New Upper Bound for Space-Time Block Coded Spatial Modulation

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***Abstract*—In this paper, we present a new upper bound for the *bit error probability* (BEP) of the so-called Space Time Block Coded Spatial Modulation (STBC-SM) system introduced by E. Basar *et al.* in [1] over a quasi-static Rayleigh fading channel. Based on the Verdu's theorem [2], the concept of the spatial constellation (SC) codewords and maximum likelihood (ML) decoder in [3], the upper bound is obtained by eliminating a number of redundant *pairwise error probabilities* (PEPs). Our approach leads to a new upper bound, which is tighter than the union bound. Consequently, it allows us to evaluate the bit-error performance of STBC-SM systems more exactly, particularly when the signal-to-noise power ratio (SNR) is sufficiently high.**

***Index Terms*—STBC-SM, PEP, ML detection, bit error rate, new upper bound, union bound, codeword matrix, quasi-static Rayleigh fading.**

I. Introduction

To improve the performance and spectral efficiency of Multiple-Input Multiple-Output (MIMO) systems, various techniques have been researched and developed. Among them, the STBC-SM proposed in [1] has been known as a typical approach of exploiting the strong points of both the Alamouti's space-time block code (STBC) [4] and the spatial modulation (SM) [5]. Now, we will gradually summarize the origin of the STBC-SM and their significant qualifications in comparison with the other MIMO systems.

In [4], a simple, yet efficient, transmit diversity scheme was introduced by Alamouti. This scheme is then considered to be the first space-time block code featuring the so-called orthogonality property. Later, the useful orthogonality structure was generalized by Tarokh *et al.* [6] for larger numbers of transmit antennas, resulting in a class of orthogonal STBCs (OSTBCs). OSTBCs are attractive for two reasons: 1) they achieve full diversity; and 2) they enable single-stream maximum likelihood (ML) decoders to be implemented at the receiver. As a consequence, they have been applied in the advanced wireless communication systems such as the third generation (3G) and fourth generation (4G). Recently, Mesleh *et al.* proposed the Spatial Modulation (SM) technique [5]. In SM, the bit sequence is carried not only by the modulated symbol (i.e., QAM or PSK constellation symbol) but also by the indices of the activated antennas. In addition, since there is one transmit antenna being activated at a time, the inter-channel interference (ICI) and inter-antenna synchronization problems associated with the conventional MIMO techniques are completely avoided.

After the invention of OSTBC and SM techniques, it is natural to think of combining the two techniques in such a way that their advantages are fully utilized. One of such scheme was first proposed by Basar *et al.* in [1] and is widely known as Space Time Code Block Spatial Modulation (STBC-SM). In a STBC-SM system, information bits are conveyed by both the Alamouti codeword and the indices of transmit antennas. This leads to a spectral efficiency of $\left(\frac{1}{2}\log_2 c + \log_2 M\right)$ *bits per channel use* (bpcu), where M is the constellation size and $c = \left\lfloor \begin{pmatrix} n_T \\ 2 \end{pmatrix} \right\rfloor_{2^p}$ with p is a positive integer [1]. Besides, the system attains the second order transmit diversity. However, the design of STBC-SM codewords is not simple, especially when the number of transmit antennas is large. In [7], Le *et al.* introduced the High-rate Space-Time Block Coded Spatial Modulation, which uses the concept of spatial constellation (SC) codeword matrices. This scheme offer an increase in spectral efficiency as compared to the Basar's scheme for the same number of transmit antennas. However, the design of SC codewords still requires the use of computer search. Therefore, finding a new method of SC codeword design without relying on computer is of necessity. In [3], Le *et al.* proposed SM-OSTBC systems with Non-Vanishing Determinants (NVD). As shown in [3], the construction of SC codewords is systematic without using a computer search and applicable to the cases that either all or a subset of n_T transmit antennas are activated, provided that n_T is even and greater than or equal 4. The SM-OSTBC also achieves the same diversity order as the STBC-SM while providing higher spectral efficiency, which is $n_T - 2 + \log_2 M$ bpcu. Moreover, because of the orthogonality of the Alamouti STBC codeword matrix, the single-stream low-complexity ML decoder is achieved. According to [3], the SM-OSTBC subsumes the STBC-SM as the special case.

In all the aforementioned papers, the BER performances of systems were evaluated via the utilization of both the Monte-Carlo simulation and the theoretical union bound. However, in numerous cases, the union bound may be very loose and not close to the simulation curves, especially in sufficient low signal-to-noise power ratio (SNR) regions and/or with a large number of the SM-STBC codewords. This can lead to an inaccurate evaluation of the BER performance of systems in case of using the union bound. Hence, it is necessary to study and construct a new upper bound for the BER of the SM-OSTBC and STBC-SM schemes. In [2], S. Verdu proposed a new upper bound for the simple BPSK system in the presence of additive white Gaussian noise based on the method of error sequence decomposition. This idea was then exploited by Ngo *et al.* to build an expurgated union bound (i.e., new upper bound) for the space-time code systems via excluding the superfluous code-matrices [8]. It is worth noting that the original idea in [8] was based on the generation of the Verdu's theorem, which was presented by Biglieri *et al.* in [9].

In this paper, based on the Verdu's theorem in [2] we construct again a new upper bound for the bit error probability of the STBC-SM system over the quasi-static Rayleigh fading MIMO channel. Firstly, we present a novel theorem that allows us to exclude unnecessary pairwise error probabilities (PEPs) from the union bound. We also provide an equation to precisely compute the number of excluded PEPs. Then, based on this theorem, the new upper bound is given in a closed-form expression. The numerical results in comparison with simulation results are provided to show how the new bound is tighter than the union bound.

The rest of the paper is organized as follows. Section II presents system model. In section III, the maximum likelihood detection and the union bound of STBC-SM system are introduced. The new upper bound and its closed form are presented in section IV. Section V provides numerical and simulation results to make a comparison between the new bound and the union bound. Finally, section VI concludes the paper.

Notation: Throughout this paper, bold capital letters are used for matrices, $(.)^H$ and $(.)^*$ are the Hermitian transpose and complex conjugation, respectively. For a complex number z, $\Re\{z\}$ and $\Im\{z\}$ denote the real part of z and the image part of z, respectively. $\|.\|$ and $tr(.)$ stand for the Frobenius norm and the trace of a matrix, respectively. $E[.]$ represents the expectation or the average value and the probability of an event is denoted by $P(.)$.

II. System Model

Consider an STBC-SM system with n_T transmit antennas and n_R receive antennas over a quasi-static Rayleigh fading MIMO channel, the received $n_R \times 2$ signal matrix $\mathbf{Y}$ can be expressed as [1]:

$$\mathbf{Y} = \sqrt{\gamma}\mathbf{HC} + \mathbf{N} \tag{1}$$

where $\mathbf{C}$, $\mathbf{H}$ and $\mathbf{N}$ respectively denote $n_T \times 2$ STBC-SM transmission matrix, $n_R \times n_T$ channel matrix and $n_R \times 2$ noise matrix. The entries of $\mathbf{H}$ and $\mathbf{N}$ are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. We assume that $\mathbf{H}$ is unchanged within a codeword of two symbol intervals and independently varied from one codeword to another. $\mathbf{C}$ is normalized such that the ensemble average of the trace of $\mathbf{C}^H\mathbf{C}$ is equal to 2, i.e., $E[tr(\mathbf{C}^H\mathbf{C})] = 2$. γ is the average SNR at each receive antenna.

Using the concept of SC codewords [7], [3], each transmission matrix $\mathbf{C}$ in (1) can be represented by the product of $\mathbf{S}$ and $\mathbf{X}$, i.e., $\mathbf{C} = \mathbf{SX}$. Where the 2×2 matrix $\mathbf{X}$ is the Alamouti STBC codeword created from two M-QAM or M-PSK constellation symbols x_1 and x_2 selected from the modulation constellation U_x with $M = 2^m$ signal points. The $n_T \times 2$ matrix $\mathbf{S}$ is a SC codeword that belongs to the set of total c SC codeword $\mathbf{S}_1$, $\mathbf{S}_2$, ..., $\mathbf{S}_c$, $c = 2^l$. Then, the $n_R \times 2$ received signal matrix $\mathbf{Y}$ in (1) can be rewritten as:

$$\mathbf{Y} = \sqrt{\gamma}\mathbf{HSX} + \mathbf{N}. \tag{2}$$

III. ML Detection and Union Bound

A. Maximum Likelihood Detection

In this subsection, we formulate the ML detector for the STBC-SM systems under the assumption that perfect channel state information is available at the receiver. Let U_X, U_S be the search spaces of the Alamouti codewords $\mathbf{X}$ and the SC codewords $\mathbf{S}$, respectively. The ML detector exhaustively searches over all codewords $\mathbf{C}$ and chooses $\hat{\mathbf{C}} = \hat{\mathbf{S}}\hat{\mathbf{X}}$ that satisfies [3]:

$$(\hat{\mathbf{S}}, \hat{\mathbf{X}}) = \arg \min_{\mathbf{S}\in U_S, \mathbf{X}\in U_X} \|\mathbf{Y} - \sqrt{\gamma}\mathbf{HSX}\|^2. \tag{3}$$

For a given $\mathbf{S}_p \in U_S, p = 1, 2, ..., c$, let us define the corresponding $n_R \times 2$ equivalent matrix $\mathbf{H}_p = \mathbf{HS}_p$. Then, (2) can be reduced to the following equivalent system:

$$\mathbf{Y} = \sqrt{\gamma}\mathbf{H}_p\mathbf{X} + \mathbf{N} \tag{4}$$

Applying the ML detector to (4), we are able to detect $\mathbf{X}$ for a given $\mathbf{S}_p$ to obtain $\hat{\mathbf{X}}_p$ as follows:

$$\hat{\mathbf{X}}_p = \arg \min_{\mathbf{X}\in U_X} \|\mathbf{Y} - \sqrt{\gamma}\mathbf{H}_p\mathbf{X}\|^2. \tag{5}$$

Since the Alamouti's STBC codeword can be represented in the form of a linear dispersion code as:

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} = \sum_{i=1}^{4} \{\Re\{x_i\}\mathbf{A}_i + j\Im\{x_i\}\mathbf{B}_i\}, \tag{6}$$

The detection of $\hat{\mathbf{X}}_p$ amounts to the detection of the signal pair $(\hat{x}_{1,p}, \hat{x}_{2,p})$ based on the following equation [3]:

$$(\hat{x}_{1,p}, \hat{x}_{2,p}) = \arg \min_{\mathbf{X}\in U_X} h_p(\mathbf{Y}, \mathbf{X}). \tag{7}$$

where

$$h_p(\mathbf{Y}, \mathbf{X}) = \sum_{i=1}^{2} |x_i - x_{i,p}|^2, \tag{8}$$

$$x_{i,p} = \frac{\Re\{tr(\mathbf{Y}^H\mathbf{H}_p\mathbf{A}_i\}}{\sqrt{\gamma}\|\mathbf{H}_p\|^2} - j\frac{\Im\{tr(\mathbf{Y}^H\mathbf{H}_p\mathbf{B}_i\}}{\sqrt{\gamma}\|\mathbf{H}_p\|^2} \tag{9}$$

Let us define

$$L_p = \gamma\|\mathbf{H}_p\|^2 \left[h_p\left(\mathbf{Y}, \hat{\mathbf{X}}_p\right) - K_p\right]. \tag{10}$$

where

$$K_p = \sum_{i=1}^{2} |x_{i,p}|^2, i = 1, 2 \quad (11)$$

The index of $\mathbf{S}_p$ can be recovered according to:

$$\hat{p} = \arg\min_p L_p. \quad (12)$$

Finally, the transmitted codewords are obtained as follows:

$$\hat{\mathbf{S}} = \mathbf{S}_{\hat{p}}, \hat{\mathbf{X}} = \begin{bmatrix} \hat{x}_{1,\hat{p}} & -\hat{x}_{2,\hat{p}}^* \\ \hat{x}_{2,\hat{p}} & \hat{x}_{1,\hat{p}}^* \end{bmatrix}.$$

From $\hat{\mathbf{S}}$ and $\hat{\mathbf{X}}$ estimated above, we can estimate the transmission matrix $\hat{\mathbf{C}}$ that corresponds to the $(2m + l)$ data bits.

B. The Union Bound

In this subsection, we formulate the union bound in a closed form for the STBC-SM scheme. This bound can be derived by evaluating their pairwise error probability (PEP) similarly to the evaluation in [1] and [7].

The PEP $P(\mathbf{C}_i \to \mathbf{C}_j)$ is the probability of deciding STBC-SM matrix $\mathbf{C}_j$ given that matrix $\mathbf{C}_i$ is transmitted. The PEP, conditioned on the channel $\mathbf{H}$, is given by [6]

$$P(\mathbf{C}_i \to \mathbf{C}_j|\mathbf{H}) = Q\left(\sqrt{\frac{\gamma}{2} d^2(\mathbf{C}_i, \mathbf{C}_j)}\right) \quad (13)$$

where $Q(x) = \frac{1}{2\pi}\int_x^\infty e^{-\frac{1}{2}t^2} dt$ is the Gaussian tail probability, and $d^2(\mathbf{C}_i, \mathbf{C}_j)$ is the modified Euclidean distance between $\mathbf{C}_j$ and $\mathbf{C}_i$.

Let us define $\lambda_n, n = 1, 2$ as the 2 eigenvalues of the codeword distance matrix $\mathbf{C}(i,j) = (\mathbf{C}_i - \mathbf{C}_j)^H(\mathbf{C}_i - \mathbf{C}_j)$. Since $\mathbf{C}(i,j)$ is nonnegative-definite Hermitian matrix [6], $\lambda_n \geq 0$. Following the approach in [7], [10], the PEP of the STBC-SM system is obtained as follows:

$$P(\mathbf{C}_i \to \mathbf{C}_j) = \frac{1}{\pi}\int_0^{\pi/2} \prod_{n=1}^{2} \left[1 + \frac{\gamma\lambda_n}{4\sin^2\theta}\right]^{-n_R} d\theta. \quad (14)$$

Each of the transmitted $2m + l$ data bits corresponds to one of a total $T = cM^2 = 2^{2m+l}$ STBC-SM codeword matrices $\mathbf{C}_1, \mathbf{C}_2, ..., \mathbf{C}_T$, i.e., $2m+l = \log_2 T$. By using the PEP in (14), the union bound for the average bit error probability (BEP) is given by:

$$P_e \leq \frac{1}{(2m + l)T} \sum_{i=1}^{T}\sum_{j=1}^{T} r_{i,j} P(\mathbf{C}_i \to \mathbf{C}_j) \quad (15)$$

where $r_{i,j}$ is the number of error bits, caused by estimating $\mathbf{C}_j$ when $\mathbf{C}_i$ is transmitted. In fact, $r_{i,j}$ is Hamming distance between $\mathbf{C}_j$ and $\mathbf{C}_i$.

Because $P(\mathbf{C}_i \to \mathbf{C}_j) = P(\mathbf{C}_j \to \mathbf{C}_i), r_{i,j} = r_{j,i}$ and $r_{i,i} = 0$, the union bound in (15) can be reduced to:

$$P_e \leq \frac{2}{(2m + l)T} \sum_{i=1}^{T-1}\sum_{j=i+1}^{T} r_{i,j} P(\mathbf{C}_i \to \mathbf{C}_j). \quad (16)$$

IV. New Upper Bound

In this section, we present a novel theorem and the corresponding proof in order to derive the new upper bound for the STBC-SM scheme.

Theorem: *Given a transmitted STBC-SM codeword matrix $\mathbf{C}_i = \mathbf{S}_p\mathbf{X}_t$ with $\mathbf{X}_t$ defined as follows:*

$$\mathbf{X}_t = \begin{bmatrix} a_t + jb_t & -c_t + jd_t \\ c_t + jd_t & a_t - jb_t \end{bmatrix} = \begin{bmatrix} x_{1,t} & -x_{2,t}^* \\ x_{2,t} & x_{1,t}^* \end{bmatrix}$$

where $x_{1,t}, x_{2,t} \in U_x$, $\mathbf{S}_p \in U_S$, $\mathbf{X}_t \in U_X$ and $t = 1, 2, ..., 2^{2m}$. Define

$$V_t = \{\mathbf{X}_k \in U_X | x_{1,k} \neq x_{1,t}, x_{2,k} \neq x_{2,t}\} \quad (17)$$

and

$$U_i = U_C - \{\mathbf{C}_j \in U_C | \mathbf{C}_j = \mathbf{S}_p\mathbf{X}_v, \mathbf{X}_v \in V_t\} \quad (18)$$

U_C is the set of the total T codeword matrices $\mathbf{C}$. The new upper bound for the bit error probability of the STBC-SM scheme is given as follows:

$$P_e \leq \frac{1}{(2m + l)T} \sum_{i=1}^{T}\sum_{\mathbf{C}_j \in U_i} r_{i,j} P(\mathbf{C}_i \to \mathbf{C}_j). \quad (19)$$

Proof: Assume that the receiver decides the signal matrix $\mathbf{C}_j = \mathbf{S}_p\mathbf{X}_k \in U_C - U_i$ when the codeword $\mathbf{C}_i = \mathbf{S}_p\mathbf{X}_t$ is transmitted, where $\mathbf{X}_k \neq \mathbf{X}_t$, i.e., $x_{1,k} \neq x_{1,t}, x_{2,k} \neq x_{2,t}$. According to (7), we can clearly observe that as both $\mathbf{C}_i$ and $\mathbf{C}_j$ have the same $\mathbf{S}_p$, the detector estimates $\mathbf{X}$ as $\mathbf{X}_k$ instead of $\mathbf{X}_t$.

In order to prove (19), we need to prove that the PEP $P(\mathbf{C}_i \to \mathbf{C}_j)$ is irrelevant in the calculation of the average error probability P_e.

Let us define:

$$\mathbf{X}_n = \begin{bmatrix} x_{1,k} & -x_{2,t}^* \\ x_{2,t} & x_{1,k}^* \end{bmatrix} \in U_X - V_t \quad (20)$$

and

$$\mathbf{X}_q = \begin{bmatrix} x_{1,t} & -x_{2,k}^* \\ x_{2,k} & x_{1,t}^* \end{bmatrix} \in U_X - V_t. \quad (21)$$

From Section III, we can see that $h_p(\mathbf{Y}, \mathbf{X}_k)$ is the minimum value among $\{h_p(\mathbf{Y}, \mathbf{X}) | \mathbf{X} \in U_X\}$ because $\mathbf{X}_k$ is the solution of (7). In order to exclude $P(\mathbf{C}_i \to \mathbf{C}_j)$ from the union bound (16), we have to prove either

$$h_p(\mathbf{Y}, \mathbf{X}_n) \leq h_p(\mathbf{Y}, \mathbf{X}_t) \quad (22)$$

or

$$h_p(\mathbf{Y}, \mathbf{X}_q) \leq h_p(\mathbf{Y}, \mathbf{X}_t). \quad (23)$$

This is equivalent to showing that:

$$\{\mathbf{Y}|h_p(\mathbf{Y}, \mathbf{X}_k)\, min\} \subset \{\mathbf{Y}|h_p(\mathbf{Y}, \mathbf{X}_n) \leq h_p(\mathbf{Y}, \mathbf{X}_t)\} \quad (24)$$

or

$$\{\mathbf{Y}|h_p(\mathbf{Y}, \mathbf{X}_k)\, min\} \subset \{\mathbf{Y}|h_p(\mathbf{Y}, \mathbf{X}_q) \leq h_p(\mathbf{Y}, \mathbf{X}_t)\}. \quad (25)$$

Similar to the proof of Verdu's theorem [2], we first investigate the following equation:

$$h_p(\mathbf{Y},\mathbf{X}_k) - h_p(\mathbf{Y},\mathbf{X}_n) - h_p(\mathbf{Y},\mathbf{X}_q) + h_p(\mathbf{Y},\mathbf{X}_t) = \\ = \sum_{i=1}^{2}|x_{i,k}-x_{i,p}|^2 - \sum_{i=1}^{2}|x_{i,n}-x_{i,p}|^2 - \\ - \sum_{i=1}^{2}|x_{i,q}-x_{i,p}|^2 + \sum_{i=1}^{2}|x_{i,t}-x_{i,p}|^2.$$

From equations (20) and (21), we can see that $x_{1,n} = x_{1,k}, x_{1,q} = x_{1,t}, x_{2,n} = x_{2,t}, x_{2,q} = x_{2,k}$. Thus, we obtain:

$$h_p(\mathbf{Y},\mathbf{X}_k) - h_p(\mathbf{Y},\mathbf{X}_n) - h_p(\mathbf{Y},\mathbf{X}_q) + h_p(\mathbf{Y},\mathbf{X}_t) = 0. \quad (26)$$

Since $h_p(\mathbf{Y},\mathbf{X}_k) \le h_p(\mathbf{Y},\mathbf{X}_q)$, or equivalently, $h_p(\mathbf{Y},\mathbf{X}_q) - h_p(\mathbf{Y},\mathbf{X}_k) \ge 0$, it follows from (26) that

$$h_p(\mathbf{Y},\mathbf{X}_t) - h_p(\mathbf{Y},\mathbf{X}_n) = h_p(\mathbf{Y},\mathbf{X}_q) - h_p(\mathbf{Y},\mathbf{X}_k) \ge 0. \quad (27)$$

This means that (22) is proved.

Similarly, (23) is proved since $h_p(\mathbf{Y},\mathbf{X}_n) - h_p(\mathbf{Y},\mathbf{X}_k) \ge 0$. Therefore, the theorem is proved. ■

Next, we will compute the number of the PEPs which is excluded from the union bound (16). According to (17), we choose $x_{1,k}, x_{2,k} \in U_x$ such that $x_{1,k} \neq x_{1,t}, x_{2,k} \neq x_{2,t}$. Since $|U_x| = 2^m$, both $x_{1,k}$ and $x_{2,k}$ have $2^m - 1$ choices. Thus, we obtain:

$$|V_t| = (2^m - 1)^2. \quad (28)$$

In addition, from (18), the cardinality of U_i is equal to:

$$|U_i| = |U_C| - |V_t| \\ = 2^{2m+l} - (2^m - 1)^2. \quad (29)$$

Therefore, the total number of PEPs to be evaluated in the new upper bound (19) is given by

$$T(T - (M-1)^2) = 2^{2m+l}(2^{2m+l} - (2^m - 1)^2). \quad (30)$$

In other words, there are a total of $2^{2m+l}(2^m - 1)^2$ PEPs to be eliminated from the union bound (16). As a consequence, when the modulation order is large, the number of excluded PEPs, increase. In such cases, we expect that the new upper bound gets tighter than the union bound.

V. Numerical Results and Comparison

In this section, we compare the new bound with the union bound and simulation results a number of STBC-SM configurations. For the sake of convenience, a MIMO system with n_T transmit antennas and n_R receive antennas is denoted as (n_T, n_R) system.

In Fig. 1, the union bound, the new upper bound and simulation results for the BER of the $(4, 1)$ STBC-SM system using 16-QAM modulation are illustrated. As we can see from Fig. 1 that both the union bound and the new bound are very close to the simulation curve in sufficiently high SNR regions. However, the new bound is closer to the simulation curve. At BER $= 10^{-2}$, the new bound is tighter than the union bound by an SNR gain of approximately 0.5 dB.

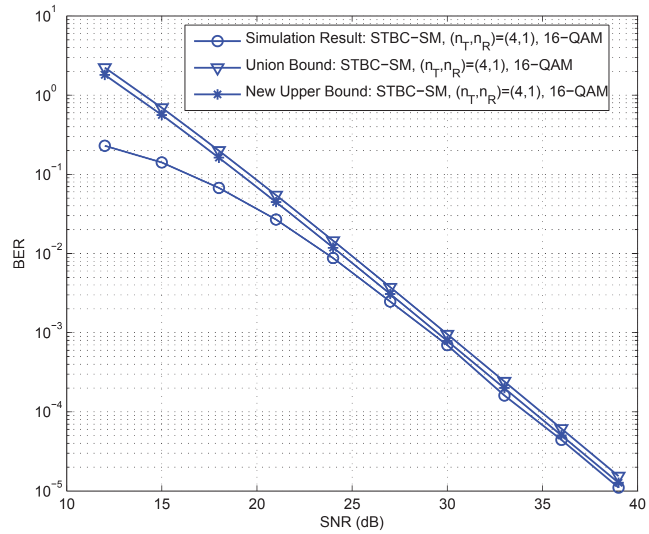

Fig. 1. The new upper bound, the union bound and simulation curve for the BER of the $(4, 1)$ STBC-SM system using 16-QAM modulation.

Fig. 2 shows the union bound, the new upper bound and simulation results for the BER of $(4, 2)$ STBC-SM system using 64-QAM modulation. We again observe from Fig. 2 that the new upper bound is closer to the simulation curve than the union bound. Specifically, at BER $= 10^{-2}$, the new bound is tighter than the union bound by approximately 1 dB in SNR.

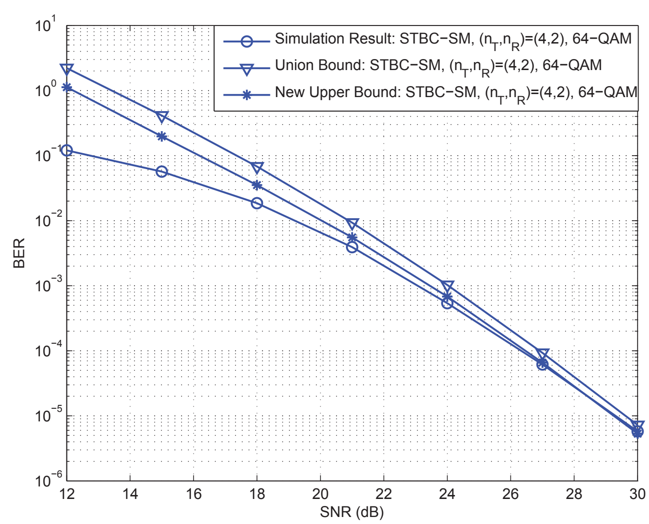

Fig. 2. The new upper bound, the union bound and simulation curve for the BER of the $(4, 2)$ STBC-SM system using 64-QAM modulation.

From both Fig. 1 and Fig. 2 we can also see that the higher the modulation order is used, the tight the new upper bound is. This confirms our expectation presented in Section IV.

VI. Conclusion

In this paper, based on the Verdu's theorem in [2], we derive a new upper bound for the bit error probability of the STBC-SM scheme. It is shown via analytical and simulation results that our new bound is not only tighter but also less computationally complex than the union bound. It is worth emphasizing that the new upper bound can also be applied to the high-rate STBC-SM in [7] or the SM-OSTBC in [3]. However, the new upper bound is still loose in the low SNR regions and not remarkably tighter than the union bound when l is comparable to m. These issues are left for our future work.

REFERENCES

[1] E. Basar, U. Aygolu, E.Panayrc, and H. V. Poor, "Space-time block coded spatial modulation," *IEEE Trans. Commun.*, vol. 59, no. 3, pp. 823-832, Mar. 2011.

[2] S. Verdu, "Maximum likelihood sequence detection for intersymbol interference channel : A new upper bound on error probability," *IEEE Trans. Inform. Theory.*, vol. IT-33, no. 1, pp. 62-68, Jan. 1987.

[3] Minh-Tuan Le, Vu-Duc Ngo, Hong-Anh Mai, Xuan Nam Tran, and Marco Di Renzo, "Spatial Modulated Orthogonal Space-Time Block Codes with Non-Vanishing Determinants" *IEEE Trans. Commun.*, vol. 62, no. 1, pp. 85-99, Jan. 2014.

[4] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1988.

[5] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228-2241, July 2008.

[6] Vahid Tarokh, Nambi Seshadri, and A. R. Calderbank, "Space-Time codes for high data rate wireless communication: performance criterion and code construction" *IEEE Trans.Inf Theory*, vol. 44, no. 2, pp. 744-765, Mar. 1998.

[7] Minh-Tuan Le, Vu-Duc Ngo, Hong-Anh Mai, and Xuan Nam Tran, "High rate space time block coded spatial modulation," in *Proc. 2012 Int. Conf. Advanced Technol. Commun.*, pp. 278-282.

[8] Vu-Duc Ngo, Hae-Wook Choi, and Sin-Chong Park, "An Expurgated Union Bound for Space-Time Code Systems," *LNCS*, vol. 3124, pp. 156-162, 2004.

[9] E. Biglieri, "Expurgating the Union Bound to Error Probability: A Generalization of the Verdu-Shields Theorem," *ISIT 1997, Ulm, Germany*, June 29 July 4.

[10] M. K. Byun and B. G. Lee, "New bounds of pairwise error probability for space-time codes in Rayleigh fading channels" in *Proc. Wireless Commun. Netw. Conf.*, pp. 17-21, Mar. 2002.

[11] H. A. Mai, T. C. Dinh, X. N. Tran, M. T. Le, and V. D. Ngo, "A novel spatially-modulated orthogonal space-time block code for 4 transmit antennas," *2012 IEEE Int. Symposium Signal Process. Inform. Technol.*

[12] Rachard E.Blahut, "Digital Transmission of Information," *Addison-Wesley Publishing Company 1990.*