Novel Approaches for Performance Enhancement of High Rate-Spatial Modulation System

Van-Son Trinh*, Xuan-Nghia Nguyen*‡, Minh-Tuan Le †‡, Xuan-Nam Tran §, Vu-Duc Ngo*‡
*Hanoi University of Science and Technology, Vietnam
†Ha Noi Department of Science and Technology, Viet Nam
§Le Quy Don Technical University, Viet Nam
‡Mobifone Corporation, Viet Nam
Email:{trinhvanson92,tuan.hdost}@gmail.com, nghianx@mobifone.vn, namtx@mta.edu.vn, duc.ngovu@hust.edu.vn

***Abstract*—This paper proposes an enhanced sub-optimal detector for the High Rate-Spatial Modulation (HR-SM) systems presented in [1] by incorporating the ISQRD detector previously proposed in [2] with a new search space. The proposed decoder, called Enhanced ISQRD, achieves not only significant improvement in the bit error rate (BER) performance but also complexity reduction as compared to the ISQRD detector. In addition, we propose a new bit mapping approach that allows the HR-SM system to improve its BER performance. Simulation results and complexity analysis are presented to verify the effectiveness of the proposed approaches in terms of performance improvement and detection complexity reduction when they are applied to the HR-SM systems.**

***Index Terms*—MIMO Detector, High Rate-Spatial Modulation, HR-SM detector, ISQRD detector, EISQRD detection algorithm.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) wireless communication systems that use multiple transmit and receive antennas were theoretically shown to have higher spectral efficiency than the conventional single antenna systems in [3], [4]. MIMO has been used in many wireless standards such as IEEE 802.11n (Wi-Fi), IEEE 802.11ac (Wi-Fi), HSPA+ (3G), Mobile WiMAX (4G), LTE (4G) and shown to be a potential candidate for future wireless technologies.

Spatial Modulation (SM) MIMO system, which was invented by Mesleh *et al.* in [5], [6], has recently drawn significant research interest by numerous researchers. In the SM, the transmitter activates only one antenna element at each symbol period, thereby completely eliminating inter-channel interference (ICI). Information bits are conveyed by both QAM/PSK-modulated symbols and the indices of transmit antennas. Currently, various SM-based MIMO schemes have been developed [1], [7]- [15], among which the High Rate-Spatial Modulation, proposed by Phuong Nguyen *at al.* [1], provides a considerable increase of spectrum efficiency as compared to the SM in [5], the GSM in [10]- [11], and the STBC-SM in [13]. This scheme utilized the concept of Spatial Constellation (SC) codewords introduced in [14] to generate transmit signal vectors (i.e., HR-SM codewords) by multiplying the SC codewords with QAM/PSK symbols. Consequently, the data bits are carried by both the SC codewords and the modulated symbols.

In order to detect HR-SM codewords, a ML detector was presented in [1]. The ML detector enables the HR-SM schemes to achieve optimal BER performance. However, it requires excessive computational complexity. In [2], Dong Nguyen *et al.* proposed two low-complexity detectors called MSQRD and MBLAST which are based on modifying MMSE-SQRD [16] and MMSE-BLAST [17] algorithms respectively. These detectors achieve dramatically complexity reduction, yet at cost of considerable performance degradation as compared to their ML counterpart. The MBLAST provides higher BER performance than the MSQRD because it detects the signals in the optimal order from the strongest one to the weakest one. Another detector that was also proposed in [2] is the so-called ISQRD. The ISQRD is actually the MSQRD algorithm implemented by making a full search for the first signal layer. It achieves higher performance compared to the MSQRD and MBLAST detectors for the same level of complexity as sufficiently-low modulation order was used. Nevertheless, the BER performance of ISQRD is still significantly lower than that of the ML one. Besides, the ISQRD becomes more complex than the MSQRD and MBLAST detectors as the modulation order increases.

In this paper, we first show that the ISQRD does not fully utilize special property of signal encoding for signal recovery at the receiver. We then construct a new signal search space and apply it to the ISQRD to create an enhanced detector, called Enhanced ISQRD (EISQRD). The proposed decoder is shown to be capable of improving system performance as well as offering low decoding complexity compared to the ISQRD. Besides, we propose a new approach of mapping information bits into SC codewords in order to reduce BER based on the conventional Gray mapping method. Numerical analysis and simulation results are provided to verify the performance and complexity of the EISQRD in comparison with the ISQRD as well as the performance improvement of the new mapping technique.

The rest of the paper is organized as follows. In Section II, the system model and notation is introduced. Section III recalls the ISQRD detector and presents the new detection algorithm as well as the proposed mapping technique. Complexity of the EISQRD algorithm is investigated in Section IV. The simulation results and performance comparison are given in

978-1-5090-1698-3/16/$31.00 ©2016 IEEE

section V. The last section concludes the paper.

II. System Model

We consider a specific HR-SM system with n_T transmit and n_R receive antennas $(n_R > n_T)$ in a quasi-static Rayleigh fading MIMO channel, as illustrated in Fig. 1. In the HR-SM system, $l+m$ data bits are fed into the transmitter. There are $l = 2 \times (n_T - 1)$ bits mapped into a $n_T \times 1$ SC codeword vector, $\mathbf{s} \in \Omega_s$, where Ω_s is the spatial constellation, which has K elements. The remaining m bits are modulated by using an M-QAM or M-PSK modulator to achieve signal x in signal constellation Ω_x. As proposed in [1], the SC codeword is a $n_T \times 1$ vector designed by fixing the first element with 1 and assigning the remaining elements with values selected from the set {±1;±j} according to the bits mapped onto, $\mathbf{s} = [1, s_1, ..., s_{n_T-1}]^T$,[1] where $s_i \in \{\pm 1, \pm j\}$, $i = (1 \div n_T - 1)$, $j^2 = -1$. Therefore, in this system, there are a total of $K = 4^{n_T-1}$ SC codewords belonging to the spatial constellation Ω_s. The transmit codeword (or HR-SM codeword), $\mathbf{c}$, is created by multiplying $\mathbf{s}$ by x, such as $\mathbf{c} = \mathbf{s} \times x$, so that $\mathbf{c} = [x, x \times s_1, ..., x \times s_{n_T-1}]^T = [c_1, c_2, ..., c_{n_T}]^T$ (the elements in codeword $\mathbf{c}$ are called *layers*). The process of forming $\mathbf{c}$ makes it belong to the signal constellation Ω_x[2]. These n_T codeword layers are transmitted respectively via n_T transmit antennas within a symbol period.

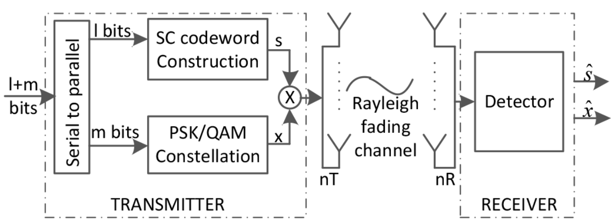

Fig. 1. Model of a HR-SM system with n_T transmit and n_R receive antennas

At the receiver, the receive signal matrix, $\mathbf{Y}$, is given as:

$$\mathbf{Y} = \sqrt{\frac{\gamma}{n_T E_s}} \mathbf{H}.\mathbf{c} + \mathbf{N} \tag{1}$$

or

$$\mathbf{Y} = \bar{\boldsymbol{H}}.\mathbf{s}.x + \mathbf{N} \tag{2}$$

where $\mathbf{H}$ is the $n_R \times n_T$ channel matrix; $\mathbf{N}$ is $n_R \times 1$ noise vector; $\bar{\boldsymbol{H}} = \sqrt{\frac{\gamma}{n_T E_s}} \mathbf{H}$ is the equivalent channel matrix; γ is the average SNR at each receive antenna; and E_s is the average symbol energy of x. The entries of $\mathbf{H}$ and $\mathbf{N}$ are assumed to be independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit variance.

The HR-SM codeword $\mathbf{c}$ will be detected at the receiver by using a detector. After that, we will receive again the modulated signal $\hat{x}$, and the SC codeword $\hat{\boldsymbol{s}}$, and from that we can decode to data bits.

[1]In this paper, $(.)^T$, $(.)^H$ denote matrix transport and Hermitian transpose, respectively.

[2]This statement is true if the M-QAM constellations are square, i.e., $M = 2^{2n}$ for some natural numbers n.

III. The Enhanced ISQRD detector

A. The ISQRD detection algorithm

The ISQRD detection algorithm is one of three detectors proposed in [2]. Each of these three detectors achieve lower complexity in comparison with the ML detector, while the ISQRD detector obtains the highest BER performance. In this system, the channel matrix $\mathbf{H}$ is assumed to be known; for example, it can be estimated from the training sequence bits. The idea of the ISQRD algorithm is as follows:

From equation (2) we get:

$$\mathbf{t} = \mathbf{Y} - \mathbf{H}_1 \times x = \mathbf{H}_t \times \bar{\boldsymbol{c}} + \mathbf{N} \tag{3}$$

where $\mathbf{H}_1$ and $\mathbf{H}_t$ is deployed from $\bar{\boldsymbol{H}}$ as $\bar{\boldsymbol{H}} = [\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_{n_T}]$; $\mathbf{H}_1$ is the first column of $\bar{\boldsymbol{H}}$; $\mathbf{H}_t = [\mathbf{H}_2, ..., \mathbf{H}_{n_T}]$ is the remaining columns of $\bar{\boldsymbol{H}}$; and $\bar{\boldsymbol{c}} = [c_2, ..., c_{n_T}]^T$. From (3), one can observe that the MSQRD algorithm can be applied to detect vector $\bar{\boldsymbol{c}}$ (see more in [2] and [16]). However, x in equation (3) is unknown at the receiver, and the solution is to perform a full search for x. With each $x \in \Omega_x$, we get a vector $\bar{\boldsymbol{c}}$, then we compute the Euclidean distance of detection vector $\mathbf{c} = [x; \bar{\boldsymbol{c}}]$ using the function $d = ||\mathbf{Y} - \bar{\boldsymbol{H}} \times \mathbf{c}||^2 = ||\mathbf{t} - \mathbf{H}_t \times \bar{\boldsymbol{c}}||^2$. Finally, we compare these distances to decide the final recovered vector $\mathbf{c}$, which has the minimum Euclidean distance. The ISQRD algorithm is summarized in Tab. I.

TABLE I
ISQRD Detection Algorithm

Input: $\mathbf{y}, \bar{\boldsymbol{H}}$
Output: $\hat{x}, \hat{\boldsymbol{s}}$

1. Decompose $\mathbf{D}_t = \begin{bmatrix} \mathbf{H}_t \\ \frac{1}{\sqrt{E_s}} \mathbf{I}_{n_T-1} \end{bmatrix}$ using MMSE-SQRD algorithm to get $\mathbf{Q}$, $\mathbf{R}$, and the permutation vector $\mathbf{p}$.
2. Detection and Cancellation:
   for m = 1 : M
      compute $\mathbf{t}_m = \mathbf{y} - \mathbf{H}_1 \times x_m$ and $\mathbf{v} = \mathbf{Q}^H \begin{bmatrix} \mathbf{t}_m \\ 0 \end{bmatrix}$
      for $k = n_T - 1 : -1 : 1$
         if $(k == n_T - 1)$
            $c_{m,k} = \arg\min_{c \in \Omega_x} ||v_k - c \times r_{k,k}||^2$
         else
            for $l = k+1 : n_T - 1$
               $v_k = v_k - r_{l,k} \times c_{m,l}$
            end
         end
      end
      Compute $d_m = ||\mathbf{t}_m - \mathbf{H}_t \times \bar{\boldsymbol{c}}_{m,\mathbf{p}}||^2$
   end
3. Find $\hat{m} : \hat{m} = \arg\min_m d_m$
4. Obtain the recovered modulated signal $\hat{x} = x_{\hat{m}}$ and the recovered SC codeword, $\hat{\boldsymbol{s}}$, from $\bar{\boldsymbol{c}}_{\hat{m},\mathbf{p}}$

B. The Enhanced ISQRD detection algorithm

The ISQRD algorithm has not yet fully exploited the rule of generating the transmit codeword $\mathbf{c}$ to detect transmitted signals. Concretely, if the first signal layer is x, the the remaining signal layers, which are the entries of $\bar{c}$, should belong to the set $\Psi_s = \{x \times s, s \in \{\pm 1; \pm j\}\}$. Therefore, when applying the MSQRD algorithm to detect the vector $\bar{\boldsymbol{c}} = [c_2, ..., c_{n_T}]^T$, instead of searching the elements of $\bar{\boldsymbol{c}}$ in the transmitted signal constellation Ω_x, we just search its elements in the set Ψ_s, which is a subset of Ω_x. This provides us with two benefits. First, the complexity of the detection algorithm is reduced. Second, the performance is improved because the search space contains more probable symbols. In other words, Ψ_s is the right set to search for the elements of vector $\bar{\boldsymbol{c}}$. If x is the transmitted symbol, then the elements of $\bar{\boldsymbol{c}}$, found in the set Ψ_s, tend to be the corrected ones. The proposed EISQRD detection algorithm is summarized in Tab. II.

TABLE II
EISQRD DETECTION ALGORITHM

Input: $\mathbf{y}$, $\bar{\boldsymbol{H}}$
Output: $\hat{x}$, $\hat{\boldsymbol{s}}$

1. Decompose $\mathbf{D}_t = \begin{bmatrix} \mathbf{H}_t \\ \frac{1}{\sqrt{E_s}}\mathbf{I}_{n_T-1} \end{bmatrix}$ using MMSE-SQRD algorithm to get $\mathbf{Q}$, $\mathbf{R}$, and the permutation vector $\mathbf{p}$.
2. Detection and Cancellation:

```
for m = 1 : M
    compute t_m = y − H_1 × x_m and v = Q^H [t_m; 0]
    for k = n_T − 1 : −1 : 1
        if (k == n_T − 1)
            c_{m,k} = arg min_{c∈Ψ_s} ||v_k − c × r_{k,k}||^2
        else
            for l = k + 1 : n_T − 1
                v_k = v_k − r_{l,k} × c_{m,l}
            end
        end
    end
    Compute d_m = ||t_m − H_t × c̄_{m,p}||^2
end
```

3. Find $\hat{m}$: $\hat{m} = \arg\min_m d_m$
4. Obtain the recovered modulated signal $\hat{x} = x_{\hat{m}}$ and the recovered SC codeword from $\bar{\boldsymbol{c}}_{\hat{m},\mathbf{p}}$, $\hat{\boldsymbol{s}} = \frac{1}{x_{\hat{m}}}\{x, \bar{\boldsymbol{c}}_{\hat{m},\mathbf{p}}\}$

C. Proposed approach of mapping information bits into SC codewords

In HR-SM system, the transmit codeword is generated as $\mathbf{c} = \mathbf{s} \times x$, where x is a modulated symbol drawn from a M-QAM or M-PSK signal constellation Ω_x; $\mathbf{s}$ is SC codeword vector, $\mathbf{s} = [1, s_1, ..., s_{n_T-1}], s_i \in \{\pm 1, \pm j\}, i = (1 \div n_T - 1)$, and $j^2 = -1$. For example, in a 4×4 HR-SM system, we can have a codeword $\mathbf{c} = [x, -x, jx, -jx]^T$, whose elements are presented in Fig. 2a. Let us consider three elements $c_1 = s_1 x$; $c_2 = s_2 x$ & $c_3 = s_3 x$ of transmitted codeword $\mathbf{c}$. It is clear that:

$$\text{If } d(c_1, c_2) > d(c_1, c_3) \quad \text{then } P(c_1 \to c_2|c_1) < P(c_1 \to c_3|c_1)$$

where $d(c_i, c_j) = (c_i - c_j)^2, (i, j = 1 \div 3)$, is the Euclidean distance of between the two signal points c_i and c_j; $P(c_i \to c_j|c_i)$ denotes the probability that the detected signal is c_j while the actual transmitted signal is c_i.

For a given x, we have $d(c_i, c_j) = (c_i - c_j)^2 = (s_i - s_j)^2 x^2$. Therefore, the above proposition is equivalent to:

$$\text{If } d(s_1, s_2) > d(s_1, s_3) \quad \text{then } P(c_1 \to c_2|c_1) < P(c_1 \to c_3|c_1) \tag{4}$$

The proposition in (4) implies that Gray mapping technique could be use to map information bits into the elements of an SC codeword as demonstrated in Fig. 2b.

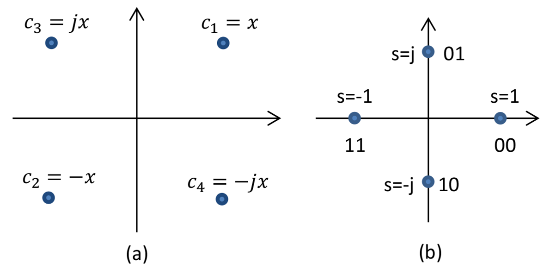

Fig. 2. The proposed bit mapping approach to generate SC codewords

IV. COMPLEXITY ANALYSIS

In this section, we investigate the complexity of EISQRD detection algorithm in comparison with ML detection (presented in [1]) and ISQRD algorithms. The complexity of the algorithms is estimated into flops unit. We assume that a real addition, a real multiplication, a real division, a real square root are considered as one flops, 2 flops, 3 flops and 4 flops, respectively. A complex addition is equal to two real additions, a complex multiplication is composed of four real multiplications and two real additions, a module calculation of a complex number is composed of two real multiplications and one real addition. A division of a complex number to a real number is equal two real divisions. The complexity of ML detector is equal to:

$$f_{ML} = MK(24n_R + 6) + n_R K(12n_T - 2) \tag{5}$$

where $K = 4^{n_T-1}$. The complexities of ISQRD and EISQRD detectors are given by:

$$f_{ISQRD} = f_{MMSE-SQRD} + f_{main-ISQRD} \tag{6}$$

$$f_{EISQRD} = f_{MMSE-SQRD} + f_{main-EISQRD} \tag{7}$$

where $f_{MMSE-SQRD}$ is complexity of the MMSE-SQRD algorithm; $f_{main-ISQRD}$ is the complexity of the main part (from step 2 to step 4 in Table I) of ISQRD detector; $f_{main-EISQRD}$ is the complexity of the main part (from step 2 to step 4 in Table II) of EISQRD detector. These component complexities are given by:

$$f_{MMSE-SQRD} = 12n_T^3 + 12n_T^2 n_R - 40n_T^2 - 18n_R n_T + 6n_R + 46n_T - 18 \tag{8}$$

$$f_{main-ISQRD} = M(17M(n_T - 1) + 6n_T^2 + 24n_R n_T - 5n_R - 20n_T + 12) \tag{9}$$

$$f_{main-EISQRD} = M(68(n_T - 1) + 6n_T^2 + 24n_Rn_T - 5n_R - 20n_T + 12) \quad (10)$$

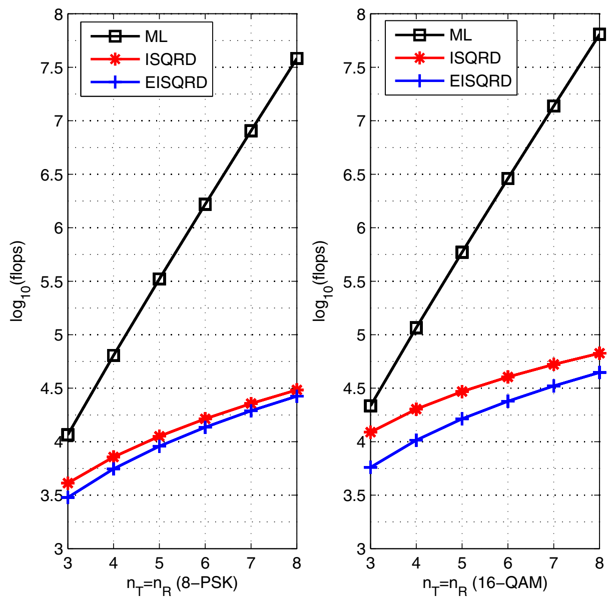

Fig. 3. Complexities of ML, ISQRD and EISQRD detectors for HR-SM systems

Fig. 3 shows the complexities of the ML, ISQRD and EISQRD detectors for HR-SM systems with $n_T = n_R$. The figure on the left is for 8-PSK modulation and the one on the right is for 16-QAM modulation. As we can see from Fig. 3, both the ISQRD and EISQRD detectors significantly reduces detection complexities in comparison with the ML one. In addition, the EISQRD has low complexities than the ISQRD one in both cases. For the same antenna configuration, the higher modulation order is, the lower complexity the EISQRD can offer.

V. Performance comparison

In this section, we investigate the performance of the EISQRD detector, in terms of BER, when it is applied to HR-SM schemes and make a comparison those of the ISQRD, MSQRD, and the ML ones. Fig. 4 shows the BER curves of the ISQRD, the EISQRD, the MSQRD and the ML detectors for a HR-SM scheme with $n_R = n_T = 4$ using 16-QAM modulation. The EISQRD has two BER curves corresponding to the cases of using and not using the proposed mapping approach. As we can see from Fig. 4, utilizing the proposed mapping approach allows the HR-SM to improve its performance. Moreover, the EISQRD detector significantly improves the BER performance of the HR-SM as compared to those of the ISQRD and the MSQRD. For instance, at $BER = 10^{-2}$ dB, the EISQRD obtains about 3 dB and 6 dB of SNR gain as compared to the ISQRD and the MSQRD respectively. The BER curve of EISQRD is relatively closed to that of the ML decoder when SNR is low. The gap between these two curves becomes more significant as SNR>18 dB.

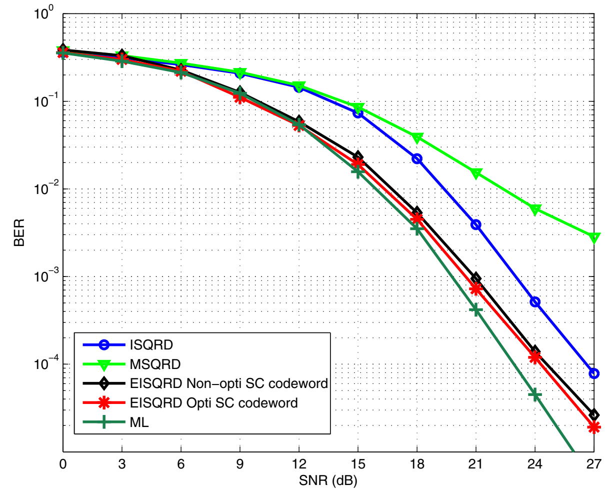

Fig. 4. BERs of different detectors for HR-SM scheme when nR=nT=4 and using 16-QAM modulation

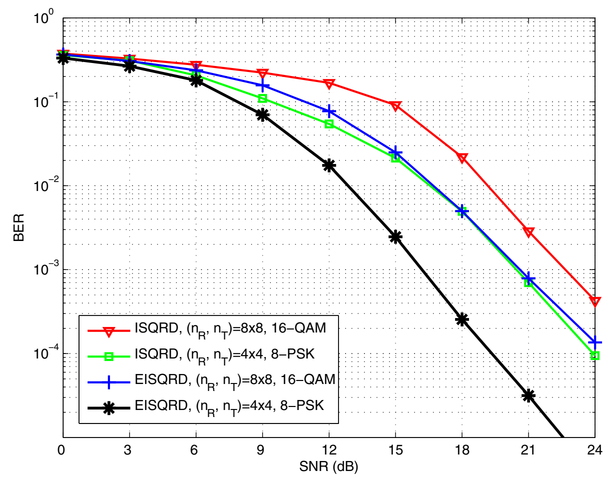

Fig. 5. BERs of ISQRD and EISQRD detectors for different HR-SM schemes

Fig. 5 further compares the performance of the EISQRD detector with that of the ISQRD detector when they are applied to HR-SM schemes with different antenna configurations and modulation schemes. The first one is a HR-SM scheme with $n_R = n_T = 4$, using 8-PSK modulation and the other is a HR-SM scheme with $n_R = n_T = 8$, using 16-QAM modulation. It can be observed from the figure that the EISQRD remarkably outperforms the ISQRD.

From the analytical and simulation results in Fig. 3, Fig. 4, and Fig. 5, it can be concluded that the EISQRD noticeably outperforms the ISQRD with respect to both performance and complexity. The higher modulation order the system uses, the more efficiency of the EISQRD when comparing to the ISQRD.

VI. Conclusion

In this paper, we present the enhanced version of ISQRD detector for signal detection in HR-SM schemes, called EISQRD detector. We also propose a more efficient bit mapping method when mapping information bits into SC codewords. The analytical and simulation results show that the EISQRD detector provides remarkable reduction of the detection complexity and considerable increase in the BER performance compared to the ISQRD detector. The proposed detector and bit mapping method make HR-SM schemes more practical and potential for future wireless communication systems.

ACKNOWLEDGEMENT

This work is sponsored by National Foundation for Science and Technology Development (Nafosted) under project number 102.02-2015.23

References

[1] Thu-Phuong Nguyen, Minh-Tuan Le, Vu-Duc Ngo, Xuan-Nam Tran, Hae-Wook Choi, "Spatial Modulation for High-Rate Transmission Systems," in *Vehicular Technology Conference (VTC Spring), IEEE*, pp. 1-5, May 2014.

[2] Tien-Dong Nguyen, Xuan-Nam Tran, Trung-Minh Do, Vu-Duc Ngo, Minh-Tuan Le, "Low-complexity detectors for High-Rate Spatial Modulation," in *ATC, IEEE*, Hanoi, pp. 652-656, Oct. 2014.

[3] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", in *Wireless Personal Communications*, vol. 6, no. 6, pp. 311-335, March 1998.

[4] E. Telatar, "Capacity of multi-antenna Gaussian channels", in *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 558-595, Nov./Dec. 1999.

[5] R. Mesleh, H. Haas, Chang Wook Ahn, Sangboh Yun, "Spatial Modulation - A New Low Complexity Spectral Efficiency Enhancing Technique," in *IEEE International Conference on Communications and Networking in China*, pp. 1-5, Oct. 2006.

[6] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," in *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228-2241, July 2008.

[7] J. Jeganathan, A. Ghrayeb, L. Szczecinski, A. Ceron, "Space shift keying modulation for MIMO channels," in *IEEE Transaction on Wireless Communications*, vol. 8, no. 7, pp. 3692-3703, July 2009.

[8] J. Jeganathan, A. Ghrayeb, L. Szczecinski, "Generalized space shift keying modulation for MIMO channels," in *PIRMC, IEEE*, pp. 1-5, Sep. 2008.

[9] Jinlin Fu, Chunping Hou, Wei Xiang, Lei Yan, Yonghong Hou, "Generalised spatial modulation with multiple active transmit antennas," in *GLOBECOM Workshops (GC Wkshps), IEEE*, pp. 839-844, Dec. 2010.

[10] A. Younis, N. Serafimovski, R. Mesleh, H. Haas, "Generalised spatial modulation," in *Proc. of IEEE ASILOMAR*, pp. 1498-1502, Nov. 2010.

[11] Jinlin Fu, Chunping Hou, Wei Xiang, Lei Yan, Yonghong Hou, "Generalised spatial modulation with multiple active transmit antennas," in *GLOBECOM Workshops (GC Wkshps), IEEE*, pp. 839-844, Dec. 2010.

[12] Jintao Wang, Shuyun Jia, Jian Song "Generalised Spatial Modulation System with Multiple Active Transmit Antennas and Low Complexity Detection Scheme," in *IEEE Transaction on Wireless Communications*, vol. 11, no. 4, pp. 1605-1615, March 2012.

[13] E. Basar, U. Aygolu, E. Panayirci, H.V. Poor, "Space-Time Block Coded Spatial Modulation," in *IEEE Transaction on Communications*, vol. 59, no. 3, pp. 823-832, March 2011.

[14] Minh-Tuan Le, Vu-Duc Ngo, Hong-Anh Mai and Xuan-Nam Tran "High-rate Space-Time Block Coded Spatial Modulation," in *ATC, IEEE*, pp. 278-282, March 2011.

[15] Minh-Tuan Le, Vu-Duc Ngo, Hong-Anh Mai, Xuan Nam Tran, and Marco Di Renzo, "Spatially Modulated Orthogonal Space-Time Block Codes with Non-Vanishing Determinants," in *IEEE Transaction on Communications*, vol. 62, no. 1, pp. 85-99, Jan. 2014.

[16] Dirk Wubben, Ronald Bohnke, Volker Kuhn, Karl-Dirk Kammeyer, "MMSE Extension of V-BLAST based on Sorted QR Decomposition," in *Proc. of IEEE VTC*, vol. 1, pp. 508512, Oct. 2003.

[17] B. Hassibi, "An Efficient Square-Root Algorithm for BLAST," in *Proc. IEEE ICASSP*, vol. 2, pp. II737-II740, June 2000.