New Upper Bound for High-Rate Spatial Modulation Systems using QAM Modulation

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***Abstract*—In this paper, we present a new upper bound for the *bit error probability* (BEP) of the so-called High-Rate Spatial Modulation (HR-SM) system using QAM modulation introduced by Thu Phuong Nguyen *et al.* in [1], over a quasi-static Rayleigh fading channel. Our approach based on the Verdu's theorem [2], the concept of the spatial constellation (SC) codewords and maximum likelihood (ML) decoder in [1] results in the new upper bound which is tighter than the union bound due to eliminating a number of redundant *pairwise error probabilities* (PEPs). Therefore, by using the new upper bound rather than the union bound, we can evaluate the bit-error performance of HR-SM systems more exactly, especially when the signal-to-noise power ratio (SNR) is sufficiently high.**

***Index Terms*—HR-SM, bit error probability, ML detection, bit error rate, new upper bound, union bound, codeword matrix, quasi-static Rayleigh fading.**

I. Introduction

Using multiple transmit and receive antennas in wireless communication systems has been theoretically and practically shown to considerably improve spectral efficiency compared with conventional signal antenna systems [3], [4]. Since then, various transmission techniques for Multiple Input Multiple Output (MIMO) systems have been introduced by numerous researchers.

Recently, a technique known as Spatial Modulation (SM) was proposed by Mesleh *et al.* in [5]. In SM, only one out of n_T transmit antennas is activated at every symbol time, and a quadrature amplitude modulation (QAM) symbol or a phase shift keying (PSK) modulation symbol is transmitted via this activated antenna, thus resulting in avoiding inter-channel interference (ICI) completely. In addition, information bits are conveyed not only by the QAM or PSK modulated symbols but also by the index of the activated antenna. A special case of SM is Space Shift Keying (SSK) proposed by Jeganathan *et al.* in [6]. In SSK, information bits are carried by only the antennas indices, not by QAM or PSK modulated symbols, to obtain the more simple design of system and the decrease in decoding complexity. Both the SSK and SM schemes exploit the multiplexing gain of multiple transmit antennas, however, they do not achieve any transmit diversity gains. Furthermore, SM/SSK schemes can not attain a very high spectral efficiency due to only a logarithmic increase of data rate with the number of transmit antennas. To solve this problem, the authors introduced the Generalized SM (GSM) scheme in [7], [8]. In GSM, the spectral efficiency is no longer limited to $\log_2 n_T$ bits/s/Hz as that of SM/SSK schemes because the number of activated antennas can be arbitrarily greater than 1.

In [9], Basar *et al.* proposed a so-called Space-Time Block Coded Spatial Modulation (STBC-SM), which achieves simultaneously both diversity and multiplexing gains based on combining STBC (i.e., the Alamouti's STBC) with spatial modulation. In [11], the High-rate STBC-SM scheme was proposed with the novel concept of spatial constellation (SC) codeword matrices by Le *et al.*. The spectral efficiency of this scheme increases by 0.5 bits/s/Hz compared to STBC-SM scheme for the same number of transmit antennas as in [11], while keeping the diversity order unchanged. Very recently, in [12], Le *et al.* proposed a MIMO scheme which is Spatial Modulated Orthogonal Space-Time Block Coding (SM-OSTBC), based on the concept of SC codewords [11]. The SM-OSTBC scheme achieves a lot of advantages in comparison with the previous MIMO schemes, especially its spectral efficiency increases linearly with the number of transmit antennas n_T. This means that it provides higher spectral efficiency than both the STBC-SM and High-rate STBC-SM schemes for the same number of transmit antennas. Moreover, all these three schemes can use the single-stream low-complexity ML decoders, hence reducing the signal processing complexity at the receiver.

In [1], Thu Phuong Nguyen *et al.* proposed a novel MIMO scheme known as High-rate Spatial Modulation (HR-SM) by using the concept of SC codewords [11]. In this scheme, transmitted signal vectors (i.e., HR-SM codewords) are created by multiplying SC codewords with QAM or PSK constellation symbols. Whereby, in HR-SM, information bits are carried by both SC codewords and constellation symbols resulting in a substantial increase in the spectral efficiency compared to the SM/SSK/GSM schemes. In addition, the low-complexity ML detector of the HR-SM scheme can be performed at the receiver. In fact, the SM/SSK/GSM schemes are considered as the special case of the HR-SM scheme.

In order to evaluate the bit error rate (BER) performance of the HR-SM schemes [1], the Monte Carlo simulation and the theoretical union bound for the BER were used. Nevertheless, the union bound is not close to the simulation curve in numerous cases, especially when the SNR is low enough and/or the number of HR-SM codewords is large. This can lead to an inexact evaluation of the BER performance of systems in case of using the union bound. Hence, it is essential to derive a new upper bound for the BER of the HR-SM schemes. In [2], based on the method of error sequence decomposition, S. Verdu presented a new upper bound for the simple BPSK system in the presence of additive white Gaussian noise. Then, in [13], Ngo *et al.* exploited this idea to construct an expurgated union bound (i.e., new upper bound) for the space-time code (STC) systems via eliminating the superfluous code-matrices. It is worth noting that in [14], Biglieri *et al.* introduced the generalization of the Verdu's theorem which is the original idea in [13].

In this paper, based on the Verdu's theorem in [2] we build again a new upper bound for the bit error probability of the HR-SM system using QAM modulation, over the quasi-static Rayleigh fading MIMO channel. Here, we just consider the HR-SM using QAM constellation symbols because our approach depends on the position of constellation symbols. Firstly, we propose a novel theorem that allows us to eliminate unnecessary pairwise error probabilities (PEPs) from the union bound. We also derive an equation to exactly calculate the number of excluded PEPs. Then, based on this proposed theorem, the new upper bound is given in a closed-form expression. The numerical results in comparison with simulation results are presented to show how the new upper bound is tighter than the union bound.

The rest of the paper is organized as follows. Section II presents system model. In section III, we introduce the maximum likelihood decoder and the union bound of HR-SM system. The new upper bound and its closed form are presented in section IV. Section V provides numerical and simulation results to make a comparison between the new bound and the union bound. Finally, section VI concludes the paper.

Notation: Throughout this paper, bold capital and lowercase letters are respectively used for matrices and column vectors, $(.)^H$ is the Hermitian transpose. $E[.]$ represents the expectation or the average value and the probability of an event is denoted by $P(.)$. For a complex number z, $\Re\{z\}$ and $\Im\{z\}$ denote the real part of z and the image part of z, respectively. $\|.\|$ and $tr(.)$ stand for the Frobenius norm and the trace of a matrix, respectively.

II. System Model

Consider a HR-SM system with n_T transmit antennas and n_R receive antennas and a quasi-static Rayleigh fading channel. At every symbol period, $(m+l)$ bits enter the HR-SM transmitter. The first l bits are mapped into a $n_T \times 1$ SC codeword $\mathbf{s}$, out of K SC codewords $(K = 2^l)$ in the spatial constellation U_s. The remaining m bits are mapped into a M-QAM constellation symbol x, out of $M = 2^m$ symbols in the modulation constellation U_x. The $n_T \times 1$ HR-SM codeword $\mathbf{c}$ is generated by multiplying $\mathbf{s}$ with x, i.e., $\mathbf{c} = \mathbf{s}x$, and is subsequently transmitted via n_T transmit antennas within a symbol interval. The $n_R \times 1$ received signal vector $\mathbf{y}$ is expressed as [1]:

$$\mathbf{y} = \sqrt{\gamma}\mathbf{Hc} + \mathbf{n} = \sqrt{\gamma}\mathbf{Hs}x + \mathbf{n} \tag{1}$$

where $\mathbf{H}$ and $\mathbf{n}$ respectively denote $n_R \times n_T$ channel matrix and $n_R \times 1$ noise matrix. The entries of $\mathbf{H}$ and $\mathbf{N}$ are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. We assume that $\mathbf{H}$ is unchanged within a symbol interval of a HR-SM codeword and independently varied from one codeword to another. $\mathbf{c}$ is normalized such that the ensemble average of the trace of $\mathbf{c}^H\mathbf{c}$ is equal to 1, i.e., $E[tr(\mathbf{c}^H\mathbf{c})] = 1$. γ is the average SNR at each receive antenna.

Following the SC codeword construction presented in [1], a maximum spectral efficiency of the HR-SM scheme is equal to $(2(n_T - 1) + \log_2 M)$ bits/s/Hz, which is linear with the number of transmit antennas, instead of $(\log_2 n_T + \log_2 M)$ as offered by the SM scheme in [5]. This means that the HR-SM scheme in [1] saves a large number of transmit antennas in comparison with SM scheme for a given spectral efficiency.

III. ML Detection and Union Bound

A. Maximum Likelihood Detection

In this subsection, we modify the ML decoding algorithm presented in [1] as in [12], under the assumption that perfect channel state information is available at the receiver. The ML decoder for the HR-SM scheme exhaustively searches over all codewords $\mathbf{c}$ and chooses $\hat{\mathbf{c}} = \hat{\mathbf{s}}\hat{x}$ that satisfies [12]:

$$(\hat{\mathbf{s}}, \hat{x}) = \arg \min_{\mathbf{s}\in U_s, x\in U_x} \|\mathbf{y} - \sqrt{\gamma}\mathbf{Hs}x\|^2. \tag{2}$$

For a given $\mathbf{s}_k \in U_s, k = 1, 2, ..., K$, we define the corresponding $n_R \times 1$ equivalent matrix $\mathbf{h}_k = \mathbf{Hs}_k$. Then, (1) can be reduced to the following equivalent system:

$$\mathbf{y} = \sqrt{\gamma}\mathbf{h}_k x + \mathbf{n} \tag{3}$$

The ML decoding rule in (2) becomes the ML decoding rule for x conditioned on $\mathbf{s}_k$, as follows:

$$\widehat{x}_k = \arg \min_{x\in U_x} \|\mathbf{y} - \sqrt{\gamma}\mathbf{h}_k x\|^2. \tag{4}$$

Let us define:

$$\bar{x}_k = \frac{\Re\left\{\mathbf{y}^H\mathbf{h}_k\right\}}{\sqrt{\gamma}\|\mathbf{h}_k\|^2} - j\frac{\Re\left\{\mathbf{y}^H\mathbf{h}_k\right\}}{\sqrt{\gamma}\|\mathbf{h}_k\|^2}, \tag{5}$$

$$R_k = \|\mathbf{y}\|^2 - \gamma\|\mathbf{h}_k\|^2|\bar{x}_k|^2, \tag{6}$$

$$d_k(\mathbf{y}, x) = |x - \bar{x}_k|^2. \tag{7}$$

Then, we represent (4) as:

$$\|\mathbf{y} - \sqrt{\gamma}\mathbf{h}_k x\|^2 = \gamma\|\mathbf{h}_k\|^2 d_k(\mathbf{y}, x) + R_k. \tag{8}$$

Because R_k does not depend on x, we can reduce (4) to the following equivalent equation:

$$\widehat{x}_k = \arg \min_{x\in U_x} d_k(\mathbf{y}, x). \tag{9}$$

With each $\widehat{x}_k$ detected above, we define:

$$L_k = \gamma\|\mathbf{h}_k\|^2 \left(d_k(\mathbf{y}, \widehat{x}_k) - |\bar{x}_k|^2\right). \tag{10}$$

The index of $\mathbf{s}_k$ can be recovered according to:

$$\widehat{k} = \arg \min_{k=1,2,...,K} L_k. \tag{11}$$

Finally, the SC codeword and the constellation symbol are estimated as follows:

$$\widehat{\mathbf{s}} = \mathbf{s}_{\widehat{k}}, \widehat{x} = \widehat{x}_{\widehat{k}}. \tag{12}$$

From $\widehat{\mathbf{s}}$ and $\widehat{x}$ estimated above, we recover $(m + l)$ data bits.

B. Union Bound

According to [1], [11], the union bound (i.e., upper bound) of the HR-SM scheme is given by:

$$P_e \leq \frac{1}{(m+l)N} \sum_{i=1}^{N} \sum_{j=1}^{N} P(\mathbf{c}_i \rightarrow \mathbf{c}_j) w_{i,j} \tag{13}$$

where, $P(\mathbf{c}_i \rightarrow \mathbf{c}_j)$ is the pairwise error probability (PEP) of deciding $\mathbf{c}_j$ while $\mathbf{c}_i$ is transmitted. The PEP $P(\mathbf{c}_i \rightarrow \mathbf{c}_j)$ is evaluated by averaging $P(\mathbf{c}_i \rightarrow \mathbf{c}_j|\mathbf{H})$ over the channel matrix $\mathbf{H}$ as presented in [11], [15]. $N = KM = 2^{m+l}$ is the total number of the HR-SM codewords and $w_{i,j}$ is the number of erroneous bits between $\mathbf{c}_i$ and $\mathbf{c}_j$.

IV. New Upper Bound

In this section, we present a novel theorem and the corresponding demonstration so as to derive the new upper bound for the HR-SM scheme using QAM modulation.

Firstly, we define the concept of *"close symbol"* as follows: Let $x, x' \in U_x$, where U_x is a QAM constellation. Then x and x' are said to be "close symbols" if and only if the Euclidean distance between x' and x is minimum.

Theorem: *Given a transmitted HR-SM codeword matrix* $\mathbf{c}_i = \mathbf{s}_k x_n, k = 1, 2, ..., K, i = 1, 2, ..., N$ *and* $n = 1, 2, ..., M$. *Let us define* V_n *is the set of close symbols of* x_n *and* $\bar{V}_n = U_x - V_n$ *is the set of remaining symbols that are not close symbols of* x_n. *Define:*

$$U_i = U_c - \left\{\mathbf{c} \in U_c | \mathbf{c} = \mathbf{s}_k x_v, x_v \in \bar{V}_n\right\} \tag{14}$$

where U_c *is the set of the total* N *HR-SM codewords* $\mathbf{c}$. *The new upper bound for the bit error probability of the HR-SM scheme is given as follows:*

$$P_e \leq \frac{1}{(m+l)N} \sum_{i=1}^{N} \sum_{\mathbf{c}_j \in U_i} P(\mathbf{c}_i \rightarrow \mathbf{c}_j) w_{i,j}. \tag{15}$$

Proof: Assume that $\mathbf{y}$ is the received signal vector and then the receiver decides the codeword $\mathbf{c}_j = \mathbf{s}_k x_v \in U_C - U_i$ while the codeword $\mathbf{c}_i = \mathbf{s}_k x_n$ is transmitted, where $x_v \in \bar{V}_n$ and $x_v \neq x_n$. By the definition of $\mathbf{c}_i$ and $\mathbf{c}_j$ above, we can clearly see that as both $\mathbf{c}_i$ and $\mathbf{c}_j$ have the same $\mathbf{s}_k$, the detector estimates x as x_v instead of x_n.

Let us define $V_n = \{x_{n_1}, x_{n_2}, \ldots, x_{n_Q}\}$, where Q only can be equal to 2, 3 or 4. This can be clearly seen from the construction of the QAM constellation symbols.

In order to prove (15), we need to prove that the PEP $P(\mathbf{c}_i \rightarrow \mathbf{c}_j)$ is redundant in the evaluation of the average error probability P_e.

Following section III, we can see that $d_k(\mathbf{y}, x_v)$ is the minimum value among $\{d_k(\mathbf{y}, x)|x \in U_x\}$ because x_v is the solution of equation (9). To eliminate $P(\mathbf{c}_i \rightarrow \mathbf{c}_j)$ from the upper bound (13), we have to prove that there exists at least a index $n_q \in \{n_1, n_2, ..., n_Q\}$ such that [16]:

$$\begin{gathered}\left\{\mathbf{y}|d_k(\mathbf{y}, x_v) = \min_{x \in U_x} d_k(\mathbf{y}, x)\right\} \\ \subset \left\{\mathbf{y}|d_k\left(\mathbf{y}, x_{n_q}\right) \leq d_k(\mathbf{y}, x_n)\right\}\end{gathered} \tag{16}$$

This is equivalent to show that:

$$d_k\left(\mathbf{y}, x_{n_q}\right) \leq d_k(\mathbf{y}, x_n) \tag{17}$$

For each $n_q \in \{n_1, n_2, ..., n_Q\}$, let us define:

$$V_{n,n_q} = \left\{\mathbf{y}|d_k\left(\mathbf{y}, x_{n_q}\right) \leq d_k(\mathbf{y}, x_n)\right\} \tag{18}$$

and

$$E_n = E - \left\{\mathbf{y}|d_k(\mathbf{y}, x_n) = \min_{x \in U_x} d_k(\mathbf{y}, x)\right\} \tag{19}$$

where the set E contains every point on the QAM modulation plane. Here, to prove (17), we need to show that:

$$\bigcup_{n_q = n_1, n_2, ..., n_Q} V_{n,n_q} = E_n. \tag{20}$$

We consider the plane E as a Voronoi diagram generated by M QAM modulation symbols.

For the case of $Q = 2$, i.e., x_n has 2 close symbols x_{n_1} and x_{n_2} as shown in Fig. 1, we can easily see that the Voronoi region generated by x_n corresponds to $E - E_n$ as the blue region and the remaining white region corresponds to E_n. The red vertical line divides the plane E into 2 half-plane, one of them that contains x_{n_1} corresponds to V_{n,n_1}. Similarly, we also define the region of V_{n,n_2}. By observing Fig. 1, we can easily verify the accuracy of (20).

Similar to the case of $Q = 2$, we also successfully prove that (20) is true in case of $Q = 3$ or $Q = 4$.

Hence, the theorem is proved completely. ■

Next, we will formulate the number of excluded PEPs. Among the total of $M = 2^m$ QAM constellation symbols, where $m \geq 2$ is an even number, there are 4 symbols which have 2 close symbols, $4(\sqrt{M}-2)$ symbols which have 3 close symbols and $(\sqrt{M}-2)^2$ symbols which have 4 close symbols. For instance, with 16-QAM modulation (i.e., $M = 16$), the number of symbols having 2, 3 or 4 close symbols respectively is 4, 8, and 4. Each symbol having 2, 3 or 4 close symbols, the number of excluded non-zero PEPs is $M - 3$, $M - 4$ and $M - 5$, respectively. Thus, the total number of excluded non-zero PEPs is calculated as follows:

$$\begin{gathered}K(4(M-3) + 4(\sqrt{M}-2)(M-4) + (\sqrt{M}-2)^2(M-5)) \\ = K(M^2 - 5M + 4\sqrt{M})\end{gathered} \tag{21}$$

while, the number of non-zero PEPs of the union bound (13) is $N(N - 1) = KM(KM - 1)$.

As a result, the number of excluded PEPs increases when the modulation order is large. In such cases, we expect that the new upper bound is much tighter than the union bound.

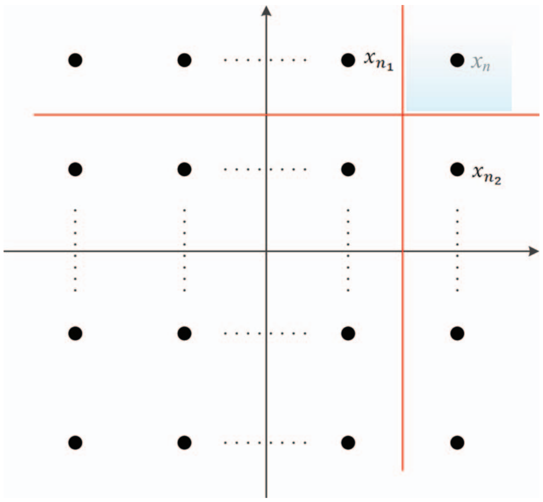

Fig. 1. The construction of QAM constellation symbols with 2 close symbols of x_n.

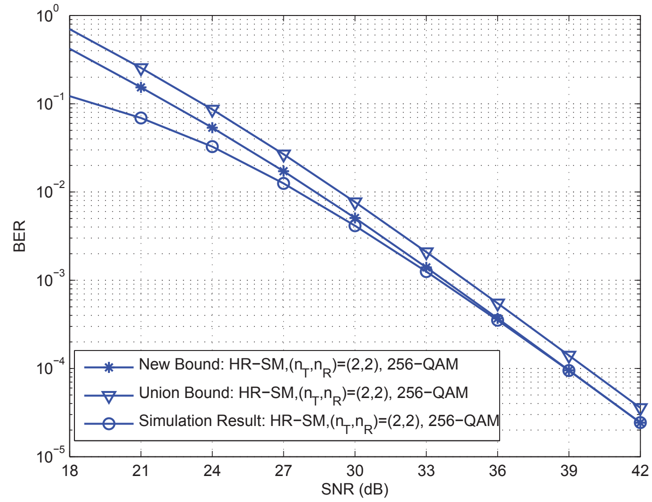

Fig. 3. The new upper bound, the union bound and simulation curve for the BER of the $(2, 2)$ HR-SM system using 256-QAM modulation.

V. Numerical Results and Comparison

In this section, we compare the new bound with the union bound and simulation results of a number of HR-SM configurations. For convenience, we denote a MIMO system with n_T transmit antennas and n_R receive antennas as the (n_T, n_R) system.

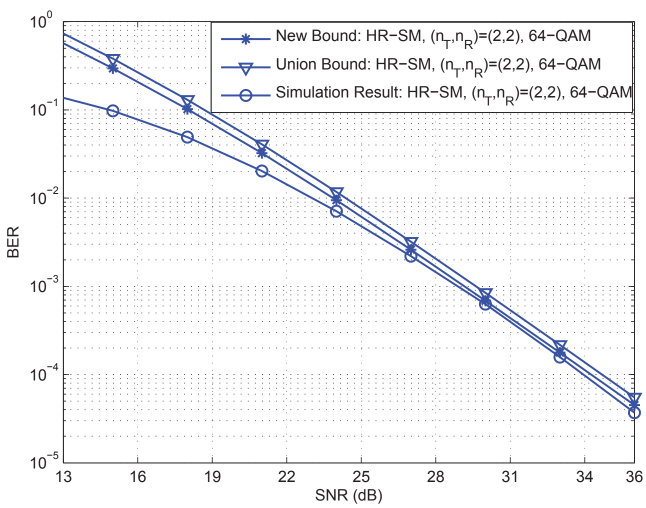

Fig. 2. The new upper bound, the union bound and simulation curve for the BER of the $(2, 2)$ HR-SM system using 64-QAM modulation.

In Fig. 2, the new upper bound, the union bound and simulation result for the BER of the $(2, 2)$ HR-SM system using 64-QAM modulation are shown. As indicated in Fig. 2, we can see that both the union bound and the new bound are very close to the simulation curve in sufficiently high SNR regions, however, in low SNR regions, they are still loose. We obseve that the new bound is closer to the simulation curve and at BER $= 10^{-3}$, it is tighter than the union bound by an SNR gain of approximately 0.5 dB.

Fig. 3 compares the new upper bound with the union bound and simulation result for the BER of $(2, 2)$ HR-SM system using 256-QAM modulation. We again see from Fig. 3 that the new upper bound is remarkably closer to the simulation curve than the union bound. Obviously, by using the higher modulation order, the new bound in Fig. 3 is much better than that in Fig. 2. In particular, at BER $= 10^{-4}$, the new bound is tighter than the union bound by approximately 1 dB in SNR.

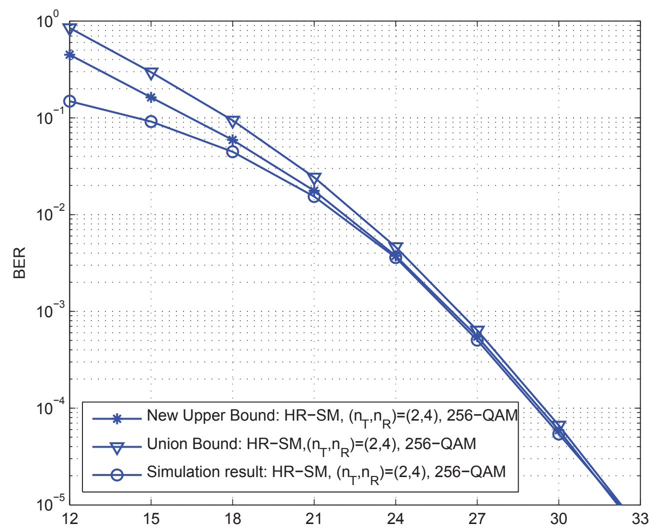

Fig. 4. The new upper bound, the union bound and simulation curve for the BER of the $(2, 4)$ HR-SM system using 256-QAM modulation.

The comparison for the $(2, 4)$ HR-SM system using 256-QAM modulation is shown as in Fig. 4. The new upper bound is very close to the simulation curve, especially when the SNR is greater than 20 dB. We also can look at BER $= 10^{-1}$, the new bound is tighter than the union bound about 1.5 dB in SNR.

According to all three figures above, we can also conclude that the higher the modulation order is used, the tighter the new upper bound is. This again verifies our anticipation presented in section IV.

VI. Conclusion

In this paper, based on the Verdu's theorem in [2], we propose a new upper bound for the bit error probability of the HR-SM system using QAM modulation. It is shown via analytical and simulation results that our new bound is not only tighter but also less computationally complex than the union bound. However, the new upper bound is still loose in the low SNR regions and not considerably tighter than the union bound when l is comparable to m. These issues are left for our future research.

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