_{12}t}$ if $a_{11} \neq a_{12}$ or

$x_1(t) = z_{11}e^{a_{11}t} + z_{12}te^{a_{11}t}$ if $a_{11} = a_{12}$.

For $x_2(t)$, either $x_2(t) = z_{21}e^{a_{21}t} + z_{22}e^{a_{22}t}$ if $a_{21} \neq a_{22}$ or

$x_2(t) = z_{21}e^{a_{21}t} + z_{22}te^{a_{21}t}$ if $a_{21} = a_{22}$.

where z_{11}, z_{12} z_{21}, and z_{22} are constant parameters which depend on initial conditions.

Since $\mathbf{K}_D, \mathbf{K}_P$ are chosen as above, the real parts of all $a_{11}, a_{12}, a_{21}, a_{22}$ are negative values, Therefore, both $x_1(t)$ and $x_2(t)$ converge on zero asymptotically i.e. $\mathbf{x} \to \mathbf{0}$. Consequently, $\mathbf{y} \to \mathbf{y}_{desired}$. Otherwise, $y_1 \to C$ and $y_2 \to 0$. Point P (see Fig. 2) in turn tracking the target (point D) asymptotically. This would lead to that the requirement of the control problem stated above is satisfied.

Remark: Due to nonholonomic constraints, the WMR can not move to approach to the target along its lateral direction. So, when $y_1 = 0$, particularly point D (target) belongs to the line joining the center of the wheels (Fig. 2), it is impossible to control the WMR to approach to the target.

Because of estimating risk of a case in which y_1 converges on zero. We have prevented y_1 from converging on 0 in such a way that y_1 is forced to come into a constant $C > 0$. Therefore, our control law performs three tasks simultaneously. Those three tasks consist of:

- Ensuring matrix **h** in (44) is always invertible.
- Compensating slips comprising lateral slip, longitudinal slip of the right wheel, and longitudinal slip of the left wheel.
- Controlling the WMR tracking to a moving target with errors converge on zeros.

IV. SIMULATION

The whole control system is simulated with parameters described in TABLE I. Also, C = 0.3 m (see Fig. 2), $\mathbf{K}_P = \mathbf{K}_D = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ can be selected.

Next, the simulation is performed in two cases:

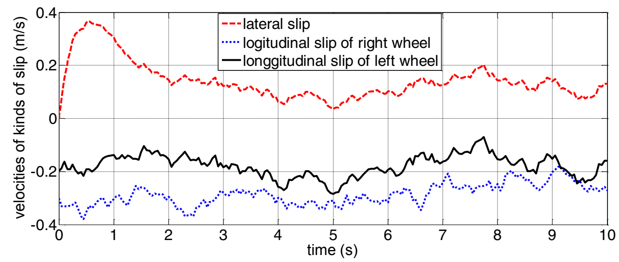
A. Case 1: The target D(x_D, y_D) moves with linear velocity V_D = 2 m/s in a straight line which has a direction angle $\theta_D = \pi/6$.

B. Case 2: The target, D(x_D, y_D), moves in a circular path as follows:

$$(x-3)^2 + (y-3)^2 = 3^2$$

with a linear velocity V_D = 0.9 m/s.

Without loss of generality, we suppose that lateral slip, longitudinal slips in both the cases are presented as Fig. 4.



Figue 4. Slips in both **Case 1** and **Case 2**.

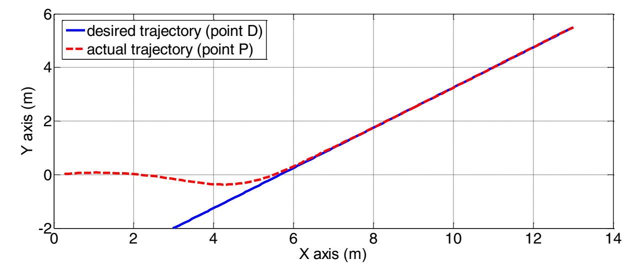


Figure 5. Desired and actual trajectory of point P in **Case 1**.

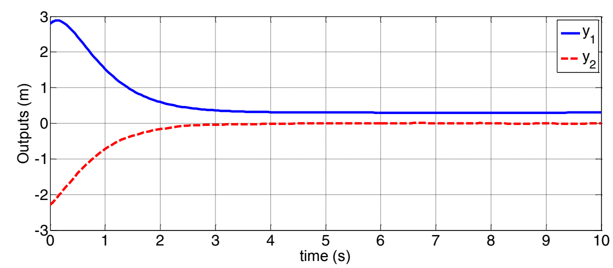


Figure 6. The Outputs in **Case 1**.

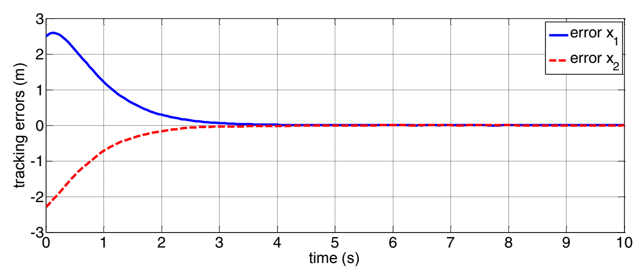


Figure 7. Errors $\mathbf{x} = \mathbf{y} - \mathbf{y}_{desired}$ in **Case 1**.

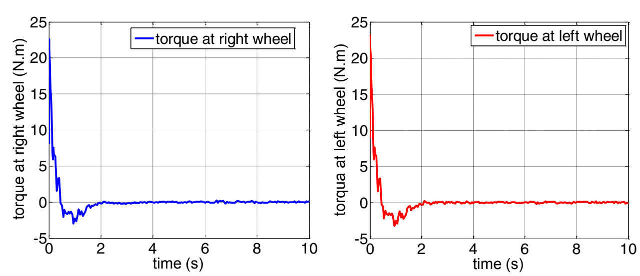


Figure 8. Torques at axes of wheels in **Case 1**.

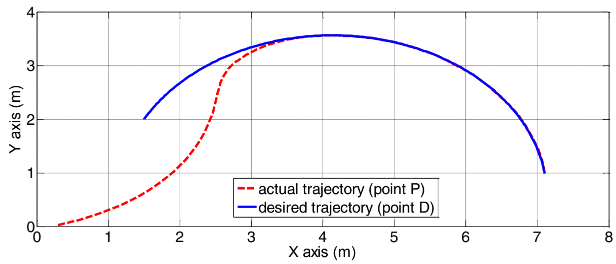


Figure 9. Actual and desired trajectory of point P in **Case 2.**

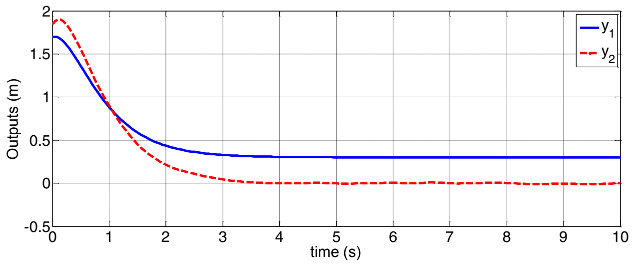


Figure 10. the outputs ***y*** in **Case 2.**

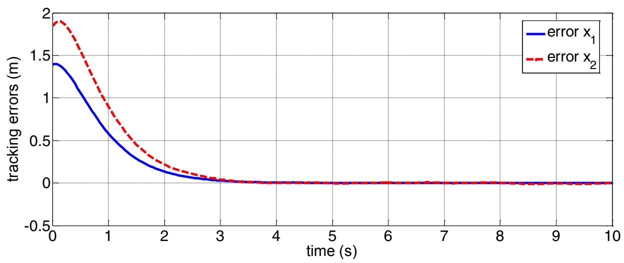


Figure 11. tracking errors $\mathbf{x} = \mathbf{y} - \mathbf{y}_{desired}$ in **Case 2.**

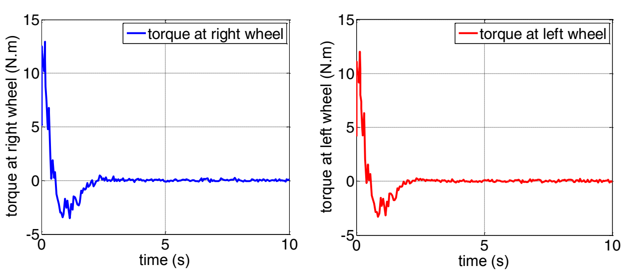


Figure 12. Torques at the right and left wheel in **Case 2**.

Figures including 5, 7, 9, and 11 show the correctness and the performance of the control law in both cases. In Figures 6, 10, $y_1 \to C$ when $t \to \infty$. Otherwise, $y_1 > 0$ with $\forall t > 0$, so matrix $\mathbf{h}$ in (45) is always invertible. Fig.8 and Fig.12 express the torques in both cases which are always finite and smooth. Consequently, the control law described by (44) and the scheme as Fig.3 is feasible.

V. Conclusion

This paper proposes a systematic method to build kinematic, dynamic models to model the motion of a nonholonomic wheeled mobile robot when there are slips including lateral slip, longitudinal slip of the right wheel, and longitudinal slip of the left wheel. Next, a control law is suggested to control the WMR track to a target moving with a desired trajectory. The control law is designed with the input-output feedback linearization method, which makes the tracking errors converge on zero asymptotically.

TABLE I. SYMBOLS AND QUANTITY

Symbol	Quantity	Value
r	Radius of wheel	0.098 (m)
b	Half of the distance between two wheels	0.2 (m)
I_M	The inertial moment of platform about the vertical axis through point M (center of mass)	0.02 (kg.m^2)
I_W	The inertial moment of wheel about the rotational axis	0.003 (kg.m^2)
I_D	The inertial moment of each wheel about its diameter axis	0.005 (kg.m^2)
m_M	Mass of platform	22 kg
m_W	Mass of each wheel	2 kg

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