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New algorithm to minimise kinematic tool path errors around 5-axis machining singular points

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The singular points of a given 5-axis CNC machine could be found in the domain of the joint variables of the machine. In the neighbourhood of a singular point, even for a small change of the tooltip position, an enormous change of axis displacements of the machine is often required. This causes a large deviation between the real cutting path and the desired tool path, and the machining surface could be destroyed. This paper provides with an analytical scheme for identifying singular configuration of 5-axis CNC machines. In particular, an efficient and robust algorithm is proposed to compute the cutter path across the neighbourhood of the singular points identified such that the computed cutter path tracks the desired tool path within a controllable error. Numerical examples and real cutting parts are carried out and discussed to show the effectiveness and the efficiency of the presented method.

Keywords: 5-axis CNC machining; tool path planning; singularity in CNC machining

1. Introduction

The main function of any CAM programme is to compute the cutter trajectory (tool path) in the work space, basing the input on the part surface modelling, the surface quality required (machining error), the cutter definition, the tool path pattern, etc. The tool path is defined as a piecewise curve passing through CL points (cutter location points) that represents in standardised ASCII format $(x_i, y_i, z_i, i_i, j_i, k_i)$ where (x_i, y_i, z_i) are the coordinates of the tool tip, and (i_i, j_i, k_i) the direction cosines of the tool axis orientation correspondingly.

According to the numerical controller (NC) unit integrated in the machine, there exist two main possibilities offered to the user to describe the tool trajectory, which is transmitted to the NC unit (Tournier et al. 2006). The first solution is to implement the inverse kinematic transformation (IKT) in real time by the NC unit and the tool trajectory expressed in the reference frame is directly sent. In this case, the postprocessor is used to translate the programme generated by the CAM software into APT language (ISO3592) or into G-code (ISO6983) language. The main advantage of this approach is to keep the consistency within the numerical chain since the programme would be the same whatever the machine tool structure used. Another solution is to use a dedicated postprocessor the role of which is to compute the IKT. Hence, displacement orders on each axis are calculated in the articular space and are transmitted to the NC unit. This approach is more difficult to manage since we have to control the effective relative movements between the tool and the part and each axis velocity. In essence, the main goal of the postprocessor is to transform the CL data $(x_i, y_i, z_i, i_i, j_i, k_i)$ into the set of 5-axis displacements as (X, Y, X, A, B) or (X, Y, Z, A, C) or (X, Y, Z, B, C) (Figure 1). The notation of the rotation axes are either (A, B) or (B, C) or (A, C) that depend on the structure of specific machine.

The inverse kinematics transformation and the postprocessor construction for different types of 5-axis CNC machines have been considered in a number of researches (Bohez, Makhanov, and Sonthipermpoon 2000; Bohez 2002; She 2002; She and Huang 2008; My 2010), but a few of them deals with the singularity.

The reason for the singularity is that the exact solution of the inverse kinematics equation does not exist at singular configurations, since the forward kinematics equation of 5-axis CNC machine is nonlinear. In this situation, the axis displacements of the machine cannot be computed exactly according to given CL points. Therefore, the cutter could move along the tool path incorrectly. It is one of the serious machining problems that may destroy the machining surface.

Figure 2 shows a simple example of a machined surface on which the zigzag tool path and the corresponding CL data are generated in ProEngineer CAM software. Notice that a ball nose end cutter without tool axis inclination is used

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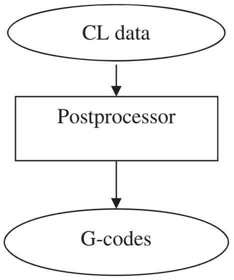

Figure 1. Postprocessor for 5-axis CNC machine.

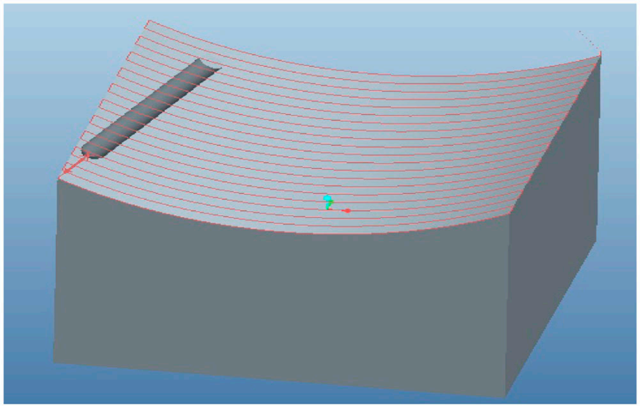

Figure 2. Machined part and its tool path generated by CAM.

for calculating the tool path. Figure 3 shows the simulation of the real cutting path calculated with the 5-axis CNC machine DMU 50e.

As shown in Figure 3, when the cutter cuts across the middle points (singular point), very big errors (the unwanted cutting passes) left on the machining surface.

In the literature, the mentioned problem has been considered in some researches (Jung et al. 2002; Affouard et al. 2004; Munlin, Makhanov, and Bohez ; Tournier et al. 2006; Sørby 2007) but none of them considers the mathematical formulation of singularity that could be meaningful to find out singular configuration analytically, even helpful for a new machine design. Moreover, they almost use techniques of geometrical approximation and subdivision for regenerating the tool path near the singular points. The method presented in (Jung et al. 2002) uses the tool retracts to avoid the

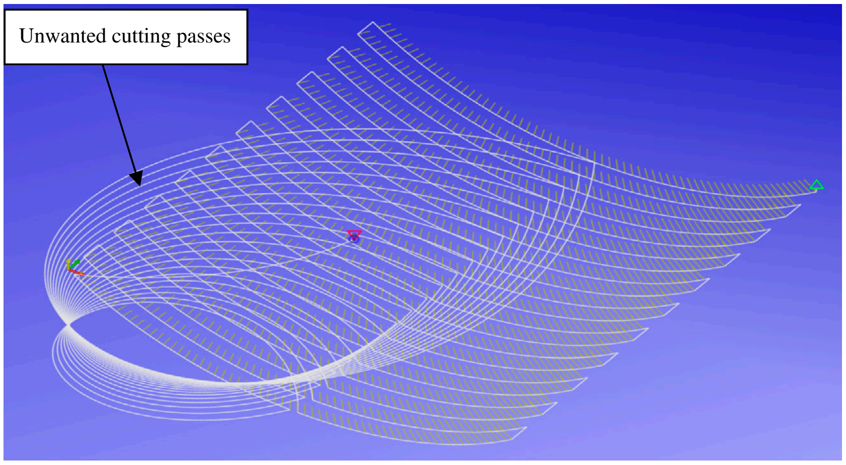

Figure 3. Simulation of real cutting path.

points near the singular configuration. This modification causes interrupts in the cutting path, and it is therefore undesirable in simultaneous 5-axis machining. The singularity is also discussed by Sørby (2007) who develops a postprocessor for the non-orthogonal rotary axis machines. Sørby (2007) revealed that the problem is related to the solution of the inverse kinematics and the positioning of the machine axes near a kinematic singularity. This is for the offline postprocessing as a part of NC programme preparation. Based on the technique of geometrical approximation, an algorithm is proposed for computing the cutter trajectory across the singular points. To illustrate the singularity intuitively, a simple machining surface is taken into account and machined on MDU 50e CNC machine. This example is just for demonstrating the case; however, it could be also handled by other commercial CAD/CAM software. Additionally, Munlin, Makhanov, and Bohez consider a method for reducing the machining error near singularities, by optimising the sequence of the rotation axis. They explain in detail that the existent of the problem even in the case of the tool axis slightly inclined to avoid the undercuts and decrease the machining time when machining concave surface of small curvature. Affouard et al. (2004) insist that the singularity problem could be occurred in high-speed machining of complex part surface with large curvature radii or flank milling for realisation of ruled surface. Therefore, they propose a method to deform the tool path so that the tool does not traverse the singular cone during machining.

In particular, Tournier et al. (2006) cope with the singularity while machining the leading edge of a hydraulic blade with the best quality possible in order to avoid manual polishing. Considering two types of specific 5-axis CNC machines and analysing possible machining strategies in which the tool axis orientation is changed, they clarify that it is an interesting issue which can be found not only in the machining of blade surfaces but also in the machining of any type of form presenting strong evolutionary curvature areas.

Though the mentioned methods are capable of solving the problem, none of them considers the mathematical modelling of the generalised singularity of 5-axis CNC machine. Almost solutions are based on the techniques of geometrical approximation and subdivision for regenerating the tool path near the singular points. In this context, further researches could be proposed to focus on the generalised modelling of the problem and the optimal solution to the problem.

Extending the researches presented, this paper considers the same problem but utilises a different approach to model and solve the problem. By means of the differential motion concept, the singularity of 5-axis CNC machines is treated in a general and efficient way. As a result, the singular points are analytically identified, and a robust algorithm is designated for computing the cutter trajectory across the points such that the error between the cutter path computed and the desired tool path is controlled within a prescribed limitation. In our approach, the analytical model, the method to find the feasible solution, and the control of the tool path error are addressed and discussed. For the purpose of illustrating the singularity and the analysis in an intuitive way, we consider a simple concave surface machined by a ball nose end cutter with tool axis oriented along with the normal vector of the surface.

2. Kinematic singularity of 5-axis CNC machine

Consider a general kinematic chain of 5-axis CNC machine shown in Figure 4. We denote $\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5]^T \in Q^5 \subset R^5$ as an axis (joint) variable vector in the joint space, Q^5, where (q_1, q_2, q_3) are the notations of three translational joint variables, and (q_4, q_5) are the notations of the rotational joint variables. We also denote $\mathbf{r} = [x \quad y \quad z \quad i \quad j]^T \in W_p^5 \subset R^5$ representing a CL point computed by CAMs in the work space, W_p^5.

The forward kinematics equation of 5-axis CNC machines can be written as

$$\mathbf{r} = \mathbf{f}(\mathbf{q}), \tag{1}$$

where $\mathbf{f}$ is a nonlinear mapping: $\mathbf{f} : Q^5 \to W_p^5$.

We also denote $\mathbf{J}(\mathbf{q}) = \frac{\partial \mathbf{f}}{\partial \mathbf{q}} \in R^{5\times 5}$ as the Jacobian matrix with respect to Equation (1).

Differentiating Equation (1) yields

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \text{ or} \\ \mathrm{d}\mathbf{r} &= \mathbf{J}(\mathbf{q})\mathrm{d}\mathbf{q} \end{aligned}. \tag{2}$$

With a small change of $\mathbf{r}$ and $\mathbf{q}$, Equation (2) can be rewritten in the following form.

$$\delta\mathbf{r} = \mathbf{J}(\mathbf{q})\delta\mathbf{q} \tag{3}$$

To implement the inverse kinematics transformation, we can solve either Equation (1) for $\mathbf{q}$ or Equation (3) for $\delta\mathbf{q}$. Given the change of the tool position on the tool path (CL point) $\delta\mathbf{r}$, the axis displacements of the machine $\delta\mathbf{q}$ can be computed as

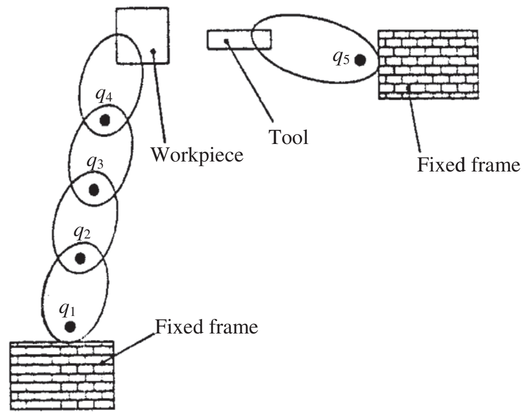

Figure 4. Kinematic chain of 5-axis CNC machine.

$$\delta\mathbf{q} = \mathbf{J}^{-1}(\mathbf{q})\delta\mathbf{r}. \tag{4}$$

For both cases of either the nonlinear function of Equation (1) or the linear function of Equation (3), the condition for existence of the inverse function is

$$\mathrm{Det}\,\mathbf{J}(\mathbf{q}) \neq 0$$

Therefore, for a given 5-axis CNC machine, if $\mathrm{Det}\,\mathbf{J}(\mathbf{q}) = 0$, Equation (4) degenerates, $\delta\mathbf{q}$ cannot be analytically calculated for a given $\delta\mathbf{r}$, and it is the so-called kinematical singular configuration of the machine.

Notice that if the forward kinematics equation of a 5-axis CNC machine is given, the Jacobian matrix corresponding to the equation is then derived, and the singular points in the joint space $\mathbf{q}_s$ can be determined by solving the following equation:

$$\mathrm{Det}\,\mathbf{J}(\mathbf{q}) = 0 \tag{5}$$

Also, the corresponding singular points in the work space (the singular CL points) can be computed via the forward kinematic relationship.

$$\mathbf{r}_s = \mathbf{f}(\mathbf{q}_s) \tag{6}$$

Example 2.1. Consider a typical orthogonal 5-axis configuration, MAHO 600e. The forward kinematics equation is described as follows (Bohez, Makhanov, and Sonthipermpoon 2000).

$$\mathbf{r} = \mathbf{f}(\mathbf{q}) = \begin{bmatrix} q_1 Sq_5 Cq_4 + q_2 Sq_4 + (Z_{0f} + q_3) Cq_4 Cq_5 + X_{01} \\ q_1 Sq_5 Sq_4 - q_2 Cq_4 + (Z_{0f} + q_3) Sq_4 Cq_5 + Y_{01} \\ q_1 Cq_5 - (Z_{0f} + q_3) Sq_5 + Z_{03} + Z_{01} \\ Cq_4 Cq_5 \\ Sq_4 Cq_5 \end{bmatrix} \tag{7}$$

Notice that $C*$ and $S*$ are denoted for cosine and sine functions; X_{01}, Y_{01}, Z_{01} and Z_{0f} are the machining set-up constants.

In this case, Equation (5) can be rewritten in the following formulation:

$$\mathrm{Det}\,\mathbf{J}(\mathbf{q}) = Sq_5 Cq_5 = 0 \tag{8}$$

Solving Equation (8) yields $\mathbf{q}_s = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & -\frac{\pi}{2} \end{bmatrix}^T$.

The corresponding singular CL points are computed as

$$\mathbf{r}_s = \begin{bmatrix} -q_1 Cq_4 + q_2 Sq_4 + X_{01} \\ -q_1 Sq_4 - q_2 Cq_4 + Y_{01} \\ Z_{0f} + q_3 + Z_{03} + Z_{01} \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

Therefore, for the machining system under consideration, if CL points in the CL point list computed by CAM having its value is presented as CL $= [x \quad y \quad z \quad 0 \quad 0 \quad 1]^T$, it is a singular CL point.

Example 2.2. As for the typical non-orthogonal 5-axis CNC machine, Deckel MAHO DMU 50e, we can use the same procedure for finding out the singularity. The forward kinematics equation is written as follows (Sørby 2007).

$$\mathbf{r} = \mathbf{f}(\mathbf{q}) = \begin{bmatrix} q_1 Cq_5\left[\Delta_z^2(1-Cq_4)+Cq_4\right] + q_1 Sq_4\left[\Delta_y\Delta_x(1-Cq_4)-\Delta_z Sq_4\right] + q_2 Cq_5\left[\Delta_y\Delta_x(1-Cq_4)+\Delta_z Sq_4\right] \\ +q_2 Sq_4\left[\Delta_y^2(1-Cq_4)-Sq_4\right] + q_3 Cq_5\left[\Delta_z\Delta_x(1-Cq_4)+\Delta_y Sq_4\right] + q_3 Sq_5[\Delta_z\Delta_x(1-Cq_4)-\Delta_x Sq_4] + G_x \\ -q_1 Sq_5\left[\Delta_x^2(1-Cq_4)+Cq_4\right] + q_1 Cq_4\left[\Delta_y\Delta_x(1-Cq_4)-\Delta_z Sq_4\right] - q_2 Sq_5\left[\Delta_y\Delta_x(1-Cq_4)+\Delta_z Sq_4\right] \\ +q_2 Cq_4\left[\Delta_y^2(1-Cq_4)-Sq_4\right] - q_3 Sq_5\left[\Delta_z\Delta_x(1-Cq_4)+\Delta_y Sq_4\right] + q_3 Cq_5[\Delta_z\Delta_x(1-Cq_4)-\Delta_x Sq_4] + G_y \\ q_1\left[\Delta_z\Delta_x(1-Cq_4)+\Delta_y Sq_4\right] + q_3\left[\Delta_z\Delta_y(1-Cq_4)-\Delta_x Sq_4\right] + q_3\left[\Delta_z^2(1-Cq_4)+Cq_4\right] + G_z \\ Cq_5\left[\Delta_x\Delta_z(1-Cq_4)-\Delta_y Sq_4\right] + Sq_5\left[\Delta_y\Delta_z(1-Cq_4)-\Delta_x Sq_4\right] \\ -Sq_5\left[\Delta_x\Delta_z(1-Cq_4)-\Delta_y Sq_4\right] + Cq_5\left[\Delta_y\Delta_z(1-Cq_4)-\Delta_x Sq_4\right] \end{bmatrix} \tag{10}$$

In Equation (6), G_x, G_y, G_z, Δ_x, Δ_y, Δ_z are constants of the machine and the workpiece.

Equation (5) becomes

$$\operatorname{Det}\mathbf{J}(\mathbf{q}) = \frac{1}{4} Sq_4(1+Cq_4) = 0. \tag{11}$$

Solving Equation (11), the singular point is found out as $\mathbf{q}_s = [q_1 \quad q_2 \quad q_3 \quad 0 \quad q_5]^T$. Therefore, all the CL points having its value as CL $= [x \quad y \quad z \quad 0 \quad 0 \quad 1]^T$ are the singular CL points.

3. Inverse kinematics at singularity

As discussed before, at singularities, the analytical solution of Equation (4) does not exist since $\mathbf{J}(\mathbf{q})^{-1}$ is not defined. The axis displacements of the machine, $\mathbf{q}$, cannot be computed according to the given singular CL points. In order to overcome this difficulty, the following approximation method is proposed to find out a feasible solution for Equation (4) numerically.

Suppose that, in the neighbourhood of a singularity, the singular CL point and its neighbours are chosen as control points to parameterise a continuous curve $\mathbf{r}_d(t)$; this curve is called the desired path of the cutter when cutting across the singular point. It is also assumed that the derivative $\dot{\mathbf{r}}_d(t)$ is defined in the same domain of the parameter t.

It is noticeable that if the derivative of the joint variables vector $\dot{\mathbf{q}}(t)$ is computed, the axis displacements $\mathbf{q}(t)$ can be determined by integrating $\dot{\mathbf{q}}(t)$ numerically. Therefore, in the case of singularity, if the $\dot{\mathbf{q}}(t)$ is determined more conveniently than $\mathbf{q}(t)$, the inverse kinematics could consider $\dot{\mathbf{q}}(t)$ as unknown, instead of $\mathbf{q}(t)$. To control the machining error, the calculation of $\dot{\mathbf{q}}(t)$ must be taken in such a way that the cutter track the desired tool path, $\mathbf{r}_d(t)$, and the error $\mathbf{e} = \mathbf{r}_d(t) - \mathbf{f}(\mathbf{q}(t)) \to 0$.

Taking a look back to Equation (2); it is a set of five linear algebraic equations with five unknowns, the components of vector $\dot{\mathbf{q}}$. In order to find out a feasible solution for $\dot{\mathbf{q}}$ in the case that $\operatorname{Det}\mathbf{J}(\mathbf{q}) = 0$, we consider a cost function as

$$\Im(\dot{\mathbf{q}}, \lambda) = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{W}\dot{\mathbf{q}} + \boldsymbol{\lambda}^T[\dot{\mathbf{r}}_d - \mathbf{J}\dot{\mathbf{q}}], \tag{12}$$

where $\mathbf{W}$ is a symmetric positive definite weighting matrix prescribed, and $\boldsymbol{\lambda}$ is the vector of Lagrangian multipliers.

Differentiating Equation (12) yields

$$\frac{\partial}{\partial\dot{\mathbf{q}}}\Im(\dot{\mathbf{q}}, \boldsymbol{\lambda}) = \mathbf{W}\dot{\mathbf{q}} - \mathbf{J}^T\boldsymbol{\lambda}. \tag{13}$$

The value of the cost function minimises if $\mathbf{W}\dot{\mathbf{q}} - \mathbf{J}^T\boldsymbol{\lambda} = 0$. Thus, we can write

$$\dot{\mathbf{q}} = \mathbf{W}^{-1}\mathbf{J}^T\boldsymbol{\lambda}. \tag{14}$$

Substituting (14) into (2) yields

$$\dot{\mathbf{r}}_d = \mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T\boldsymbol{\lambda}. \tag{15}$$

Solving for $\boldsymbol{\lambda}$ yields

$$\boldsymbol{\lambda} = \left[\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T\right]^{-1}\dot{\mathbf{r}}_d \tag{16}$$

Substituting (16) into (14), the optimal solution of $\dot{\mathbf{q}}$ can be obtained as

$$\begin{aligned}\dot{\mathbf{q}} &= \mathbf{W}^{-1}\mathbf{J}^T\left[\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T\right]^{-1}\underline{\mathbf{r}}_d \\ &= \mathbf{J}^{+}\dot{\mathbf{r}}_d\end{aligned} \tag{17}$$

The matrix $\mathbf{J}^{+} = \mathbf{W}^{-1}\mathbf{J}^T\left[\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T\right]^{-1}$ is called the weighting pseudo inverse matrix of $\mathbf{J}$.

Integrating $\dot{\mathbf{q}}$ obtains

$$\mathbf{q}(t) = \mathbf{q}(0) + \int_0^t \dot{\mathbf{q}}(t)\mathrm{d}t \tag{18}$$

The value of $\dot{\mathbf{q}}(t)$ calculated by Equation (17) satisfies Equation (2), but $\mathbf{q}(t)$ obtained by Equation (18) could not satisfy Equation (1) due to the accumulated error. This means that the error $\mathbf{e} = \mathbf{r}_d(t) - \mathbf{f}(\mathbf{q}(t))$ may exceed the prescribed tolerance. In order to overcome this issue, an adaptive computation law is needed to maintain that $\mathbf{e} = \mathbf{r}_d(t) - \mathbf{f}(\mathbf{q}(t)) \to 0$ for every steady state.

Remark. In the neighbourhood of a singular point of a 5-axis CNC machine, if the joint variable vector, $\mathbf{q}(t)$, is obtained by integrating its derivative, $\dot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})(\dot{\mathbf{r}}_d + \mathbf{K}\mathbf{e})$, where $\mathbf{K}$ is a diagonal positive definite matrix, the deviation between the desired tool path and the calculated cutter path $\mathbf{e} = \mathbf{r}_d(t) - \mathbf{f}(\mathbf{q}(t)) \to 0$; $\mathbf{q}(t)$ is called the feasible solution of the inverse kinematics in the vicinity of the singular point, and $\dot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})(\dot{\mathbf{r}}_d + \mathbf{K}\mathbf{e})$ is called the computation law of the cutter path.

Proof. Multiplying both sides of the computation law $\dot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})(\dot{\mathbf{r}}_d + \mathbf{K}\mathbf{e})$ by $\mathbf{J}(\mathbf{q})$ yields

$$\mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{K}\mathbf{e} + \dot{\mathbf{r}}_d$$

We can write

$$\begin{aligned}\mathbf{K}\mathbf{e} &= \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} - \dot{\mathbf{r}}_d \\ &= \dot{\mathbf{r}} - \dot{\mathbf{r}}_d \\ &= -\dot{\mathbf{e}}\end{aligned} \tag{19}$$

The following ordinary differential equation is obtained.

$$\dot{\mathbf{e}} + \mathbf{K}\mathbf{e} = 0 \tag{20}$$

The solution of the differential Equation (20) has the form as following:

$$\mathbf{e}(t) = \mathbf{e}(0)e^{-k_{ii}t}, \tag{21}$$

where k_{ii} are the diagonal components of the positive definite matrix $\mathbf{K}$.

Since $k_{ii} > 0$, $\mathbf{e}(t) \to 0$ when t is large enough. □

Remark 2. implies that if $\mathbf{q}(t)$, the axis displacements, is integrated from the differential equation $\dot{\mathbf{q}} = \mathbf{J}^{+}(\mathbf{q})(\dot{\mathbf{r}}_d + \mathbf{Ke})$, the cutter of the machine will keep track the given tool path $\mathbf{r}_d(t)$, and the error $\mathbf{e}(t)$ between the real cutting path and the desired path reaches to zero.

Based on the computation law and Equation (18), the following block diagram of the computational algorithm is constructed as follows.

In Figure 5, the algorithm diagram is built with respect to the programming structure and the toolboxes provided in the software Matlab Simulink. At any time step, t_i, $\mathbf{e}(t_i)$ is obtained by comparing the value of the point $\mathbf{r}_d(t_i)$ on the given desired tool path $\mathbf{r}_d(t)$ and the value of the point computed via the forward kinematic function $\mathbf{f}(\mathbf{q}(t_i))$. The component $(\dot{\mathbf{r}}_d(t_i) + \mathbf{Ke}(t_i))$ of computation law is determined and multiplied by $\mathbf{J}^{+}(\mathbf{q}(t_i))$. In this manner, the time derivative of the joint variable vector is obtained as $\dot{\mathbf{q}}(t_i) = \mathbf{J}^{+}(\mathbf{q}(t_i))(\dot{\mathbf{r}}_d(t_i) + \mathbf{Ke}(t_i))$. Finally, the *Integrator* toolbox is used to compute $\mathbf{q}(t_i)$ as shown in the figure.

4. Implementation and real cut part

The proposed algorithm is implemented in Matlab Simulink R2012a software. The programming model in Matlab Simulink environment is built as follows.

Notice that two *Interpreted Matlab Fcn* toolboxes are used to implement $\mathbf{J}^{+}(\mathbf{q})$ and $\mathbf{f}(\mathbf{q})$, respectively. These functions are coded in Matlab programming syntaxes. The other toolboxes used in the model are available in the toolbox library of Simulink (see Figure 6).

To show the advantage of the presented method, two machining experiments are implemented, based on the same CL data. In the first machining experiment, the singularity are not considered; in the second one, the singular error arcs are eliminated using the algorithm.

Presenting in Figure 7 is the machining surface designed in commercial ProEngineer CAD/CAM software. Used for the part designing as well as real machining, a workpiece having the dimension of 200 × 100 × 100 (mm) is prepared,

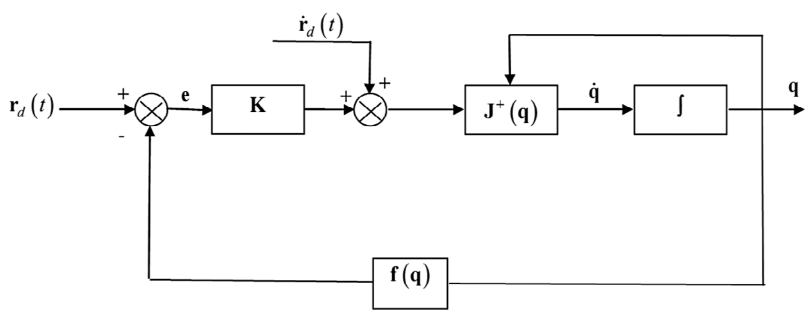

Figure 5. Algorithm diagram for inverse kinematics at singularity.

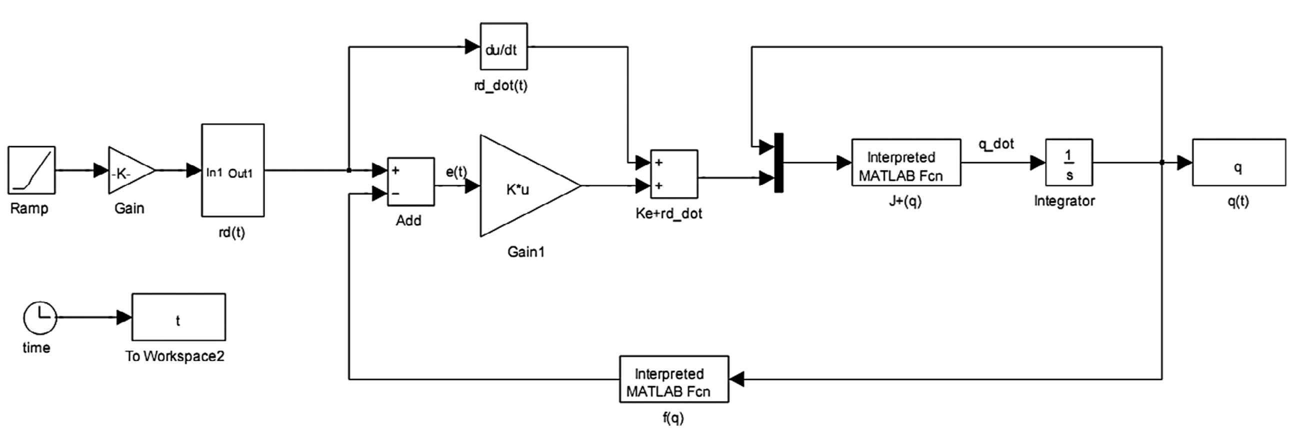

Figure 6. The programme built in Matlab Simulink for inverse kinematics at singularity.

of which the origin of the workpiece coordinate system is set at its under left corner. Deckel Maho DMU 50e 5-axis CNC machine and a ball nose end cutter R12 are selected for all the experiments.

Since the zigzag tool path pattern is applied, the tool path constitutes passes along with the length of the workpiece (Figure 7). For each pass, a singular CL point can be found in the neighbourhood of the middle point. Corresponding to the first pass, the CL point records including singular point outputted by the software are detailed as following:

```
...
GOTO / 85.2126924355, 0.0000000000, 75.6274611417,  $
0.0228461259, 0.0000000000, 0.9997389932
GOTO / 89.4573112738, 0.0000000000, 75.5587126750,  $
0.0095524399, 0.0000000000, 0.9999543744
GOTO / 91.584973112, 0.0000000000, 75.5525126712,  $
0.0029524212, 0.0000000000, 0.9999993011
GOTO / 93.7125102552, 0.0000000000, 75.5463108483,  $
-0.0037136479, 0.0000000000, 0.9999931044
GOTO / 97.9770137799, 0.0000000000, 75.5903952836,  $
-0.0169503302, 0.0000000000, 0.9998563328
...
```

Based on this CL points, the tool tip trajectory is computed using the inverse kinematics transformation as usual. Since there exists a singular point, a big unwanted error arc on the machining surface is caused that is depicted in Figure 8. For the whole tool path planning, several error arcs are existed so that in real cutting process, the cutter cuts over the surface and it seems to be destroyed. See Figure 9.

To eliminate the error, the algorithm is applied for recalculating $\{\mathbf{q}\} = \left\{ \begin{bmatrix} X & Y & Z & B & C \end{bmatrix}^T \right\}$ and generating G codes near the singular points. Figure 10 shows the cutter pass with the error arc eliminated. The corresponding error $|\mathbf{e}(t)| = \sqrt{\mathbf{e}_x^2(t) + \mathbf{e}_y^2(t) + \mathbf{e}_z^2(t)}$ is calculated and its history is shown in Figure 11. The maximum error is also determined as $|\mathbf{e}(t)|_{\max} = 0.0667\,\text{mm}$.

Figures 11 and 12 demonstrate the normal error curve $|\mathbf{e}(t)|$ for the cases that w_{ii} andk_{ii} are set with different values. As can be seen in the figures, $|\mathbf{e}(t)| \to 0$ with $k_{ii} > 0$. Furthermore, if the gains k_{ii} increase, the magnitude of the error will decrease over the time domain.

Finally, the real cut part is implemented on DMU 50e machine; the machining picture is shown in Figure 13 where the cutter cuts along smooth trajectory without any error arc.

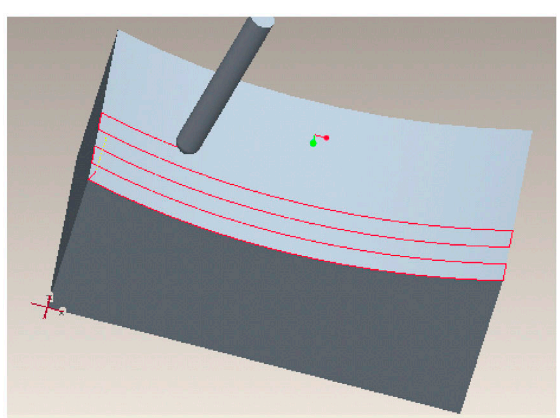

Figure 7. Machining surface and the tool path in ProEngineer software.

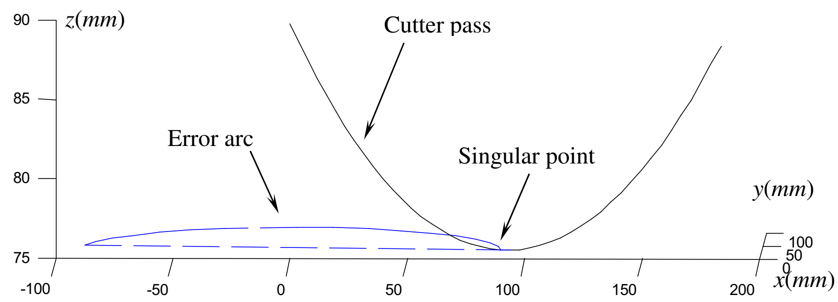

Figure 8. Machining error arc in the neighbourhood of the singular point.

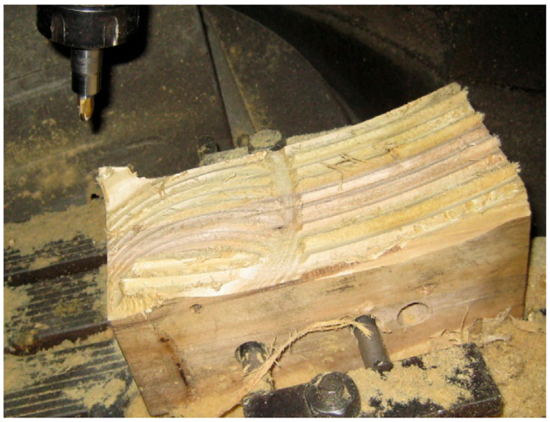

Figure 9. Real cutting passes with big errors.

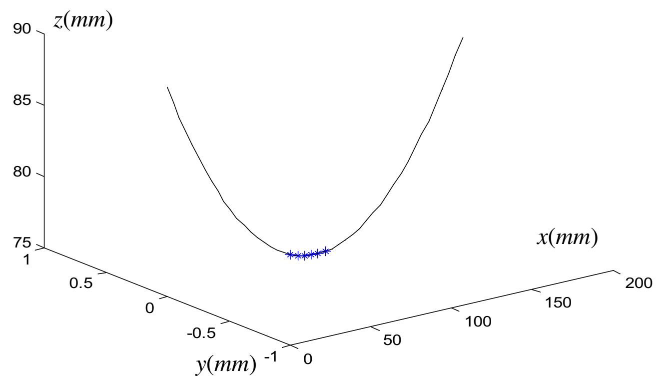

Figure 10. Cutting pass with the error arc eliminated.

Notice that, in this example of implementation, the segment of desired tool path, $\mathbf{r}(u)$, across the singular point is parameterised in the form of Bezier curve as

$$\mathbf{r}(u(t)) = \mathbf{P}_0(1-u(t))^2 + 2\mathbf{P}_1 t(1-u(t)) + \mathbf{P}_2 u^2(t),$$

where $u \in [0,1]$, $\mathbf{P}_0$, $\mathbf{P}_1$ and $\mathbf{P}_2$ are adjacent CL points in the neighbourhood of the singular CL point. The function $u = f(t)$ represents the relationship between the parameter u and time t. Depending on the velocity profile along the curve $\mathbf{r}(u)$, $f(t)$ can be determined by some interpolation procedure.

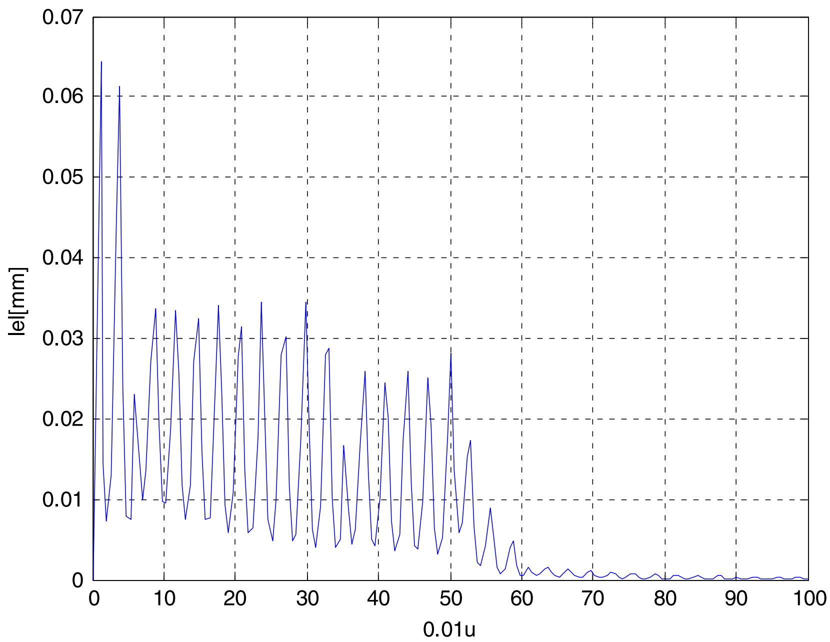

Figure 11. The curve $|\mathbf{e}(t)|$, with $\mathbf{W} = \mathrm{diag}[1 \quad 1 \quad 1 \quad 1000 \quad 1000]$ and $\mathbf{K} = \mathrm{diag}[6 \quad 6 \quad 6 \quad 6 \quad 6]$.

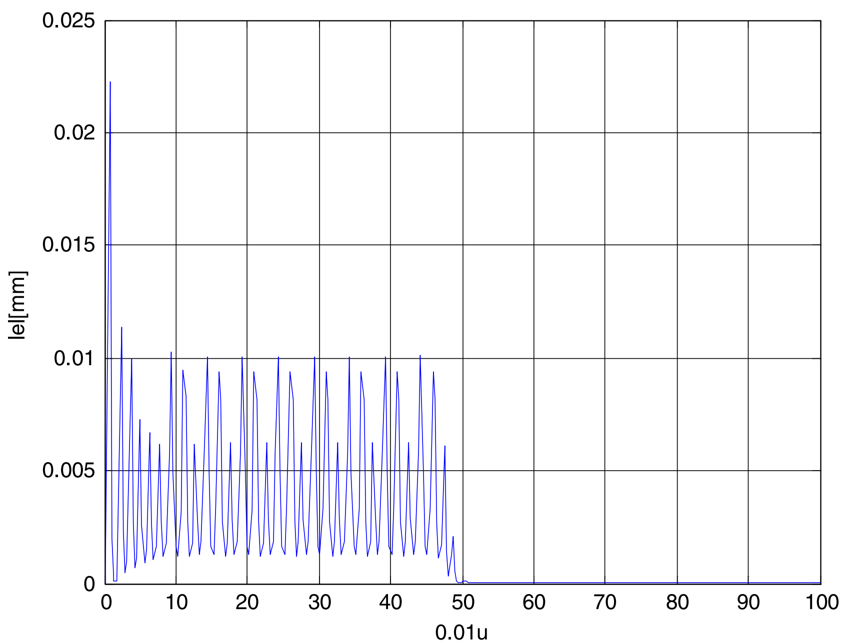

Figure 12. The curve $|\mathbf{e}(t)|$, with $\mathbf{W} = \mathrm{diag}[1 \quad 1 \quad 1 \quad 1000 \quad 1000]$ and $\mathbf{K} = \mathrm{diag}[10 \quad 10 \quad 10 \quad 20 \quad 20]$.

Without loss of the generality, $u(t)$ can be written as follows:

$$u(t) = \begin{cases} 0 & \text{if } t \le t_0 \\ \alpha t & \text{otherwise} \end{cases},$$

where t_0 is a delay time (after this time, the steady state is defined), and α is a given constant.

Figure 13. Real cut part with smooth cutting passes.

$$\mathbf{P}_0 = [\,89.4573 \quad 0.0 \quad 75.5587 \quad 0.00955 \quad 0.0\,]^T, \quad \mathbf{P}_1 = [\,91.58497 \quad 0.0 \quad 75.5525 \quad 0.00295 \quad 0.0\,]^T, \text{ and}$$

$$\mathbf{P}_2 = [\,93.7125 \quad 0.0 \quad 75.5463 \quad -0.0037136 \quad 0.0\,]^T.$$

We set $\mathbf{K} = \mathrm{diag}[\,5 \quad 5 \quad 5 \quad 5 \quad 5\,]$, and $\mathbf{W} = \mathrm{diag}[\,1 \quad 1 \quad 1 \quad 1000 \quad 1000\,]$. The simulation time is set as [0, 100 s]. We set $\alpha = 0.01$, and $t_0 = 1.5$ s.

5. Conclusions

The differential motion approach for modelling and analysing the kinematic singularity of the general 5-axis CNC machine was used to analyse and control the tool path error. The singular points are identified analytically based on the forward kinematic equations and its Jacobian matrix. This is different from the previous works in 5-axis machining research area. This would be useful for analysing the singularity of any 5-axis CNC machines, even for new design of 5-axis CNC machine. The proposed algorithm for solving the inverse kinematics at singularities shows its robustness in producing smooth cutter trajectories crossing the neighbourhood of the singular points. In this case, the error between the desired tool path and the real cutting path is reduced and evaluated which is very important factor in fabricating high-quality products.

Disclosure statement

No potential conflict of interest was reported by the authors.

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