applied optics

To enhance imaging performance of hybrid imaging systems by using two asymmetrical phase masks

VAN NHU LE,¹ SHOUQIAN CHEN,^{1,*} ZHIGANG FAN,¹ AND NGHIA MINH PHAM²

¹Research Center for Space Optics Engineering, Harbin Institute of Technology, Heilongjiang, Harbin 150001, China ²Le Quy Don Technical University, 236 Hoang Quoc Viet Street, Hanoi, Vietnam *Corresponding author: shouqian.chen@hit.edu.cn

Received 6 August 2015; revised 16 November 2015; accepted 1 December 2015; posted 5 January 2016 (Doc. ID 247505); published 9 February 2016

We propose the use of two asymmetrical phase masks combined with the subtracted imaging method to enhance the signal-to-noise ratio in wavefront coding systems. This subtracted imaging technique is similar to the variable pinhole diameter in confocal microscopy. Two different phase modulations of same phase masks are employed to promote the magnitude of the optical transfer function (OTF). The ratio factor is used to control the phase variation between two phase masks. The noise of decoded images is suppressed owing to the higher magnitude of the OTF than the wavefront coding systems with a phase mask. A tangent phase mask as an example is used to demonstrate our concept. Simulated results show that the performance promotion controls noise amplification of decoded images while maintaining a depth-of-field extension. © 2016 Optical Society of America

OCIS codes: (110.1758) Computational imaging; (110.0110) Imaging systems.

http://dx.doi.org/10.1364/AO.55.001067

1. INTRODUCTION

Wavefront coding is a powerful technology which can be used to extend depth of field in the incoherent imaging systems [1]. By placing a phase mask in the pupil plane, we can reduce impact of defocus error and the invariable point spread function (PSF) or optical transfer function (OTF) can be yielded to maintain the stale imaging properties over a wide range of defocus. The encoded images are deblurred by applying only a deconvolution kernel to achieve the sharply decoded images. Because phase modulation is introduced, the deconvolution kernel is an enlarged PSF or a lower magnitude of the OTF. As a result, noise amplifications are produced in the decoding process.

Noise amplification is an intrinsic property in the opticaldigital hybrid imaging systems. The effect of additive white noise at the detector in the wavefront coding systems has been analyzed and a noise factor is used to quantify noise amplification [2]. The spectral signal-to-noise ratio (SNR) of an imaging system with an arbitrary pupil function has been used to show the limit of depth-of-field extension [3]. According to the linear deconvolution filtering, when the noise amplification was calculated assuming uncorrelated Gaussian white noise, the noise gain in the process of the image restoration can be described by the RMS value of the simplified inverse filter as

NG =
$$\sqrt{\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left| \frac{H_{dl}(u_i, v_j)}{H(u_i, v_j)} \right|^2}$$
, (1)

where *H* is the encoded OTF of wavefront coding imaging systems; H_{dl} is in-focus OTF of diffraction-limited systems; *u* and *v* are the spatial frequencies; and *m* and *n* are size of filter. In Eq. (1), the noise gain is the inverse ratio to magnitude of the OTF. For the wavefront coding systems, the invariable OTF requires an increased phase modulation in the pupil plane but generates an excessive descent of encoded OTF from diffraction-limited systems. In other words, a lower magnitude of encoded OTF will result in noise amplification dramatically.

In order to suppress noise amplification, several papers have taken into account the minimum acceptable spectral SNR in the optimized process of phase masks [4,5]. Many asymmetrical phase masks are proposed to balance the signal intensity and defocus invariant properties [6–8]. Additionally, the iterative and nonlinear filtering methods have also been used to smooth noise of decoded images [9]. However, the encoded images are still recorded in a lower magnitude of the OTF. In this paper, we use two asymmetrical phase masks to promote the magnitude of an encoded OTF so that the noise amplification is fundamentally suppressed in the deconvolution filter. Our inspiration derives from virtual adaptable aperture confocal microscopy which adopts two size-variable pinholes to meet the requirement of spatial resolution and photon detecting [10]. Subtraction of two pinhole images with a suitably weighted coefficient is used to enhance intensity signals while maintaining resolution. Here, we use two phase-variable masks combined with the image subtraction technique to increase the SNR of decoded images. An intrinsic promotion of the OTF gives direct control in noise amplification while maintaining the depth-of-field extension. A tangent phase mask is used to demonstrate our concept, and the simulated results show the superiority of our proposed method.

2. THEORY

As two asymmetrical phase masks are used in our design, Fig. 1 gives a possible layout of the imaging system to obtain two intermediate images for two phase masks. In Fig. 1, the same two lenses combined with two different phase masks (PM1 and PM2) face the same scene, and two encoded images are recorded in two detectors. These two encoded images have a sight difference in signal collection since two phase masks introduce variable phase modulations. Two images are jointly processed to achieve the high SNR decoded image.

The defocused OTF is equal to the Fourier transform of the defocused PSF,

$$H(u, v; \psi) = FFT[PSF(x_0, y_0; \psi)],$$
(2)

where x_0 and y_0 are the spatial coordinates in the image plane, FFT denotes the fast Fourier transform, and ψ is the defocus parameter whose expression is

$$\psi = \frac{2\pi}{\lambda} W_{20},$$
 (3)

where W_{20} is the defocus coefficient associated with the wave aberrations, and λ is the light wavelength.

The defocused PSF for an incoherent imaging system can be expressed as

$$PSF(x_0, y_0; \psi) = |FFT[P(x, y)]|^2$$
, (4)

where x and y are the pupil coordinates, and P(x, y) is the pupil function. For a wavefront coding system in the presence of defocus, the normalized pupil function can be expressed as



Fig. 1. Layout for recording the images using two asymmetrical phase masks. PM1 and PM2 denote two phase masks, respectively.

$$P(x,y) = \begin{cases} \frac{1}{\sqrt{2}} \exp[if(x,y) + i\psi(x^2 + y^2)], & \text{if } |x| \le 1, |y| \le 1\\ 0 & \text{other} \end{cases},$$
(5)

where f(x, y) is the phase function of the phase mask. Thus, the intermediate image for wavefront coding systems in the presence of defocus can be described by

$$g(x_0, y_0; \psi) = \text{PSF}(x_0, y_0; \psi) * o(x_0, y_0) + n(x_0, y_0),$$
 (6)

where *o* is the object, * denotes the convolution operation, and *n* is the noise. We assume that the intermediate images of two phase masks are g_1 and g_2 . Our operation of two intermediate images can be described as

$$g_{\text{subtraction}}(x_0, y_0; \psi) = g_1(x_0, y_0; \psi) - \gamma g_2(x_0, y_0; \psi)$$

= [PSF₁(x₀, y₀; \u03c6) - \u03c6 PSF₂(x₀, y₀; \u03c6)]
* o(x_0, y_0) + (1 - \u03c6) n(x_0, y_0), (7)

where $g_{\text{subtraction}}$ is the subtraction of two intermediate images, and γ is the ratio factor ($\gamma > 0$). Comparing Eq. (6) with Eq. (7), the PSF of the proposed subtraction method can be expressed as

$$PSF_{Subtraction}(x_0, y_0; \psi) = PSF_1(x_0, y_0; \psi) - \gamma PSF_2(x_0, y_0; \psi).$$
(8)

Hence, the defocused OTF of the final subtractive image can be given by

$$H_{\text{Subtraction}}(u, v; \psi) = H_1(u, v; \psi) - \gamma H_2(u, v; \psi), \quad (9)$$

where H_1 and H_2 are the OTFs of two phase masks. This subtractive method will strengthen the magnitude of the high spatial frequency of the OTF, thereby amplifying the signals of the encoded images. As a result, noise amplification will be suppressed and the SNR will be promoted. According to the subtractive OTF, the final subtractive encoded image in the spatial frequency domain can be described by

$$G_{\text{subtraction}}(u, v; \psi) = H_{\text{Subtraction}}(u, v; \psi)O(u, v) + (1 - \gamma)N(u, v),$$
(10)

where O, N, and $G_{\text{subtraction}}$ are the Fourier transform of object o, noise n, and $g_{\text{Subtraction}}$, respectively. Based on the defocus invariant characteristic of the wavefront coding imaging system, the final subtractive OTF is also less sensitive to defocus. Therefore, we can use the final subtractive OTF to restore the final subtractive encoded image so that the SNR is promoted while extending the depth of field. The inverse filter as simple linear filtering executes a direct display of noise amplification. Thus, here we adopt the inverse filter in the deconvolution process. The inverse filter derived from the subtractive encoded OTF can be described by

$$F(u, v) = \frac{H_{\rm dl}(u, v)}{H_{\rm Subtraction}(u, v; \psi = 0)}.$$
 (11)

Hence, the final decoded image can be expressed as

$$I(x_0, y_0; \psi) = \mathrm{i} \mathrm{FFT}^{-1}[G_{\mathrm{subtraction}}(u, v; \psi)F(u, v)], \qquad (12)$$

where I is the final decoded image from the subtractive encoded image and iFFT⁻¹ denotes the inverse Fourier transform.

Since the subtractive method is used in our design, a high magnitude of OTF associated with the ratio factor γ can efficiently control noise amplification in the deconvolution filter. To demonstrate our concept, a tangent phase mask is adopted to modulate the wavefront and can be expressed as [8]

$$f(x, y) = ax^2 \tan(bx) + ay^2 \tan(by),$$
 (13)

where *a* and *b* are the phase mask parameters to control the magnitude of the phase deviation, with *b* as unitless and *a* in units of radians. Actually, the phase mask parameter *a* meets the relation $a = 2\pi\xi/\lambda$, where ξ is the corresponding optical path difference.

3. SIMULATED ANALYSIS

Before performing an evaluation, phase mask parameters should be optimized. In the optimization, the phase parameter is to balance the imaging performance between the similarity and the recoverability. The similarity represents a similar degree of the encoded images, and the recoverability indicates if it is easy to deblur the encoded images. Generally, the merit functions of the phase mask parameters should make the imaging properties at different defocus positions as similar as possible under the same constraint condition, which assures the recoverability of the encoded images. We establish the merit function based on the Fisher information (FI) matrix,

$$\begin{cases} \min\left[\int_{-\psi_{\max}}^{\psi_{\max}} \int \left|\frac{\partial}{\partial\psi}H(u,\psi)\right|^{2} du d\psi\right] \\ \text{subject to:} \begin{cases} \text{LB} \le \text{Para} \le \text{UB} \\ \frac{1}{2\psi_{\max}} \int_{-\psi_{\max}}^{\psi_{\max}} \int |H(u,\psi)| du d\psi \ge \text{TH} \end{cases}, \quad (14)$$



Fig. 2. Phase profiles of optimal masks. (a) PM1 and (b) PM2.



Fig. 3. MTFs of PM1 and PM2 for different defocus values of 0, 6, 12, 18, 24, and 30.

where ψ_{max} is the maximum value of the defocus parameter. Para represents the parameters of the phase masks. LB and UB are the lower and upper bounds of parameters of the phase mask, respectively. TH is a threshold to determine the minimum acceptable average magnitude of the defocused modulation transfer function (MTF).

We optimize the phase mask parameters of Eq. (13) when the minimum acceptable average magnitudes TH are 0.45 and 0.5, respectively. By using the merit function of Eq. (14), the optimal mask parameters of PM1 are (a = 11.03, b = 1.25) at TH = 0.5, and the parameters of PM2 are (a = 15.52,



Fig. 4. The in-focus MTFs of PM1, PM2, and the subtraction method. (a) $\gamma = 0.6$, (b) $\gamma = 0.8$, and (c) $\gamma = 0.9$.



Fig. 5. Simulated decoded images when the SNR is 30 dB and the ratio factor γ is 0.6. Top to bottom: PM1, PM2, and the subtraction method. Left to right: $\psi = 0$, $\psi = 15$, and $\psi = 30$.

b = 1.24) at TH = 0.45. With the optimal mask parameters, the phase profiles of two the phase masks are shown in Fig. 2.

The defocused MTFs of two tangent phase masks are shown in Fig. 3 when the defocus parameters ψ are set as 0, 6, 12, 18, 24, and 30. Compared with the conventional optical system, the magnitudes of defocused MTF curves for two phase masks have a considerable reduction but produce a nearly invariant



Fig. 6. Simulated decoded images when the SNR is 20 dB and the ratio factor γ is 0.8. Top to bottom: PM1, PM2, and the subtraction method. Left to right: $\psi = 0$, $\psi = 15$, and $\psi = 30$.





Fig. 7. Simulated decoded images when the SNR is 10 dB and the ratio factor γ is 0.9. Top to bottom: PM1, PM2, and the subtraction method. Left to right: $\psi = 0$, $\psi = 15$, and $\psi = 30$.

distribution at all defocus positions. MTFs of PM2 show less oscillation but lower magnitude, indicating that greater phase modulation is introduced than in PM1.

According to Eq. (9), the subtractive encoded OTF can be obtained by combining two individual OTFs. Figure 4 shows in-focus MTFs of PM1, PM2, and the subtractive method when the ratio factor γ is set as 0.6, 0.8, and 0.9. It can be found from Fig. 4 that the magnitudes of the MTFs have a considerable promotion, meaning that the noise will be less amplified in the deconvolution filter. Additionally, with the increase of the values of the ratio factor γ , the magnitudes of the subtractive MTFs rise but generate more serious oscillations. We can select a proper ratio factor to balance the magnitude of the MTF and its oscillation.

A more direct approach to evaluate the imaging performance of wavefront coding systems is by image simulation. Here, the test image of Lena is used to prove the subtractive methods, and Gaussian noise is introduced in the recorded images. The simulations for the restored images of PM1, PM2, and our subtractive method are shown in Figs. 5–7. The ratio factor γ is 0.6, 0.8, and 0.9, respectively. The SNRs of the intermediate images are set as 30, 20, and 10 dB, respectively. The defocus parameter ψ is set as 0, 15, and 30. It is straightforward to find that the subtractive method has an efficient suppression in noise amplification, especially in the lower SNR. In addition, a greater ratio factor will produce more serious artifacts in the restored images. We can adjust the ratio factor under the condition of different SNRs to optimize the wavefront coding systems.

4. CONCLUSION

In this paper, we proposed a method based on the subtraction of two intermediate images of two asymmetrical phase masks to suppress the noise of decoded images. Two intermediate images of same scene are recorded using the same phase but with different mask parameters. The subtractive operation is adopted between two intermediate images, and a ratio factor is used to weight the magnitude of the OTF of the final subtractive encoded image. The defocused MTFs of the subtractive method presented a higher magnitude and the noise amplification was suppressed in the deconvolution filter. The simulated images have proved our concept in noise suppression while maintain the depth of field.

Funding. National Natural Science Foundation of China (NSFC) (61205186), China Postdoctoral Science Foundation (2012M520725 and 2013T60359).

REFERENCES

- E. R. Dowski and W. T. Cathey, "Extended depth of field through wave-front coding," Appl. Opt. 34, 1859–1866 (1995).
- S. S. Sherif, E. R. Dowski, and W. T. Cathey, "Effect of detector noise in incoherent hybrid imaging systems," Opt. Lett. 30, 2566–2568 (2005).

- S. Bagheri, P. E. X. Silveira, and G. Barbastathis, "Signal-to-noiseratio limit to the depth-of-field extension for imaging systems with an arbitrary pupil function," J. Opt. Soc. Am. A 26, 895–908 (2009).
- F. Diaz, F. Goudail, B. Loiseaux, and J. P. Huignard, "Increase in depth of field taking into account deconvolution by optimization of pupil mask," Opt. Lett. 34, 2970–2972 (2009).
- F. Diaz, F. Goudail, B. Loiseaux, and J. P. Huignard, "Comparison of depth-of-focus-enhancing pupil masks based on a signal-to-noiseratio criterion after deconvolution," J. Opt. Soc. Am. A 27, 2123–2131 (2010).
- V. Le, Z. Fan, N. Minh, and S. Chen, "Optimized square-root phase mask to generate defocus-invariant modulation transfer function in hybrid imaging systems," Opt. Eng. 54, 035103 (2015).
- H. Zhao and Y. Li, "Optimized sinusoidal phase mask to extend the depth of field of an incoherent imaging system," Opt. Lett. 35, 267–669 (2010).
- V. Le, S. Chen, and Z. Fan, "Optimized asymmetrical tangent phase mask to obtain defocus invariant modulation transfer function in incoherent imaging system," Opt. Lett. **39**, 2171–2174 (2014).
- Q. X. Liu, T. Y. Zhao, W. Z. Zhang, and F. H. Yu, "Image restoration based on generalized minimal residual methods with antireflective boundary conditions in a wavefront coding system," Opt. Eng. 47, 1270051 (2008).
- H. Okugawa, "A new imaging method for confocal microscopy," Proc. SPIE 6860, 68600K (2008).