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Joint pre-processing co-channel interference cancellation for single user MIMO

Pham Thanh Hiep[1] · Vu Van Son[1]

Abstract In general, multiplexing and diversity gains of single user MIMO systems are restricted by min(M,N) where M, N denote the number of antenna elements at a transmitter and receiver, respectively. In order to increase the multiplexing/diversity gains and improve the performance of single user MIMO systems, a joint pre-processing co-channel interference cancellation (JPCIC) method is proposed. The JPCIC is analyzed in both the perfect and the imperfect channel state information. The dependence of channel capacity on the number of antenna elements in every subset, the number of subsets, transmit powers and channel estimation errors is discussed. As theoretical calculation result, the channel capacity increases when the multiplexing/diversity gains and/or the transmit power increase in a certain channel model whether the channel estimation error is absent or present. Compared to the conventional zero-forcing method, the channel capacity of JPCIC is considerably higher because of higher multiplexing/diversity gains, however, it is less robust and decreased more rapidly due to incomplete cancellation of interference terms when the channel estimation error increases. There is a trade-off between the channel capacity and the complexity of system, however, according to quick development in circuit techniques and miniaturization of devices, the JPCIC is expected to be an attractive technology for MIMO system.

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Keywords Joint pre-processing co-channel interference cancellation · Channel estimation error · Multiplexing and diversity gains · Incomplete cancellation of interference terms · Single user MIMO · Zero-forcing algorithm

1 Introduction

Multiple input multiple output (MIMO) is an attractive technology for wireless communication system. In order to increase the effect of MIMO systems, many signal processing algorithms were proposed for both linear and nonlinear receivers. The research on signal processing algorithms for MIMO also continues recently. Some well-known algorithms are zero-forcing (ZF) [1, 2], successive interference cancellation (SIC) [3, 4], minimum mean squared error (MMSE) [5, 6], dirty paper coding (DPC) [7, 8], singular value decomposition (SVD) [9–11], vertical Bell Labs layered space-time (V-BLAST) [12, 13], maximum likelihood detection (MLD) [13] and so on. The combination of these algorithms, i.e. ZF-DPC [14, 15], MMSE-SIC [16], etc. was also analyzed. Regardless of complexity and condition of channel state information (CSI) at both transmitter and receiver, in simple term of these algorithms, a channel capacity of MIMO systems is proportional to maximal number of streams, $\min(M, N)$, where M, N denote the number of antenna elements at transmitter and receiver, respectively.

Additionally, another technologies, i.e. space time block code (STBC) [12, 17–19], orthogonal frequency division multiplexing [11, 19], etc. are applied to MIMO system to increase the channel capacity and/or the robust of MIMO systems under some specified channel models. According to application of another technologies, the performance of MIMO systems is improved, however, the restriction of

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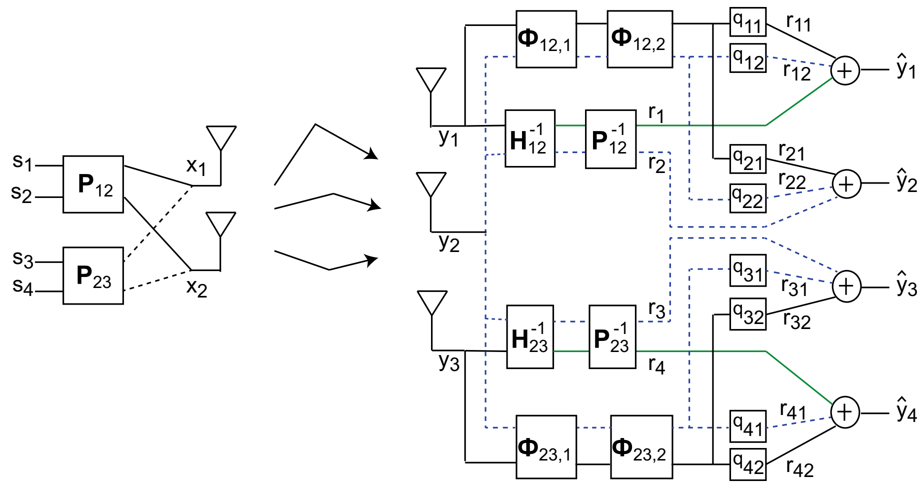
Fig. 1 System model of JPCICs where M = 2, N = 3



min(M, N) on multiplexing/diversity gains (hereafter, multiplexing gain) still has an effect. In order to greatly improve the performance of MIMO systems, the restriction of min(M, N) should be surmounted.

In this paper, we propose a method to increase the multiplexing gain of MIMO systems. In all algorithms mentioned above, the ZF is evaluated as the simplest one because of simple processing at receiver and unnecessary CSI at transmitter, therefore, the ZF is adopted in this paper. The pre-processing at transmitters is necessary to separate the received signal at receivers. Thus, this method is called as joint pre-processing co-channel interference cancellation (JPCIC). The JPCIC indicates that the multiplexing gain isn't restricted by min(M, N) and it can achieve considerable value. The maximal number of transmission streams for MIMO systems is discussed in general. Furthermore, the performance is analyzed for both perfect and imperfect CSIs. The variation in channel capacity that depends on the number of streams, the channel estimation error and the transmit power is discussed.

The notation used in this paper is as follow. Regular and bold styles respectively denote a scalar and a vector/matrix. $\mathbf{X}^{\mathcal{H}}$,$\mathbf{X}^{\mathrm{T}}$ and $\mathbf{X}^{-1}$ represent the Hermitian, transpose and pseudo-inverse operations of $\mathbf{X}$, respectively. Since a component-wise form is adopted to analyze the performance of system, let $\mathbf{X}(l)$ and $\mathbf{AB}(l)$ denote the lth row of matrix $\mathbf{X}$ and the lth row of matrix $\mathbf{Y} = \mathbf{AB}$. $\mathbb{C}^{m\times n}$ describes m rows, n columns normalized matrices from zero mean unit variance independent and identically distributed (i.i.d.) complex Gaussian entries matrices. The normalization operation is to emphasize the signal processing and keep the amplitude of signals from the variation by signal processing.

The rest of paper is organized as follows. We introduce the system model and analyze the performance of JPCIC in perfect CSI in Sect. 2. The performance analysis of imperfect CSI and calculation method of channel capacity is described in Sect. 3. Section 4 shows the numerical calculation result of JPCIC and Sect. 5 concludes the paper.

2 System model of JPCICs

2.1 A simple system model

A simple system model of JPCIC is depicted in Fig. 1, here the number of antenna elements at transmitter (M) and receiver (N) is respectively two and three. The channel response matrix is denoted by $\mathbf{H} \in \mathbb{C}^{3\times 2}$ and $\mathbf{H}$ is expressed as

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix}. \tag{1}$$

Let $\mathbf{H}_{12}$ denote channel response matrix between the first, the second antenna elements of receiver and two antenna elements of transmitter. Similarly, $\mathbf{H}_{23}$ denotes the channel response matrix between the second, the third antenna elements of receiver and two antenna elements of transmitter.

$$\mathbf{H}_{12} \equiv \begin{bmatrix} \mathbf{H}(1) \\ \mathbf{H}(2) \end{bmatrix}, \tag{2}$$

$$\mathbf{H}_{23} \equiv \begin{bmatrix} \mathbf{H}(2) \\ \mathbf{H}(3) \end{bmatrix}. \tag{3}$$

As depicted in Fig. 1, $\mathbf{P}_{12}$ and $\mathbf{P}_{23}$ are used as pre-processing for transmit signals at transmitter. Therefore, the transmit signal $\mathbf{X}$ is expressed as

$$\begin{aligned} \mathbf{X} &\equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \\ &= \mathbf{P}_{12}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \mathbf{P}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}. \end{aligned} \tag{4}$$

The received signal $\mathbf{Y}$ at receiver is as

$$\mathbf{Y} \equiv \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, = \mathbf{HX} + \mathbf{N}, \tag{5}$$

here $\mathbf{N} \in \sigma^2\mathbb{C}^{3\times1}$ represents the thermal noise vector at receiver, σ^2 is the variance of noise vector. The received signal of first and second antenna elements at receiver, $\mathbf{Y}_{12} = \mathbf{H}_{12}\mathbf{X} + \mathbf{N}_{12}$, here $\mathbf{N}_{12} \in \sigma^2\mathbb{C}^{2\times1}$ denotes the thermal noise vector of first and second antenna elements at receiver. In conventional ZF algorithms, since $s_3 = s_4 = 0$, information signals s_1, s_2 can be obtained by multiplying pseudo-inverse of $\mathbf{H}_{12}$, $\mathbf{H}_{12}^{-1}$. However, the number of streams is only 2. In the proposed system, since $s_3 \neq 0$ and $s_4 \neq 0$, each information signal should be separated meaning co-channel interference should be cancelled, however the number of streams is increased.

2.2 Co-channel interference cancellation

In this section, the co-channel interference cancellation method is explained. At first, the method to extract information signals s_1 and s_2 is explained. $\mathbf{Y}_{12}$ is multiplied by $\mathbf{H}_{12}^{-1}$, $\mathbf{P}_{12}^{-1}$ and from equations above, we have

$$\begin{aligned} \mathbf{R}_{12} \equiv \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} &= \mathbf{P}_{12}^{-1}\mathbf{H}_{12}^{-1}\mathbf{Y}_{12}, \\ &= \mathbf{P}_{12}^{-1}\mathbf{H}_{12}^{-1}\mathbf{H}_{12}\left(\mathbf{P}_{12}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \mathbf{P}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}\right) \\ &\quad + \mathbf{P}_{12}^{-1}\mathbf{H}_{12}^{-1}\mathbf{N}_{12}, \\ &= \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \mathbf{P}_{12}^{-1}\mathbf{P}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix} + \mathbf{P}_{12}^{-1}\mathbf{H}_{12}^{-1}\mathbf{N}_{12}. \end{aligned} \tag{6}$$

In (6), the first term is the desired signal, the second term is considered as interference signal that should be removed and the third term is the noise vector. As shown in Fig. 1, in order to cancel interference term in (6), the $\mathbf{Y}_{12}$ is multiplied by $\mathbf{\Phi}_{12,1} \in \mathbb{C}^{\mathbf{2\times2}}$ and $\mathbf{\Phi}_{12,2} \in \mathbb{C}^{\mathbf{2\times2}}$ which are subject to $\mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{H}_{12}\mathbf{P}_{12} = 0$, (refer to 6). After multiplying by $\mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}$, the linear processing is controlled by q_{11}, q_{12}.

$$\begin{aligned} r_{11} &\equiv q_{11}\mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{Y}_{12}(1), \\ &= q_{11}\mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\left(\mathbf{H}_{12}\mathbf{P}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix} + \mathbf{N}_{12}\right)(1). \\ &= q_{11}\left(\mathbf{K}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix} + \mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{N}_{12}\right)(1), \end{aligned} \tag{7}$$

here $\mathbf{K}_{23} = \mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{H}_{12}\mathbf{P}_{23}$. Similar to the r_{11}, the r_{12} is described by

$$r_{12} = q_{12}\left(\mathbf{K}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix} + \mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{N}_{12}\right)(2). \tag{8}$$

Therefore, the received signal of first stream, $\hat{y}_1$, is represented as follows.

$$\hat{y}_1 \equiv r_1 + r_{11} + r_{12}. \tag{9}$$

The co-channel interference is cancelled when there exists q_{11}, q_{12} that satisfy with

$$\begin{aligned} &\mathbf{P}_{12}^{-1}\mathbf{P}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(1) + q_{11}\mathbf{K}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(1) \\ &\quad + q_{12}\mathbf{K}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(2) = 0, \quad \text{for all } s_3, s_4. \end{aligned} \tag{10}$$

(refer to 7). Thus,

$$\begin{aligned} \hat{y}_1 = &s_1 + \mathbf{P}_{12}^{-1}\mathbf{H}_{12}^{-1}\mathbf{N}_{12}(1) \\ &+ q_{11}\mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{N}_{12}(1) \\ &+ q_{12}\mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{N}_{12}(2), \end{aligned} \tag{11}$$

Similar to $\hat{y}_1$, the co-channel interference term is cancelled and the received signal of second stream, $\hat{y}_2$, is obtained under condition as

$$\begin{aligned} &\mathbf{P}_{12}^{-1}\mathbf{P}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(2) + q_{21}\mathbf{K}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(1) \\ &\quad + q_{22}\mathbf{K}_{23}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(2) = 0, \quad \text{for all } s_3,\ s_4. \end{aligned} \tag{12}$$

Furthermore, instead of first and second antenna elements, the preprocessing at transmitter and the signal processing at receiver for the second and the third antenna elements are used to cancel the co-channel interference term, and then information signals s_3, s_4 can be obtained.

2.3 Maximum of multiplexing gain

As explained in the previous section, the multiplexing gain is four for 2×3 MIMO system, it is double conventional ZF systems. However, the preprocessing and signal processing for the first and the third antenna elements can be used to obtain more two multiplexing gains. It means that the maximum of multiplexing gain is $2\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, here $\begin{pmatrix} i \\ j \end{pmatrix}$ is the binomial coefficient indexed by i and j.

In general, the number of antenna elements at receiver is N, the number of pairs of two antenna elements that can extract a different information signal is $\begin{pmatrix} N2 \end{pmatrix}$, therefore the multiplexing gain achieves $2\begin{pmatrix} N2 \end{pmatrix}$. Furthermore, in case the number of antenna elements at transmitter is M, the number of pairs at transmitter is $\begin{pmatrix} M2 \end{pmatrix}$. As explained in the previous section, the co-channel interference cancellation processing is independent to the number of signals that are considered as interference signals. It means the co-channel interference cancellation processing is the same regardless the number of interference signals. The information signal can be extracted depending on the different preprocessing at transmitter and the channel response matrix. Consequently, if the different preprocessing at transmitter is applied to each transmit pair, all information signals can be extracted at receiver. As a result, the maximal multiplexing gain can achieve $2\binom{N}{2}\binom{M}{2}$.

Up to now, a subset that consists of two antenna elements (a pair) was discussed. However, in general, each subset can consist of p antenna elements. The index of desired subset that has p antenna elements is denoted by p_{sub} and the sum of interference subsets is denoted by $\sum_{\bar{p}_{sub}}$. In this case, the co-channel interference cancellation processing is the same as explained in the previous section, furthermore, the number of matrices $\mathbf{\Phi}_{p_{sub}} \in \mathbb{C}^{p\times p}$ also is two (6). The extension of number of antenna elements in every subset from two to p, $(p \geq 2)$, is straightforward. The maximum of multiplexing gain for $M \times N$ MIMO system is $p\binom{M}{p}\binom{N}{p}$. It can be said that the multiplexing gain of JPCIC increases when M or/and N increase whereas the multiplexing gain of conventional ZF and another algorithms only increases when both M and N increase. In Fig. 2, the increase of multiplexing gain based on N is depicted when M is fixed as 4 and p is changed, e.g. 2, 3, 4. When N is small, the system with $p = 2$ can achieves the highest multiplexing gain, on the contrary, when N increases, the system with higher p achieves the highest multiplexing gain. Moreover, the equal antenna element for both the transmitter and the receiver can achieve the highest multiplexing gain for every p (refer to Fig. 3).

The multiplexing gain of MIMO systems rapidly increases when the number of antenna elements increases. However, every subset of p antenna elements requests an independent preprocessing at transmitter as well as the signal processing at receiver. Therefore, the complexity of JPCIC system also considerably increases. The trade-off of multiplexing gain-complexity should be considered.

3 Performance analysis

3.1 Imperfect CSI system model

Up to now, the perfect CSI was assumed and the co-channel interference cancellation method has been explained. However, in actuality, the perfect CSI assumption is not always practical due to channel estimation errors. In order to characterize the imperfect CSI at the receiver, the estimated noisy channel response matrix is described as

$$\mathbf{H}_{p_{sub}} = \rho\hat{\mathbf{H}}_{p_{sub}} + \xi\bar{\mathbf{H}}_{p_{sub}}, \quad (13)$$

here $\bar{\mathbf{H}}_{p_{sub}}$ denotes the estimation error channel while $\hat{\mathbf{H}}_{p_{sub}}$ is the true one; $\bar{\mathbf{H}}_{p_{sub}}, \hat{\mathbf{H}}_{p_{sub}} \in \mathbb{C}^{p\times p}$. Moreover, $\xi = \sqrt{1-\rho^2}$ is the measure of estimation error of noisy channel. Therefore, when the channel estimation error is present, under the assumption that $\xi \ll 1$, the Taylor expansion of pseudo inverse channel matrix is represented as follows (8).

$$\mathbf{H}_{p_{sub}}^{-1} = \hat{\mathbf{H}}_{p_{sub}}^{-1}\left(\rho\mathbf{I}_p + \xi\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\right), \quad (14)$$

here $\mathbf{I}_p$ is $p \times p$ identity matrix. Thus, the received signal in (6) is described by

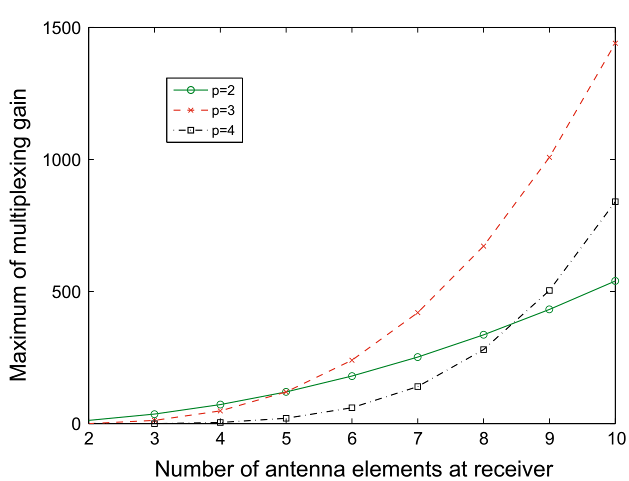


Fig. 2 The relation between maximal multiplexing gain and p, N (M is fixed as 4)

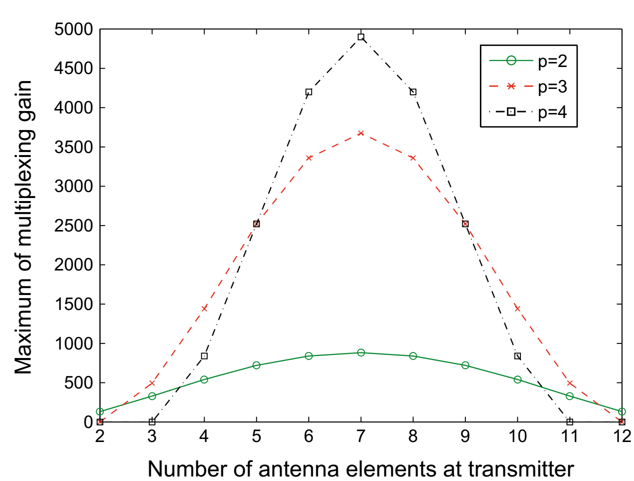


Fig. 3 The maximal multiplexing gain when total number of antenna elements is fixed as 14

$$\begin{aligned}
\mathbf{R}_{p_{sub}} &= \mathbf{P}_{p_{sub}}^{-1}\mathbf{H}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}}\mathbf{P}_{p_{sub}}\mathbf{S}_{p_{sub}}^{T} \\
&\quad + \mathbf{P}_{p_{sub}}^{-1}\mathbf{H}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}}\sum_{\bar{\mathbf{p}}_{\mathbf{sub}}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T} \\
&\quad + \mathbf{P}_{p_{sub}}^{-1}\mathbf{H}_{p_{sub}}^{-1}\mathbf{N}_{p_{sub}}, \\
&= \rho\mathbf{S}_{p_{sub}}^{T} + \xi\mathbf{P}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}}^{-1}\bar{\mathbf{H}}_{p_{sub}}\mathbf{P}_{p_{sub}}\mathbf{S}_{p_{sub}}^{T} \\
&\quad + \rho\mathbf{P}_{p_{sub}}^{-1}\sum_{\bar{\mathbf{p}}_{\mathbf{sub}}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T} \\
&\quad + \xi\mathbf{P}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}}^{-1}\bar{\mathbf{H}}_{p_{sub}}\sum_{\bar{\mathbf{p}}_{\mathbf{sub}}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T} \\
&\quad + \mathbf{P}_{p_{sub}}^{-1}\mathbf{H}_{p_{sub}}^{-1}\mathbf{N}_{p_{sub}},
\end{aligned} \tag{15}$$

here $\mathbf{S}_{p_{sub}} = [s_{p_{sub},1} \cdots s_{p_{sub},p}]$ denotes the transmit signal of p_{sub} subset. In (15), the first term is the desired signal, the second term also is the desired signal, however, since it couldn't be separated due to channel estimation errors, it is considered as interference signal. The third and the fourth terms are interference signals and the fifth term is the noise vector.

The matrices $\mathbf{\Phi}_{p_{sub},1}$ and $\mathbf{\Phi}_{p_{sub},2}$ are designed subject to

$$\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\mathbf{H}_{p_{sub}}\mathbf{P}_{p_{sub}} = \mathbf{0}. \tag{16}$$

Similar to (7), the linear control is applied.

$$\begin{aligned}
r_{ji} &\equiv q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\mathbf{Y}_{p_{sub}}(i), \\
&= q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\hat{\mathbf{H}}_{p_{sub}}\mathbf{P}_{p_{sub}}\mathbf{S}_{p_{sub}}^{T}(i) \\
&\quad + q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\hat{\mathbf{H}}_{p_{sub}}\sum_{\bar{p}_{sub}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T}(i) \\
&\quad + q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\mathbf{N}_{p_{sub}}(i),
\end{aligned} \tag{17}$$

for all $j, i = 1, \ldots, p$.

The co-channel interference cancellation condition is represented as

$$\begin{aligned}
&\mathbf{P}_{p_{sub}}^{-1}\sum_{\bar{\mathbf{p}}_{\mathbf{sub}}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T}(j) \\
&\quad + \sum_{i=1}^{p} q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\mathbf{H}_{p_{sub}}\sum_{\bar{p}_{sub}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T}(i) = 0,
\end{aligned} \tag{18}$$

for all $j = 1, \ldots, p$.

Under conditions of (16) and (18), the received signal $\hat{y}_{p_{sub},j}$ after co-channel interference cancellation processing is described as

$$\begin{aligned}
\hat{y}_{p_{sub},j} &\equiv \mathbf{R}_{p_{sub}}(j) + \sum_{i=1}^{p} r_{ji}, \\
&= \rho s_{p_{sub},j} + \xi\mathbf{P}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}}^{-1}\bar{\mathbf{H}}_{p_{sub}}\mathbf{P}_{p_{sub}}\mathbf{S}_{p_{sub}}^{T}(j) \\
&\quad - \frac{\xi}{\rho}\sum_{i=1}^{p} q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\bar{\mathbf{H}}_{p_{sub}}\mathbf{P}_{p_{sub}}\mathbf{S}_{p_{sub}}^{T}(j) \\
&\quad + \xi\mathbf{P}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}}^{-1}\bar{\mathbf{H}}_{p_{sub}}\sum_{\bar{\mathbf{p}}_{\mathbf{sub}}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T}(j) \\
&\quad - \frac{\xi^2}{\rho}\mathbf{P}_{p_{sub}}^{-1}\sum_{\bar{\mathbf{p}}_{\mathbf{sub}}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T}(j) \\
&\quad - \frac{\xi}{\rho}\sum_{i=1}^{p} q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\bar{\mathbf{H}}_{p_{sub}}\sum_{\bar{p}_{sub}}\mathbf{P}_{\bar{p}_{sub}}\mathbf{S}_{\bar{p}_{sub}}^{T}(i) \\
&\quad + \mathbf{P}_{p_{sub}}^{-1}\mathbf{H}_{p_{sub}}^{-1}\mathbf{N}_{p_{sub}}(j) + \sum_{i=1}^{p} q_{ji}\mathbf{\Phi}_{p_{sub},2}\mathbf{\Phi}_{p_{sub},1}\mathbf{N}_{p_{sub}}(i).
\end{aligned} \tag{19}$$

As shown in (19), the unavailable separation of desired signal and the incomplete cancellation of interference signals are remained. They are a reason of distortion of signal. The performance of system is estimated to deteriorate when the measure of channel estimation error, ξ, increases. However, when $\xi = 0, \rho = 1$, the result is the same as that of perfect CSI in Sect. 2.2.

3.2 Channel capacity of system

Since each received signal in all subsets can be represented as (19), the channel capacity of all streams are the same. Therefore, the channel capacity of system is described as

$$C = \sum_{j=1}^{n_{sub}}\sum_{i=1}^{p}\log_2(1+\gamma_{ji}), \tag{20}$$

here $0 < n_{sub} \leq \binom{M}{p}\binom{N}{p}$ denotes the number of subsets in $M \times N$ MIMO systems and γ_{ji} denotes the signal to interference plus noise ratio (SINR) of the received signal in ith stream of jth subset.

The receive power of all signals is assumed to equal, $e = \frac{E}{pn_{sub}}$, where E denotes the receive power when the signal is transmitted and received by single input single output (SISO) system. From (19), the SINR of received signal is calculated as follows. Notice that all matrices are normalized.

$$\gamma_{ji} = \frac{\rho^2 e}{\left(\begin{array}{c} \xi^2 e + \left(\frac{\xi}{\rho}\right)^2 e\sum_{i=1}^{p} q_{ji}^2 + \xi^2 pn_{sub}e + \frac{\xi^4}{\rho^2}pn_{sub}e \\ + \left(\frac{\xi}{\rho}\right)^2 pn_{sub}e\sum_{i=1}^{p} q_{ji}^2 + (1+\sum_{i=1}^{p} q_{ji}^2)\sigma^2 \end{array}\right)}. \tag{21}$$

Since all matrices are normalized and as described in Sect. 2.2 and (18), the power of interference terms that are included in r_1 and $r_{1i}, i = 1, \ldots, p$ are the same. Consequently, in order to cancel the interference term from r_1, $\sum_{i=1}^{p} q_{ji}^2 \approx 1$. The SINR is represented by

$$\gamma_{ji} = \frac{\rho^2 e}{\left(\frac{\xi}{\rho}\right)^2 e(\rho^2+1)(1+pn_{sub}) + \frac{\xi^4}{\rho^2}pn_{sub}e + 2\sigma^2}. \tag{22}$$

Let $\delta = \frac{E}{\sigma^2}$ denote the SNR of received signal when the signal is transmitted and received by SISO. Therefore, the γ_{ji} is rewritten as

$$\gamma_{ji} = \frac{\frac{\rho^2\delta}{pn_{sub}}}{(\frac{\xi}{\rho})^2\frac{\delta}{pn_{sub}}(\rho^2+1)(1+pn_{sub})+\frac{\xi^4}{\rho^2}\delta+2}. \quad (23)$$

The γ_{ji} is independent to j, i meaning the SINR of all streams is the same and be denoted by γ. The channel capacity is changed as

$$C = pn_{sub}\log_2(1+\gamma). \quad (24)$$

On the other hand, the SINR and the channel capacity of conventional ZF algorithm in imperfect CSI system model is represented as follows.

$$\gamma_{conv} = \frac{\rho^2\frac{\delta}{\min(M,N)}}{\xi^2\frac{\delta}{\min(M,N)}+1}, \quad (25)$$
$$C_{conv} = \min(M,N)\log_2(1+\gamma_{conv}).$$

Compare to that of JPCIC, the SINR is higher, however, the number of streams is lower if p and n_{sub} are chosen as $pn_{sub} > \min(M,N)$. The channel capacity of conventional ZF and JPCIC is compared in the next section.

4 Numerical evaluation

The number of antenna elements at transmitter and receiver is fixed, e.g. $M = 6$, $N = 4$. The result of another values of M and N can be estimated similarly. We evaluate the channel capacity of system under the variation of δ, p, n_{sub}, and ξ

4.1 Perfect CSI

In case the channel estimation error is absent meaning $\xi = 0$, $\rho = 1$, the SINR of JPCIC and conventional ZF is respectively represented as

$$\gamma = \frac{\delta}{2pn_{sub}},$$
$$\gamma_{conv} = \frac{\delta}{\min(M,N)}, \quad (26)$$

and the channel capacity is respectively calculated by (24) and (25). The channel capacity of JPCIC and conventional ZF is depicted in Figs. 4 and 5 for $p = 2$ and $p = 4$, respectively. The δ is changed from 5 to 25 dB by step 5 dB.

Channel capacities of both systems increase when the δ increases. Additionally, the channel capacity of JPCIC increases in both cases of $p = 2$ and $p = 4$ when n_{sub} increases because of increase of multiplexing gain. The channel capacity of conventional ZF is higher than that of JPCIC in low SNR region and/or when n_{sub} is small. The reason is explained as follows. It is well-known that the performance of MIMO system is deteriorated in low SNR region because of considerably small SNR of every antenna element. Since the number of streams of JPCIC is almost larger than that of conventional ZF, the performance of JPCIC is worse. Moreover, even if the number of streams of JPCIC is equal or smaller than that of conventional ZF, the SNR of every antenna element of JPCIC is still smaller due to two times addition of noise to received signal [refer to (19) and (26)]. However, the channel capacity of JPCIC rapidly increases in high SNR region, especially when p and/or n_{sub} increase. The maximal channel capacity of JPCIC system at $\delta = 25$ dB is summarized in Table. 1. The channel capacity of $p = 3$ is the largest because of having the highest multiplexing gain.

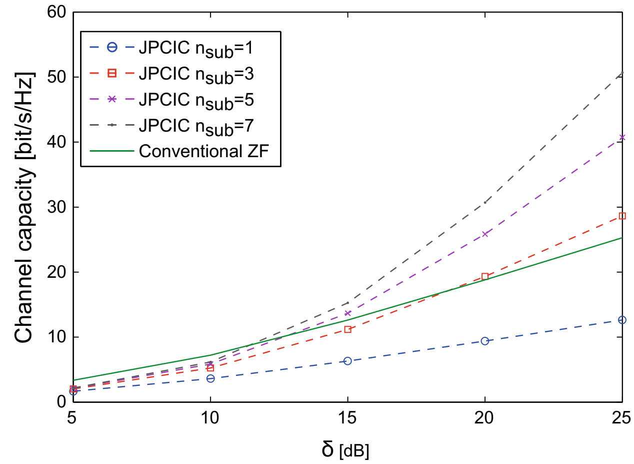


Fig. 4 The channel capacity of perfect CSI when $p = 2$

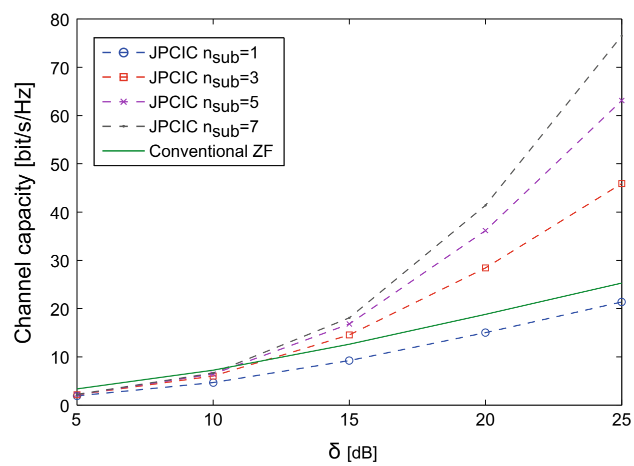


Fig. 5 The channel capacity of perfect CSI when $p = 4$

Table 1 Maximal channel capacities of JPCIC at $\delta = 25$ dB

p	Maximal n_{sub}	pn_{sub}	Maximal channel capacity (bit/s/Hz)
2	90	180	163.7
3	80	240	175.2
4	15	60	111.7

Compare to 25.2 bit/s/Hz of maximal channel capacity of conventional ZF, the maximal channel capacity of JPCIC is greatly higher. The proposal JPCIC makes the complexity increase, however it offers a method to increase the channel capacity of MIMO system.

4.2 Imperfect CSI

The dependence of channel capacity of JPCIC on p, n_{sub}, δ and ξ is depicted in Figs. 6, 7, 8 and 9. Fig. 6 shows the channel capacity of system with $p = 4$, $n_{sub} = 7$ and $\xi = 0.02, 0.1, 0.2$. Similar to system with perfect CSI, the channel capacity increases when the SNR increases. However, higher the ξ is, more tardily the channel capacity increases. The channel capacity of conventional ZF is similar.

Figures 7 and 8 depict the dependence of channel capacity on the ξ in some values of p and n_{sub}, however it can be said that the channel capacity of system which has more multiplexing gain is higher. Additionally, the channel capacity of both JPCIC and conventional ZF reduces when the ξ increases. However, the channel capacity of JPCIC reduces more rapidly and higher the channel capacity is, hastier the reduction is. It means that the conventional ZF is more robust than JPCIC when channel estimation errors are present. The reason can be

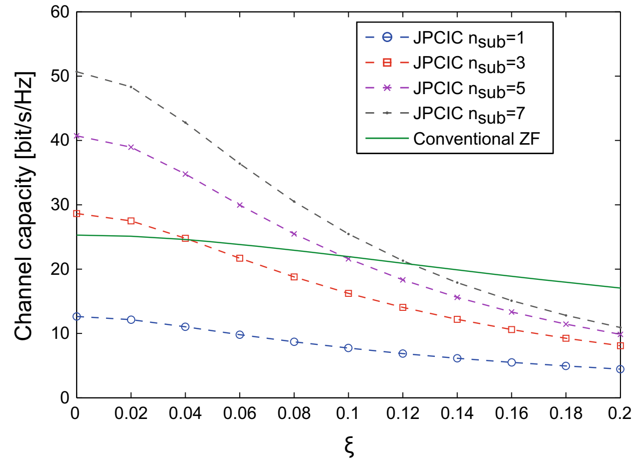


Fig. 7 The channel capacity of imperfect CSI when $p = 2, \delta = 25$ dB

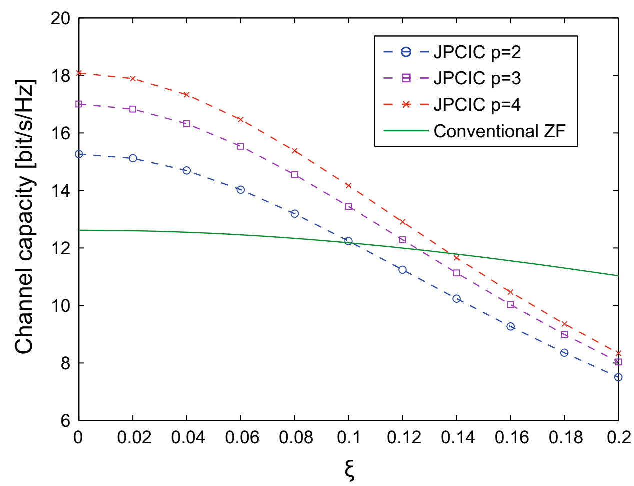


Fig. 8 The channel capacity of imperfect CSI when $n_{sub} = 7, \delta = 15$ dB

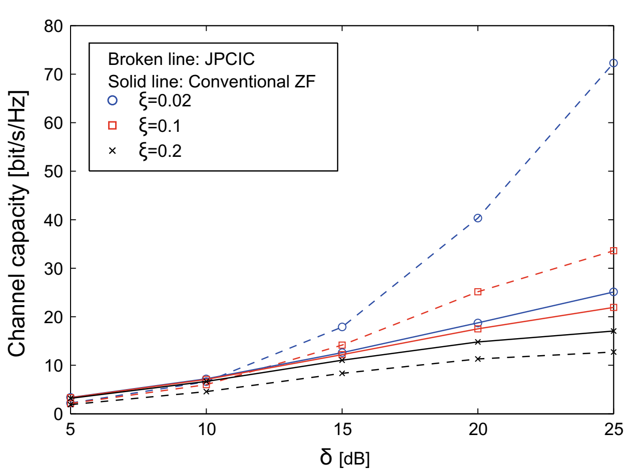


Fig. 6 The channel capacity of imperfect CSI when $p = 4, n_{sub} = 7$

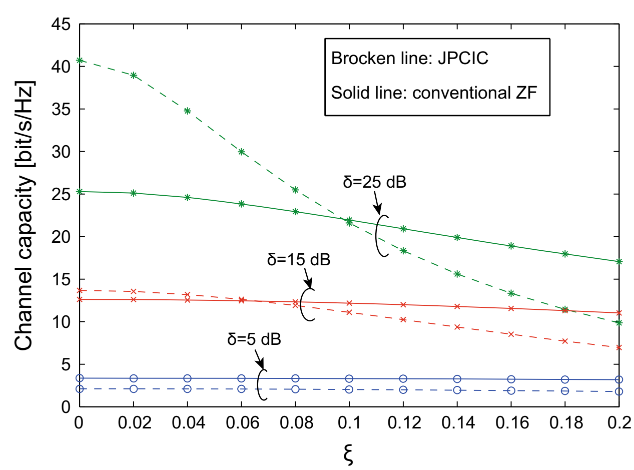


Fig. 9 The channel capacity of imperfect CSI when $p = 2, n_{sub} = 5$

explained as follows. Similar to the conventional ZF, the JPCIC is affected by unavailable separation of desired signal. Additionally, the performance of JPCIC also is deteriorated by incomplete cancellation of interference terms. As described in (21), the power of incomplete cancellation of interference terms is considerably higher than that of unavailable separation of desired signal, especially when the pn_{sub} is large. It also is the reason why the higher channel capacity of JPCIC decreases more rapidly when the ξ increases.

Fig. 9 shows the variation in channel capacity of both JPCIC and conventional ZF for some different SNRs. The channel capacity of systems whose SNR is larger is higher when the ξ is small, however, when the ξ increases, the channel capacity decreases rapidly due to high power of interference terms. Furthermore, higher the SNR is, more rapidly the channel capacity decreases. The channel capacity of JPCIC decreases more rapidly than that of conventional ZF does, the reason is that the SINR of JPCIC is not only affected by the pn_{sub}, but also considerably affected by the transmit power of every antenna element [refer to (21)] because of great increase of power of interference terms.

5 Conclusion

The JPCIC was proposed to conquer the restriction of min(M,N) on multiplexing gain of MIMO systems. The performance analysis of the JPCIC is represented in both perfect and imperfect CSI system models. According to the JPCIC, the complexity increases, however, the multiplexing gain is considerably improved. Therefore, the channel capacity of JPCIC is higher than that of conventional ZF when the channel estimation error is absent or small. However, due to unavailable separation of desired signal and incomplete cancellation of interference terms, the channel capacity of JPCIC reduces more rapidly when the channel estimation error increases, especially for high number of streams and/or high SNR. There is a trade-off between the complexity and the channel capacity, however, the JPCIC can be adopted to increase the channel capacity because of quick development in circuit techniques and miniaturization of devices.

The channel capacity of JPCIC decreases rapidly due to incomplete cancellation of interference terms when the channel estimation error increases. Therefore the channel estimation method and the JPCIC should be improved to be more robust. Moreover, the JPCIC is analyzed based on another fading channels, i.e. Rayleigh fading, Rician fading, Nakagami fading and so on in future works.

Appendix 1: Creation method of orthogonal matrices

In Sect. 2.2, the $\mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}$ is indicated to be orthogonal to $\mathbf{H}_{12}\mathbf{P}_{12}$. the design of $\mathbf{\Phi}_{12,1}$ and $\mathbf{\Phi}_{12,2}$ are represented as follows. Let

$$\begin{aligned}\mathbf{Z} &\equiv \mathbf{H}_{12}\mathbf{P}_{12}, \\ &= \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}.\end{aligned} \tag{27}$$

There are many methods to design $\mathbf{\Phi}_{12,1}$ and $\mathbf{\Phi}_{12,2}$. One of them is to create $\mathbf{\Phi}_{12,1}$ subject to

$$\begin{aligned}\mathbf{\Phi}_{12,1}(1)\begin{bmatrix} z_{11} \\ z_{21} \end{bmatrix} &= 0, \\ \mathbf{\Phi}_{12,1}(2)\begin{bmatrix} z_{11} \\ z_{21} \end{bmatrix} &= 0.\end{aligned} \tag{28}$$

Therefore,

$$\mathbf{\Phi}_{12,1}\mathbf{H}_{12}\mathbf{P}_{12} = \begin{bmatrix} 0 & \hat{z}_{12} \\ 0 & \hat{z}_{22} \end{bmatrix}. \tag{29}$$

And then, create $\mathbf{\Phi}_{12,2}$ subject to

$$\begin{aligned}\mathbf{\Phi}_{12,2}(1)\begin{bmatrix} \hat{z}_{12} \\ \hat{z}_{22} \end{bmatrix} &= 0, \\ \mathbf{\Phi}_{12,2}(2)\begin{bmatrix} \hat{z}_{12} \\ \hat{z}_{22} \end{bmatrix} &= 0.\end{aligned} \tag{30}$$

We can indicate an example as

$$\begin{aligned}\mathbf{\Phi}_{12,1} &\equiv \begin{bmatrix} -z_{21} & z_{11} \\ -z_{21} & z_{11} \end{bmatrix}. \\ \mathbf{\Phi}_{12,2} &\equiv \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}.\end{aligned} \tag{31}$$

As explained above, a vector with length of p can be designed to be orthogonal to $p-1$ vectors with the same length. Consequently, every row of matrix $\mathbf{\Phi}_{p_{sub},1}$ is created to be orthogonal to the first $p-1$ columns of $\mathbf{Z}$. Therefore,

$$\mathbf{\Phi}_{p_{sub},1}\mathbf{H}_{p_{sub}}\mathbf{P}_{p_{sub}} = \begin{bmatrix} 0 & 0 & \cdots & \hat{z}_{1p} \\ & \cdots & & \\ 0 & 0 & \cdots & \hat{z}_{pp} \end{bmatrix}, \tag{32}$$

and $\mathbf{\Phi}_{p_{sub},2}$ is designed to be orthogonal to the last column of $\mathbf{\Phi}_{p_{sub},1}\mathbf{H}_{p_{sub}}\mathbf{P}_{p_{sub}}$, $[\hat{z}_{1p}\,\hat{z}_{2p}\,\cdots\,\hat{z}_{pp}]^T$, as similar to (30).

Appendix 2: Calculation method of q_{11}, q_{12}

The (10) can be represented as

$$
\begin{aligned}
&\begin{bmatrix} \hat{p}_{11} & \hat{p}_{12} \\ \hat{p}_{21} & \hat{p}_{22} \end{bmatrix}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(1) \\
&\quad + q_{11}\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(1) \\
&\quad + q_{12}\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} s_3 \\ s_4 \end{bmatrix}(2) = 0,
\end{aligned} \tag{33}
$$

here $\hat{\mathbf{P}} \equiv \mathbf{P}_{12}^{-1}\mathbf{P}_{23}$. Therefore, it is changed as follows.

$$
(\hat{p}_{11} + q_{11}k_{11} + q_{12}k_{21})s_3 + (\hat{p}_{12} + q_{11}k_{12} + q_{12}k_{22})s_4 = 0. \tag{34}
$$

Since this equation is for all s_3 and s_4, we have

$$
\begin{aligned}
&\hat{p}_{11} + q_{11}k_{11} + q_{12}k_{21} = 0, \\
&\hat{p}_{12} + q_{11}k_{12} + q_{12}k_{22} = 0.
\end{aligned} \tag{35}
$$

As a result,

$$
\begin{aligned}
q_{11} &= \frac{\hat{p}_{12}k_{21} - \hat{p}_{11}k_{22}}{k_{11}k_{22} - k_{12}k_{21}}, \\
q_{12} &= \frac{\hat{p}_{12}k_{11} - \hat{p}_{11}k_{12}}{k_{21}k_{12} - k_{22}k_{11}},
\end{aligned} \tag{36}
$$

and matrices $\mathbf{P}_{12}$ and $\mathbf{P}_{23}$ should be designed subject to $\hat{p}_{12}k_{21} - \hat{p}_{11}k_{22} \neq 0$, $\hat{p}_{12}k_{11} - \hat{p}_{11}k_{12} \neq 0$ and $k_{21}k_{12} - k_{22}k_{11} \neq 0$. Notice that $\mathbf{K}_{23} = \mathbf{\Phi}_{12,2}\mathbf{\Phi}_{12,1}\mathbf{H}_{12}\mathbf{P}_{23}$.

Appendix 3: Taylor expansion of pseudo inverse matrix

We are going to prove that

$$
\left(\rho\hat{\mathbf{H}}_{p_{sub}} + \xi\bar{\mathbf{H}}_{p_{sub}}\right)^{-1} = \hat{\mathbf{H}}_{p_{sub}}^{-1}\left(\rho\mathbf{I}_p + \xi\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\right). \tag{37}
$$

We have

$$
\begin{aligned}
&\left(\rho\hat{\mathbf{H}}_{p_{sub}} + \xi\bar{\mathbf{H}}_{p_{sub}}\right)^{-1} \\
&\quad = \left(\rho\hat{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}} + \xi\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\hat{\mathbf{H}}_{p_{sub}}\right)^{-1}, \\
&\quad = \left(\left(\rho\mathbf{I}_p + \xi\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\right)\hat{H}_{p_{sub}}\right)^{-1}, \\
&\quad = \rho\hat{\mathbf{H}}_{p_{sub}}^{-1}\left(\mathbf{I}_p + \frac{\xi}{\rho}\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\right)^{-1},
\end{aligned} \tag{38}
$$

and the Taylor expansion is applied to.

$$
\begin{aligned}
&\left(\mathbf{I}_p + \frac{\xi}{\rho}\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\right)^{-1} \\
&\quad = \mathbf{I}_p - \frac{\xi}{\rho}\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1} + \left(\frac{\xi}{\rho}\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}\right)^2 - \cdots, \\
&\quad \approx \mathbf{I}_p - \frac{\xi}{\rho}\bar{\mathbf{H}}_{p_{sub}}\hat{\mathbf{H}}_{p_{sub}}^{-1}.
\end{aligned} \tag{39}
$$

From (38) and (39), the (37) is obtained.

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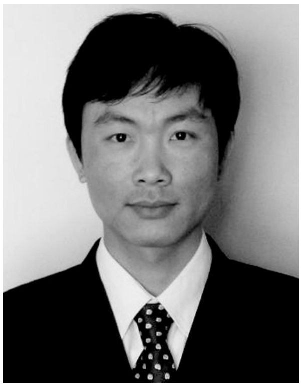

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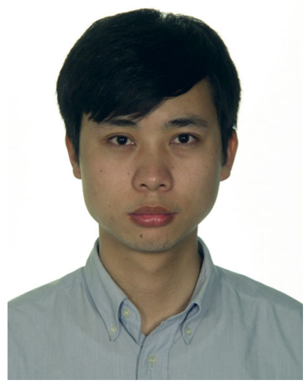

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