Interval Type-2 Fuzzy C-Means Approach to Collaborative Clustering

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***Abstract*—There have been numerous studies on using the FCM algorithm in clustering and collaboration clustering, especially in data analysis, data mining and pattern recognition. In this study, we present new methods involving interval Type-2 fuzzy sets to realize collaborative clustering. Data in which the clustering results realized at one data site impact clustering carried out at other data sites. Those methods endowed with interval type-2 fuzzy sets help cope with uncertainties present in data. The experiment with weather data sets has shown better results in comparison with the previous approaches.**

***Index Terms*—Fuzzy clustering; Collaborative Clustering; Fuzzy C-Means; Type-2 Fuzzy Sets; Cluster Validity Measures**

I. Introduction

Clustering is used to detect a sound structure or patterns in the data set, in which objects positioned within the cluster level data show a substantial level of similarity. This unsupervised technique has a long history in machine learning, pattern recognition, data mining, and many algorithms have been exploited in various applications. Clustering algorithms comes in numerous varieties including k-means and its variants [1]-[2], and a family of Fuzzy C-Mean (FCM) [3]-[5].

Pattern recognition comes with various facets of uncertainty that have be appropriately managed in real-world data. Type-2 fuzzy sets (T2 FSs) have received increased research interest over the past decade, primarily due to their potential to model aspects of uncertainty. This has led to the extension of several type-1 fuzzy clustering algorithms to Interval Type-2 Fuzzy C-Means (IT2FCM) and Interval Type-2 Possibilistic C-Means (IT2PCM) [7]-[15],[32]-[34] or some combinations of IT2 FCM and IT2PCM [16]. [30] also presented hybrid clustering algorithms by combination of IT2FCM and multiple kernels methods to enhance data classification. Sanchez et al. proposed methods for information granule formation via the concept of uncertainty-basedinformation with Interval Type-2 Fuzzy Sets [32]

Collaborative fuzzy c-means clustering was introduced by Pedrycz [17]-[20] as a vehicle to determine a structure and reveal similarity among separate data sets. There are two essential characteristics of collaborative data clustering. The first one is that the individual data cannot be transferred. Second, we can only exchange findings about the structure. Through the process of interaction in this manner, the results obtained at one site can impact clustering realized at other data site [17]-[18]. In the sequel, Coletta et al [20] extend Pedryczs method to optimize the parameters including the interaction level for all pairs of peers and the number of clusters at each place. In [23] when data sets described by multiple views, with each view having its own characterization of the data to be clustered take this advantage by applying Collaborative fuzzy c-means clustering so that we combine individual views coming from multiple clustering. M. Prasad [24] overcome some of the drawbacks of the method of Pedryczs method by introducing preprocessing phase before running collaborative phase. Zhou et al [22] proposed a novel collaborative clustering algorithm over a distributed P2P network. This algorithm searches the optimized clusters at each data site by collaborating only with prototypes of the neighboring data site. The clustering solution could be improved by applying partial supervision, which involves a subset of labeled data augmented with their class membership. This knowledge-based hints have to be included into the objective function and reflect a fact that some patterns have been labeled [19]-[25].

Another development of data collaboration clustering research is to combine the advantages of fuzzy sets and rough sets was presented by by S. Mitra et al [29]. Fu, , Tang and Cai focused on Horizontal Collaborative Fuzzy Clustering with spatial attributes data collection. The identification of data site and regional data is involved in realizing collaboration with the use of some threshold level of the membership function or entropy-based approach [25]-[26]. The assessment of the level of collaboration can be determined based on the similarity of the data between the cluster regions. The corresponding coefficient can be decided in advance or automatically adjusted after each phase of collaboration [20]-[25]. Some research was completed to handle the problem of identifying the number of clusters based on an assessment of clustering results [20]-[25] and eventually adjusting the number of clusters, remove clusters based density and the size of the cluster. Yu et al [27] presented a new approach to implementing horizontal collaborative fuzzy clustering with the knowledge provided by the prototypes instead of partition matrixes.

Collaborative fuzzy clustering algorithms using type 1 fuzzy set are not able to handle uncertainty and noise found during the process of data clustering. Uncertainty related to the input data themselves (e.g., clustering heterogeneous input data such as real numbers, intervals, and linguistic terms), it also related to the interpretation of the computed result. The collaboration has examined the impact of the data on the content area of a sector of data that can be developed in the direction of using fuzzy type 2 [19], but this is a new direction. The paper presents a new method that applies Interval Type-2 Fuzzy sets to collaborative clustering with intent to cope with uncertainty.

The paper is organized as follows: Section II offers a brief introduction to type 2 fuzzy set, interval type 2 fuzzy set and collaborative clustering; Section III proposes interval type 2 collaborative fuzzy clustering; Section 4 show the experimental results. Conclusion and future studies are covered in Section V.

II. Background

A. Type-2 Fuzzy Sets

A type-2 fuzzy set in X is denoted $\tilde{A}$, and its membership grade of $x \in X$ is $\mu_{\tilde{A}}(x,u), u \in J_x \subseteq [0,1]$, which is a type-1 fuzzy set in [0, 1]. The elements of domain of $\mu_{\tilde{A}}(x,u)$ are called primary memberships of x in $\tilde{A}$ and memberships of primary memberships in $\mu_{\tilde{A}}(x,u)$ are called secondary memberships of x in $\tilde{A}$.

Definition 2.1: A $type-2\ fuzzy\ set$, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i. e. ,

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u))|\forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (1)$$

or

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u))/(x,u), J_x \subseteq [0,1] \quad (2)$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$, see [1], [2], [31].
At each value of x, say $x = x'$, the 2-D plane whose axes are u and $\mu_{\tilde{A}}(x',u)$ is called a $vertical\ slice$ of $\mu_{\tilde{A}}(x,u)$. A *secondary membership function* is a vertical slice of $\mu_{\tilde{A}}(x,u)$. It is $\mu_{\tilde{A}}(x=x',u)$ for $x \in X$ and $\forall u \in J_{x'} \subseteq [0,1]$, i. e.

$$\mu_{\tilde{A}}(x=x',u) \equiv \mu_{\tilde{A}}(x') = \int_{u\in J_{x'}} f_{x'}(u)/u, J_{x'} \subseteq [0,1] \quad (3)$$

in which $0 \le f_{x'}(u) \le 1$. In manner of embedded fuzzy sets, a type-2 fuzzy sets is union of its type-2 embedded sets, i. e

$$\tilde{A} = \sum_{j=1}^{n} \tilde{A}_e^j \quad (4)$$

where $n \equiv \prod_{i=1}^{N} M_i$ and $\tilde{A}_e^j$ denoted the j^{th} type-2 embedded set of $\tilde{A}$, i. e. ,

$$\tilde{A}_e^j \equiv \{\left(u_i^j, f_{x_i}(u_i^j)\right), i = 1,2,...,N\} \quad (5)$$

where $u_i^j \in \{u_{ik}, k = 1,...,M_i\}$. Type-2 fuzzy sets are called an interval type-2 fuzzy sets [3] if the secondary membership function $f_{x'}(u) = 1\ \forall u \in J_x$ i. e. a type-2 fuzzy set are defined as follows:

Definition 2.2: An *interval type-2 fuzzy set* $\tilde{A}$ is characterized by an interval type-2 membership function $\mu_{\tilde{A}}(x,u) = 1$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i. e. ,

$$\tilde{A} = \{((x,u),1)|\forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (6)$$

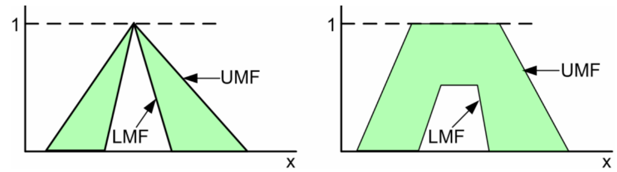

Fig. 1. The membership function of an interval type 2 fuzzy set [4]

Uncertainty of $\tilde{A}$, denoted FOU, is union of primary functions i. e. $FOU(\tilde{A}) = \bigcup_{x\in X} J_x$. Upper/lower bounds of membership function (UMF/LMF), denoted $\overline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$, of $\tilde{A}$ are two type-1 membership function and bounds of FOU which is limited by two membership functions of an type-1 fuzzy set are UMF and LMF (see Fig.1).

B. Interval Type-2 Fuzzy C-Means Clustering

In general, fuzzy memberships in interval type-2 fuzzy C means algorithm (IT2FCM) [8] is achieved by computing the relative distance among the patterns and cluster centroids. Hence, to define the interval of primary membership for a pattern, we define the lower and upper interval memberships using two different values of m . In (7), (8) and (9), m_1 and m_2 are fuzzifiers which represent different fuzzy degrees. We define the interval of a primary membership for a pattern, as the highest and lowest primary membership for a pattern. These values are denoted by upper and lower membership for a pattern, respectively. IT2-FCM is extension of FCM clustering by using two fuzziness parameters m_1, m_2 to make FOU, corresponding to upper and lower values of fuzzy clustering. The use of fuzzifiers gives different objective functions to be minimized as follows:

$$\begin{cases} J_{m_1}(U,v) = \sum_{k=1}^{N}\sum_{i=1}^{C}(u_{ik})^{m_1} d_{ik}^2 \\ J_{m_2}(U,v) = \sum_{k=1}^{N}\sum_{i=1}^{C}(u_{ik})^{m_2} d_{ik}^2 \end{cases} \quad (7)$$

in which $d_{ik} = \| x_k - v_i \|$ is Euclidean distance between the pattern x_k and the centroid v_i, C is number of clusters and N is number of patterns. Upper/lower degrees of membership, $\overline{u}_{ik}$ and $\underline{u}_{ik}$ are determined as follows:

$$\overline{u}_{ik} = \begin{cases} \dfrac{1}{\sum\limits_{j=1}^{C}\left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m_1-1)}} & if\ \dfrac{1}{\sum\limits_{j=1}^{C}\left(\frac{d_{ik}}{d_{jk}}\right)} < \dfrac{1}{C} \\ \dfrac{1}{\sum\limits_{j=1}^{C}\left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m_2-1)}} & otherwise \end{cases} \quad (8)$$

$$\underline{u}_{ik} = \begin{cases} \dfrac{1}{\sum\limits_{j=1}^{C} \left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m_1-1)}} & if \ \dfrac{1}{\sum\limits_{j=1}^{C} \left(\frac{d_{ik}}{d_{jk}}\right)} \geq \dfrac{1}{C} \\ \dfrac{1}{\sum\limits_{j=1}^{C} \left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m_2-1)}} & otherwise \end{cases} \tag{9}$$

in which $i = \overline{1,C}$, $k = \overline{1,N}$. Because each pattern has membership interval as the upper $\overline{u}$ and the lower $\underline{u}$, each centroid of cluster is represented by the interval between v^L and v^R. Cluster centroids is computed in the same way of FCM as follows:

$$v_i = (\sum_{k=1}^{N} (u_{ik})^m x_k)/(\sum_{k=1}^{N} (u_{ik})^m) \tag{10}$$

in which $i = \overline{1,C}$. After obtaining v_i^R, v_i^L, type-reduction is applied to get centroid of clusters as follows:

$$v_i = (v_i^R + v_i^L)/2 \tag{11}$$

For membership grades:

$$u_i(x_k) = (u_i^R(x_k) + u_i^L(x_k))/2, j = 1, ..., C \tag{12}$$

in which

$$u_i^L = \sum_{l=1}^{M} u_{il}/M, u_{il} = \begin{cases} \overline{u}_i(x_k) & \text{if } x_{il} \text{ uses } \overline{u}_i(x_k) \text{ for } v_i^L \\ \underline{u}_i(x_k) & otherwise \end{cases} \tag{13}$$

$$u_i^R = \sum_{l=1}^{M} u_{il}/M, u_{il} = \begin{cases} \overline{u}_i(x_k) & \text{if } x_{il} \text{ uses } \overline{u}_i(x_k) \text{ for } v_i^R \\ \underline{u}_i(x_k) & otherwise \end{cases} \tag{14}$$

Next, defuzzification for IT2FCM is made as if $u_i(x_k) > u_j(x_k)$ for $j = 1, ..., C$ and $i \neq j$ then x_k is assigned to cluster i.

C. Collaborative Fuzzy Clustering

Suppose there is "P" data sets $D[1], D[2], ..., D[P]$, which contains $N[1], N[2], ..., N[P]$ patterns data defined in the same feature space X. Each data site we group all patterns into "c" clusters. Clustering results in every area of data back to the clustering effect in the rest area, we call this process is collaborative and collaborative clustering.

The objective function for each data site when using the standard FCM algorithm comes in the well-known form

$$\sum_{k=1}^{N[ii]} \sum_{i=1}^{C} (u_{ik})^2[ii](d_{ik})^2 \tag{15}$$

with $ii = 1, 2, ..., P$.

The collaboration among each data site is done with other data site. The intensity of the interaction is described by factor β. To accommodate the collaboration effect in the optimization process, the objective function is extended into the form

$$Q_[ii] = \sum_{k=1}^{N[ii]} \sum_{i=1}^{C} (u_{ik})^2[ii](d_{ik})^2 + \beta \sum_{jj=1}^{P} \sum_{k=1}^{N[ii]} \sum_{i=1}^{C} (u_{ik}[ii] - (u_{ik}) \ [ii|jj])^2 (d_{ik})^2 \tag{16}$$

In the above formula, the first part is the "standard" objective function of the FCM algorithm. The second part reflects the impact of structural clustering of data from other sites. Among them $(u_{ik}) \ [ii|jj]$ is called the induced matrix caused by the impact of the data site ii to data site jj, calculated by the following formula:

$$(u_{ik}) \ [ii|jj] = 1/\sum_{j=1}^{C} (|x_k[ii] - v_i[jj]|/|x_k[ii] - v_j[jj]|)^2 \tag{17}$$

The collaborative clustering problem is converted to the optimization problem with the following membership constraints:

$MinQ[ii]$

s.t. $U[ii] \in U$

where U is a family of all fuzzy partition matrices,namely $U = u_k[ii] \in [0,1], \sum_{i=1}^{C} u_{ik} = 1 \forall k$ and $0 < \sum_{k=1}^{N[ii]} u_{ik}[ii] < N[ii] fori$

Using the Lagrange method to optimize the objective function, we find the matrix u and the prototypes v as the follows:

$$u_{rs} = \frac{1}{\sum_{j=1}^{C} \frac{d_{rs}^2}{d_{js}^2}} [1 - \sum_{j=1}^{C} \frac{\beta \sum_{jj=1, jj\neq ii}^{P} u_{js}[ii|jj]}{1 + \beta(P-1)}] + \frac{\beta \sum_{jj=1, jj\neq ii}^{P} u_{rs}[ii|jj]}{1 + \beta(P-1)} \tag{18}$$

$$v_{rt} = \frac{\sum\limits_{k=1}^{N[ii]} u_{rk}^2 x_{kt} + \beta \sum\limits_{jj=1, jj\neq ii}^{P} \sum\limits_{k=1}^{N[ii]} c_{rk} x_{kt}}{\sum\limits_{k=1}^{N[ii]} u_{rk}^2 + \beta \sum\limits_{jj=1, jj\neq ii}^{P} \sum\limits_{k=1}^{N[ii]} c_{rk}} \tag{19}$$

in which $c_{rk} = (u_{rk}[ii] - (u_{rk}) \ [ii|jj])^2$.

The parameter $\beta[ii|jj]$ denotes the level of collaboration between the data site ii and jj, β becomes greater, the more level of collaboration and vice versa. When the data sites similar structure, the level of collaboration will be greater or higher β values. The value of β can be acquired from experts given or calculated based on the similarity of the structure of the site. In collaborative model, the prototypes $v[jj]$ were send from data site jj to data site ii and we can compute the value of the induced objective function:

$$J \ [ii|jj] = \sum_{k=1}^{N[ii]} \sum_{j=1}^{C} (u_{ik}) \ ^2[ii|jj])|x_k - v_i[jj]|^2 \tag{20}$$

The interaction level $\beta[ii|jj]$ between two data sites ii and jj, at the collaboration stage, can be defined as

$$\beta[ii|jj] = min[1, \frac{J[ii]}{J\ [ii|jj]}] \quad (21)$$

Subsequently, the membership function and cluster centroid matrix are calculated as the equations (22) and (23).

$$u_{rs} = \frac{1}{\sum_{j=1}^{C} \frac{d_{rs}^2}{d_{js}^2}}[1 - \sum_{j=1}^{C} \sum_{jj=1, jj \neq ii}^{P} \frac{\beta[ii|jj] u_{js}[ii|jj]}{1 + \beta[ii|jj](P-1)}] + \sum_{jj=1, jj \neq ii}^{P} \frac{\beta[ii|jj] u_{rs}[ii|jj]}{1 + \beta[ii|jj](P-1)} \quad (22)$$

$$v_{rt} = \frac{\sum_{k=1}^{N[ii]} u_{rk}^2 x_{kt} + \sum_{jj=1, jj \neq ii}^{P} \sum_{k=1}^{N[ii]} b_{rk} x_{kt}}{\sum_{k=1}^{N[ii]} u_{rk}^2 + \sum_{jj=1, jj \neq ii}^{P} \sum_{k=1}^{N[ii]} b_{rk}} \quad (23)$$

in which $b_{rk} = \beta[ii|jj](u_{rk}[ii] - (u_{rk})\ [ii|jj])^2$.

III. Collaborative IT2 Fuzzy Clustering

A. Collaborative IT2 Fuzzy Clustering

In order to solve the case of the number of clusters on the data sites are different. Before each phase of collaboration, we calculate v as the new prototypes from the prototypes communicated by all remaining data sites, the number of items are the same with number of cluster of data site ii. Besides, in an effort to prove the noise reduction and uncertainty. We generalize the objective function as following form:

$$Q_[ii] = \sum_{k=1}^{N[ii]} \sum_{i=1}^{C} (u_{ik})^m [ii] (d_{ik})^2 + \beta \sum_{k=1}^{N[ii]} \sum_{i=1}^{C[ii]} u_{ik}^m (v_i[ii] - v_i\,[ii])^2 \quad (24)$$

The minimization of $Q[ii]$ is carried out with respect to the fuzzy partition $U[ii]$ and the prototypes, $v_i[ii]$. The distance d_{ik} concerns the k^{th} data (pattern) in $D[ii]$ and the i^{th} prototype, $d_{ik}^2 = \sum_{j=1}^{n} (x_{kj} - v_{ij}[ii])^2$, $v^{\sim}$ is the new prototype determined in form:

$$v_i^{\sim}[ii] = \frac{\sum_{jj=1, jj \neq ii}^{P} \beta[ii|jj] v_i^{\sim}[ii|jj]}{\sum_{jj=1, jj \neq ii}^{P} \beta[ii|jj]} \quad (25)$$

with $v_i^{\sim}[ii|jj] = \{v_k | min(\sum_{j=1}^{n} (v_{ij}[ii] - v_{kj}[jj])^2) | k = 1, ..., C\}$.

The factor β is an average of $\beta[ii|jj]$, the interaction level $\beta[ii|jj]$ between two data sites ii and jj, at a given collaboration stage, can be defined as:

$$\beta = \frac{\sum_{jj=1, jj \neq ii}^{P} \beta[ii|jj]}{(P-1)}$$
$$\beta[ii|jj] = min\{1, fracJ[ii|jj] J^{\sim}[ii|jj]\}$$
$$J^{\sim}[ii|jj] = \sum_{k=1}^{N[ii]} \sum_{j=1}^{C[jj]} u_{ik}^2[ii|jj](x_k - v^{\sim}[jj])^2$$
$$u_{ik}^2[ii|jj] = \frac{1}{\sum_{j=1}^{C[jj]} \frac{x_k[ii] - v_i[ii]}{x_k[ii] - v_i[ii]}} \quad (26)$$
$$v^{\sim}[jj] = \frac{\sum_{j=1}^{C[jj]} v_j}{c[jj]}$$

We confine ourselves to the use of the technique of Lagrange multipliers. For any data point k, $k = 1, 2, ..., N[ii]$, we reformulate the objective function to be in the form:

$$V_[ii] = \sum_{k=1}^{N[ii]} \sum_{i=1}^{C} (u_{ik})^m [ii] (d_{ik})^2 + \beta \sum_{k=1}^{N[ii]} \sum_{i=1}^{C[ii]} u_{ik}^m (v_i[ii] - v_i\,[ii])^2 - \lambda (\sum_{i=1}^{C[ii]} u_{ik}[ii] - 1) \quad (27)$$

The necessary conditions for the minimum of $V[ii]$ are expressed as $\frac{\partial V}{\partial u_{rs}} = 0$ with $r = 1, 2, ..., c; s = 1, 2, ..., N[ii]$. After computing the derivative with respect to the elements of the partition matrix we have

$$\frac{\partial V}{\partial u_{rs}} = m u_{rs}^{m-1}[ii] d_{rs}^2 + m \beta u_{rs}^{m-1} (v_r[ii] - v_r\,[ii])^2 - \lambda$$

$$u_{rs} = [\frac{\lambda}{m(d_{rs}^2 + \beta(v_r[ii] - v_r\,[ii])^2)}]^{1/(m-1)} \quad (28)$$

Given the constraint in the form $\sum_{j=1}^{C[ii]} u_{js}[ii] = 1$ we obtain

$$\sum_{j=1}^{C[ii]} [\frac{\lambda}{m(d_{rs}^2 + \beta(v_r[ii] - v_r\,[ii])^2)}]^{1/(m-1)} = 1 \quad (29)$$

$$\lambda = \frac{m}{[\frac{1}{m(d_{rs}^2 + \beta(v_r[ii] - v_r\,[ii])^2)}]^{1/(m-1)}}^{m-1} \quad (30)$$

By plugging 40 into 39 one has:

$$u_{rs} = \frac{1}{[m(d_{rs}^2 + \beta(v_r[ii] - v_r^{\sim}[ii])^2)][\frac{1}{m(d_{rs}^2 + \beta(v_r[ii] - v_r^{\sim}[ii])^2)}]^{\frac{1}{m-1}}} \quad (31)$$

Proceeding with the optimization of the objective function with regard to the prototypes, we consider now the Euclidean distance. Given the form of the distance, let us rewrite the objective function in an explicit manner.

$$Q_{[}ii] = \sum_{k=1}^{N[ii]} \sum_{i=1}^{C[ii]} (u_{ik})^m [ii] \sum_{j=1}^{n} (x_{kj} - v_{ij}[ii])^2 + \beta \sum_{k=1}^{N[ii]} \sum_{i=1}^{C[ii]} u_{ik}^m (v_i[ii] - v_i[ii])^2 \quad (32)$$

$$\frac{\partial Q}{\partial v_{rt}} = 0 \quad (33)$$

$$-2 \sum_{k=1}^{N[ii]} (u_{rk})^m (x_{kt} - v_{rt}[ii]) + 2\beta \sum_{k=1}^{N[ii]} (u_{rk})^m (v_{rt}[ii] - v_{rt}[ii]) = 0 \quad (34)$$

$$\sum_{k=1}^{N[ii]} x_{kt} - \sum_{k=1}^{N[ii]} v_{rt}[ii] - \beta \sum_{k=1}^{N[ii]} v_{rt}[ii] + \beta \sum_{k=1}^{N[ii]} v_{rt}[ii] = 0 \quad (35)$$

$$v_{rt} = \frac{\sum_{k=1}^{N[ii]} (u_{rk})^m x_{kt} + \beta \sum_{k=1}^{N[ii]} (u_{rk})^m v_{rt}[ii]}{1 + \beta) \sum_{k=1}^{N[ii]} (u_{rk})^m} \quad (36)$$

We consider two different parameter m_1, m_2 with the proposed membership functions, the parameters responsible for the width of uncertainty and the parameters responsible for the center and the support of the proposed membership function are decoupled from each other in the interval type-2 membership function, such that:

$$\overline{u_{rs}} = \frac{1}{[m(d_{rs}^2 + \beta(v_r[ii] - v_r[ii])^2)][\frac{1}{m(d_{rs}^2 + \beta(v_r[ii] - v_r[ii])^2)}]^{\frac{1}{m_1 - 1}}} \quad (37)$$

$$\underline{u_{rs}} = \frac{1}{[m(d_{rs}^2 + \beta(v_r[ii] - v_r[ii])^2)][\frac{1}{m(d_{rs}^2 + \beta(v_r[ii] - v_r[ii])^2)}]^{\frac{1}{m_2 - 1}}} \quad (38)$$

The membership function value is calculated as:

$$u_{rs} = \frac{\overline{u_{rs}} + \underline{u_{rs}}}{2} \quad (39)$$

$$\overline{v_{rt}} = \frac{\sum_{k=1}^{N[ii]} (u_{rk})^m 1 x_{kt} + \beta \sum_{k=1}^{N[ii]} (u_{rk})^m 1 v_{rt}[ii]}{1 + \beta) \sum_{k=1}^{N[ii]} (u_{rk})^m 1} \quad (40)$$

$$\underline{v_{rt}} = \frac{\sum_{k=1}^{N[ii]} (u_{rk})^m 2 x_{kt} + \beta \sum_{k=1}^{N[ii]} (u_{rk})^m 2 v_{rt}[ii]}{1 + \beta) \sum_{k=1}^{N[ii]} (u_{rk})^m 2} \quad (41)$$

and the centroid of cluster is calculated as follows:

$$v_{rt} = \frac{\overline{u_{rt}} + \underline{u_{rt}}}{2} \quad (42)$$

B. Collaborative IT2 Fuzzy Clustering Algorithrm

The essence of collaborative clustering is to explore the structures of each data site through prototype exchanges. There are two main phases, clustering data in each site by the clustering algorithm known as IT2FCM and implementation of re-clustering based on the collaboration of the data clustering results from phase 1. A general view of the processing of the proposed collaborative clustering algorithm is shown in Fig. 1

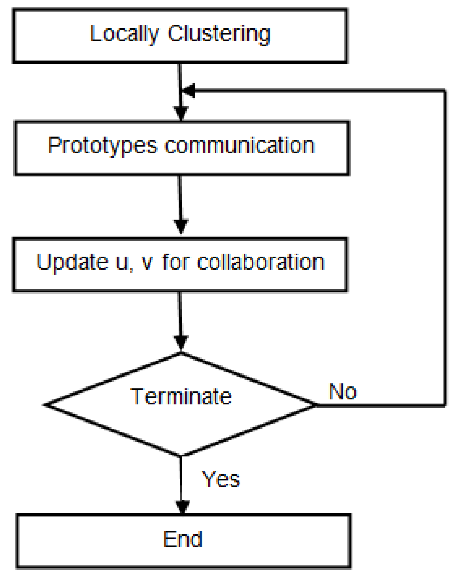

Fig. 2. The block diagram of the overall collaborative clustering algorithm.

Initially, the FCM-type algorithm is performed independently at each data site with its optimization pursuits by focusing on the local data then the prototypes of each data site is broadcasted to all other data sites. In each collaborative step, the prototype and membership matrix of each data site is recalculate and optimize from others until the end condition is matched.

CIT2FCM Clustering Algorithm

Input: the number of data site P, the number of item in each data site ii is N[ii], the number of cluster in each data site ii is c[ii], the number of attribute of data item is n, the data item in each data site X[ii] Ouput: Accuracy of the clustering.

Phase 1: locally clustering

Run IT2FCM for each data site

Phase 2: collaboration

Repeat

Communicate cluster prototypes from each data site to all others

Foreach data site D[ii]

Compute induced partition matrices

Repeat

Compute local partition matrices u by (35,36,37) or (50,51,52)

Compute local cluster prototypes v by (38,39,40) or (53,54,55)

Until the objective function is minimized

End for

Until Cluster prototype does not significantly change between two consecutive iterations

C. Validity Measures for Collaborative Fuzzy Cluster

1) Fuzzy Silhouette Width Criterion for collaborative clustering: a. Average Silhouette Width Criterion for collaborative clustering

We consider an object $j \in 1, 2, ..., N$ belonging to cluster $r \in 1, ..., c$ then the silhouette of object j is defined as follows:

$$S_j = \frac{b_{rj} - a_{rj} - d_{rj}}{max(b_{rj}, a_{rj})} \tag{43}$$

Among them a_{rj} is the average distance of the element j to all elements in the cluster r, d_{qj} is the average distance of the element j to the all elements in the cluster q, b_{rj} is the smallest value of q where $q \neq r$ or b_{rj} is the average distance to the closest cluster of cluster r. d_{ik} is the distance between element j to v_{ij} which is calculated by 3.2 and is the average of centroid of nearest cluster of cluster r in each remain data sites.

$d_{ik} = \sqrt{\sum_{j=1}^{n} (x_{kj} - v_{ij}[ii])^2}$

Obviously the larger value of S_j, in other words the value of a_{rj} and greater, the smaller value of b then the element j assigned to cluster r are more reasonable.

b. Fuzzy Silhouette Criterion

Follows [17-1], the generalized silhouette criterion for one data site ii have been defined as follows:

$$FS[ii] = \frac{\sum_{j=1}^{N} (u_{rj} - u_{qj}) S_j}{\sum_{j=1}^{N} (u_{rj} - u_{qj})} \tag{44}$$

Where u_{rj} and u_{qj} is the largest and second element in column j of the membership matrix or 2 highest membership functions of element j.

The global silhouette criterion for all data site is:

$$FS = \sum_{ii=1}^{P} FS[ii] \tag{45}$$

2) Fuzzy Sum of Squared Error (FSSE) for collaborative clustering: The one of simplest and most widely used criterion measure for clustering is Sum of Squared Error (SSE). It is defined as:

$$SSE = \sum_{k=1}^{C} \frac{1}{N_k} \sum_{\forall x_i \in C_k} |x_i - v_k|^2 \tag{46}$$

Where c is the number of clusters, N_k is the number of element in k^{th} cluster and v_k is the centroid of k^{th} cluster.

In collaborative fuzzy clustering, the probability of object i belongs to cluster k is u_{ik} and x_i as close to v_k as possible then the element x_i assigned to cluster k are more reasonable, so SSE criterion for collaborative fuzzy clustering (FSSE) of data site ii be rewritten as:

$$FSSE[ii] = \sum_{k=1}^{C[ii]} \frac{1}{N_k} \sum_{\forall x_i \in C_k} u_{ik} [|x_i - v_k|^2 + |x_i - v_k|^2] \tag{47}$$

The global FSSE criterion for all data site is:

$$FSSE = \sum_{ii=1}^{P} FSSE[ii] \tag{48}$$

IV. Experiments

To evaluate the performance of the proposed algorithm CIT2FCM, CFCM[26] clustering algorithm are chosen for comparative analysis and we used two measures in the session 3 for the quantitative assessment of two above algorithms and following datasets are used in our examples: Canadian weather energy and engineering data sets, experimental studies used parameters $m_1 = 2$ and $m_2 = 3$.

That is computer data sets of hourly weather conditions occurring at 145 Canadian locations for up to 48 years of records, starting as early as 1953, and ending for most locations in 2001. The primary purpose of these files is to provide long term weather records for the use in urban planning, siting and designing of wind and solar renewable energy systems, and designing of energy efficient buildings.

The study used Dry bulb temperature and Dew point temperature for clustering and each station data for one data site, 4 data sites and 4 stations respectively are chosen: CowleyA, EdmontonStonyPlain, EdsonA and FortChipewyanA.

Table I and II shows results of clustering algorithms CFCM and CIT2FCM and the evaluation results of the FS and FSSE are presented in the tables III and IV. The efficient algorithms have larger FS value and smaller FSSE value is CIT2FCM 1 and 2.

TABLE I
RESULTS OF CFCM ALGORITHM

No	CowleyA	EdmontonStonyPlain	EdsonA	FortChipewyanA
Class 1	18.15,3.83	19.98,6.83	16.41,7.06	14.43,6.01
Class 2	2.88,-4.26	0.82,-4.51	0.13,-5.44	-0.97,-5.36
Class 3	-4.06,-8.67	-11.81,-14.98	-12.70,-16.11	-18.74,-22.19

TABLE II
RESULTS OF CIT2FCM ALGORITHM

No	CowleyA	EdmontonStonyPlain	EdsonA	FortChipewyanA
Class 1	18.63,4.05	20.48,7.11	16.70,7.28	14.80,6.29
Class 2	3.84,-3.77	1.57,-3.99	0.78,-5.00	-0.41,-4.88
Class 3	-4.05,-8.68	-12.82,-15.95	-14.03,-17.37	-20.49,-24.06

V. Conclusion

In this study, we reviewed and discussed some algorithms for collaborative fuzzy clustering. We have developed the idea of clustering collaboration by introducing fuzzy type 2

TABLE III
COLLABORATIVE FUZZY SILHOUETTE CRITERION FOR WEATHER DATA SETS

No	Data site 1	Data site 2	Data site 3	Data site 4	FS
CFCM	2.23	3.43	3.31	3.33	12.31
CIT2FCM	3.21	2.99	2.98	3.88	13.06

TABLE IV
COLLABORATIVE FUZZY SUM OF SQUARED ERROR FOR WEATHER DATA SETS

No	Data site 1	Data site 2	Data site 3	Data site 4	FSSE
CFCM	96.24	143.35	100.26	150.52	490.37
CIT2FCM	108.40	128.54	115.72	118.82	471.48

which improved results of clustering and helped overcome the drawbacks of the conventional collaborative clustering algorithms. Also, we have developed validity measures for collaborative fuzzy clustering to compare results supplied by different methods of collaborative clustering. The experimental results demonstrated that in the experimental, the FS and FSSE yield better results that the CFCM.

The next goal is extension of various algorithms related to collaborate clustering and type-2 fuzzy sets, how to apply the proposed algorithms into other data classification such as satellite image classification.

REFERENCES

[1] T. Kanungo, D. M. Mount, N. S. Netanyahu, C. D. Piatko, R. Silverman, A. Y. Wu, An Efficient k-Means Clustering Algorithm: Analysis and Implementation, IEEE Trans. On Pattern Analysis and Machine Intelligence, Vol 24(7), pp 881-893, 2002.

[2] K.R. Zalik, An efcient k-means clustering algorithm, Pattern Recognition Letters Vol. 29, pp 1385 - 1391, 2008.

[3] J. Bezdek, Pattern Recognition with Fuzzy Objective Function Algorithms. New York: Plenum, 1981.

[4] James C.Bezdek, Robert Ehrlich, William Full, FCM: The Fuzzy c-Means Clustering Algorithm, Computers and Geosciences Vol. 10, No. 2-3, pp 191-203, 1984.

[5] J. C. Bezdek, Pattern Recognition With Fuzzy Objective Function Algorithms. Norwell, MA: Kluwer, 1981.

[6] Pedrycz W.; Izakian H, Cluster-Centric Fuzzy Modeling, IEEE transactions on fuzzy systems, Vol. 22, NO. 6, pp 1585-1597, 2014.

[7] C. Hwang and F. C. Rhee, Uncertain Fuzzy Clustering: Interval Type-2 Fuzzy Approach to C-Means, IEEE Trans. on Fuzzy Systems, Vol. 15, pp 107-120, 2007.

[8] Jerry M. Mendel: Advances in type-2 fuzzy sets and systems, Information Sciences, vol. 177, pp 84-110, 2007.

[9] Jerry M. Mendel, Robert I. John, Feilong Liu, Interval Type-2 Fuzzy Logic Systems Made Simple, Trangsactions on Systems, Vol. 14, pp 808-821.

[10] N. Karnik, J.M. Mendel and Liang Q.: Type-2 Fuzzy Logic Systems,IEEE Trans. on Fuzzy Systems, pp. 643-658, 1999.

[11] N. Karnik and J.M. Mendel.: Centroid of a Type-2 Fuzzy Set, Information Sciences, 132, pp. 195-220, 2001.

[12] Liang Q. and J.M. Mendel. Interval Type-2 Fuzzy Logic Systems: Theory and Design, IEEE Trans. on Fuzzy Systems, 535-550, 2000.

[13] J.M. Mendel , John R. I., Feilong Liu. Interval Type-2 Fuzzy Logic Systems Made Simple, IEEE Trans. on Fuzzy Systems, 808-821, 2006.

[14] Feilong Liu, An efficient centroid type-reduction strategy for general type-2 fuzzy logic system , Information Sciences, Vol. 178,2224-2236, 2008.

[15] Long Thanh Ngo, Dinh Sinh Mai, Mau Uyen Nguyen. GPU based Acceleration of Interval Type-2 Fuzzy C-Means Clustering for Satellite Imagery Land-Cover Classification, 2012 IEEE 4th International Conference of Soft Computing and Pattern Recognition (SoCPaR), p235-240, 2012.

[16] E. Rubio; O. Castillo, Interval Type-2 Fuzzy Clustering Algorithm using the Combination of the Fuzzy and Possibilistic C-Mean Algorithms,Norbert Wiener in the 21st Century (21CW), 2014 IEEE Conference, pp1-6, 2014.

[17] Witold Pedrycz, Collaborative fuzzy clustering, Pattern Recognition Letters 23, pp16751686, 2002.

[18] Witold Pedrycz, Collaborative clustering with the use of Fuzzy C-Means and its quantication, Fuzzy Sets and Systems, pp 2399 2427, 2008.

[19] Witold Pedrycz, Collaborative and knowldge based Fuzzy Clustering, ICIC International ISSN 1349-4198, pp 1-12, 2007.

[20] Luiz F. S. Coletta, Lucas Vendramin, Eduardo Raul Hruschka, Ricardo J. G. B. Campello, Witold Pedrycz Collaborative Fuzzy Clustering Algorithms: Some Refinements and Design Guidelines, IEEE Transaction on Fuzzy Systems, Vol. 20, NO. 3, pp 444-462.

[21] Witold Pedrycz and Marek Reformat, Evolutionary Fuzzy Modeling, IEEE Transaction on Fuzzy Systems, Vol. 11, NO. 5, pp 652-665, 2003.

[22] Jin Zhou,C. L. Philip Chen, Long Chen, Han-Xiong Li, Collaborative Fuzzy Clustering Algorithm in Distributed Network Environments, Fuzzy Systems-IEEE Transactions, pp 1-14.

[23] Yizhang Jiang; Fu-Lai Chung; Shitong Wang; Zhaohong Deng; Jun Wang; Pengjiang Qian, Collaborative Fuzzy Clustering From Multiple Weighted Views, IEEE TRANSACTIONS ON CYBERNETICS, pp 1-13, 2014.

[24] M. Prasad;D. L. Li;Y. T. Liu; L. Siana; C. T. Lin;A. Saxena, A Preprocessed Induced Partition Matrix Based Collaborative Fuzzy Clustering For Data Analysis, IEEE International Conference on Fuzzy Systems, pp 1553 - 1558, 2014.

[25] Fusheng Yu, Juan Tang, Ruiqiong Cai, Partially Horizontal Collaborative Fuzzy C-Means, ternational Journal of Fuzzy Systems, Vol. 9, No. 4, pp 198-204, 2007.

[26] Fusheng Yu, Juan Tang and Ruiqiong Cai, A Necessary Preprocessing in Horizontal Collaborative Fuzzy Clustering, International Conference on Granular Computing, pp 399-404, 2007.

[27] Fusheng Yu, Shengli Yu, Prototypes-based Horizontal Collaborative Fuzzy Clustering, Web Mining and Web-based Application, pp 66 - 69, 2009.

[28] Rafael Falc, Beno t Depaire and Koen Vanhoof, Ajith Abraham ,Towards a Suitable Reconciliation of the Findings in Collaborative Fuzzy Clustering, ISDA Eighth International Conference Vol 4, pp 652 657, 2008.

[29] S. Mitra, H. Banka, W.Pedrycz, RoughFuzzy Collaborative Clustering, IEEE Trangsactions on Systems, Man, and Cybernetics, Part B: Cybernetics, Vol. 36(4), pp 795-805, 2006.

[30] Dzung Dinh Nguyen, Long Thanh Ngo, Long The Pham, Witold Pedrycz, Towards hybrid clustering approach to data classification: Multiple kernels based interval-valued Fuzzy C-Means algorithms, Fuzzy Sets and Systems, in press, doi:10.1016/j.fss.2015.01.020.

[31] J. M. Mendel and R. I. B John, Type-2 Fuzzy Sets Made Simple, IEEE Transactions on Fuzzy Systems, Vol. 10, NO. 2,pp 117- 127, 2002.

[32] Mauricio A. Sanchez, Oscar Castillo, Juan R. Castro: Information granule formation via the concept of uncertainty-based information with Interval Type-2 Fuzzy Sets representation and Takagi-Sugeno-Kang consequents optimized with Cuckoo search. Appl. Soft Comput. 27: 602-609 (2015)

[33] Patricia Melin, Oscar Castillo: A review on type-2 fuzzy logic applications in clustering, classification and pattern recognition. Appl. Soft Comput. 21, pp 568-577 (2014)

[34] Elid Rubio, Oscar Castillo, Designing Type-2 Fuzzy Systems Using the Interval Type-2 Fuzzy C-Means Algorithm. Recent Advances on Hybrid Approaches for Designing Intelligent Systems, pp 37-50, 2014