Interval Type-2 Fuzzy Co-Clustering Algorithm

Van Nha Pham
MIST Institute of Science and Technology
Hoang Sam, Hanoi, Vietnam
E-mail: famvannha@gmail.com

Long Thanh Ngo
Department of Information Systems,
Le Quy Don Technical University,
236 Hoang Quoc Viet, Hanoi, Vietnam
E-mail: ngotlong@gmail.com

***Abstract*—This paper introduces a novel clustering technique by combining fuzzy co-clustering approach and interval type-2 fuzzy sets. The proposed algorithm is demonstrated through experiments on UC Berkeley image data-sets to conduct clustering on color images. The experimental results show that the clustering quality is better by evaluating using validity indexes in comparison with previous methods.**

***Index Terms*—Interval Type-2 Fuzzy Clustering, Fuzzy Co-Clustering, Image Segmentation.**

I. Introduction

Image data clustering is an unsupervised learning technique to divide the pixels into classes or clusters of similar color range. With color and multispectral images, data pixel has multiple features. The algorithm, that simultaneously clustering data points and features of data point, can be used to segment images. This algorithm is called co-clustering technique to extend the applicability and quality of clustering, combined with fuzzy logic is a need to better handle the uncertainty in the data.

Fuzzy clustering has been developed in both of theory and applications. Fuzzy C-Means Clustering (FCM) proposed by Bezdek [3] and its variants has been applied to various problems involving image segmentation [1]. Recently, type-2 fuzzy sets which are the extension of original fuzzy sets, have been developed and applied to many different problems [2], [12], [13], [14], [24], involving data clustering, due to advantage of handling uncertainty. C.Y. Yeh et al [27] introduced data-based system modeling using a type-2 fuzzy neural network with a hybrid learning algorithm. To extension the capabilities of data clustering, type-2 fuzzy sets have integrated with data clustering to achieves type-2 fuzzy clustering model. S. A. Begum et al [2] proposed a type-2 fuzzy clustering algorithm for magnetic resonance imaging segmentation. L. A. Lucas et al [11] introduced a method to classifying land cover based on general type-2 fuzzy classifiers. [12] M.U. Nguyen et al proposed an interval type-2 fuzzy subtractive clustering approach to obstacle detection of robot vision using RGB-D camera.

Fuzzy co-clustering [6], [9] is another approach for clustering complex data types as multi-dimensional, multi-feature, high size. Fuzzy co-clustering is used to classify the data type as Web data, color image data... W. Tjhi proposed various fuzzy clustering algorithms. Partitioning algorithm based on fuzzy co-clustering documents and words [18] that can initialize the fuzzy phrase structure obtained from natural distributions, meet needs restoration received information. [18] - [22] are the statistics fuzzy co-clustering algorithm, and heuristic-based fuzzy co-clustering (HFCC) and semi-supervised fuzzy co-clustering (SS-FCC) with dual partition approach, used to classify data types that have high dimensions. SS-FCC [22] proposed a semi-supervised techniques contract fuzzy clustering. In [10], authors proposed a method that comparison of imputation strategies in FNM-based and RFCM-based fuzzy co-clustering. The fuzzy co-clustering based on FNM and RFCM [15] outlined some fuzzy co-clustering algorithms. [5] was to improve the recommendation performance which is a combination of content-based and collaborative filtering approaches in a two-layer graph model getting use of web content and usage mining. [6] is considered as a new proposal in using fuzzy co-clustering model to segment color image data.

To combine the ability of handling uncertainty of type-2 fuzzy sets and processing multi-dimensional data, the paper propose an interval type-2 fuzzy co-clustering algorithm (IT2FCC). Experiments are implemented based on color image segmentation. The paper also extend the evaluation criteria to assess clustering quality. Experiments were conducted on set of sample image data includes 100 color photos, satellite images.

The paper is organized as follow: Section II provides the basic content of fuzzy set theory and type-1 fuzzy co-clustering algorithm. Section III introduces the proposed algorithm - IT2FCC. Section IV presents some experimental results. And section V concludes the paper and further works.

II. Background

This section give a briefly overview relate to interval type-2 clustering and fuzzy co-clustering.

A. Type-2 Fuzzy Sets

Definition 2.1: A $type-2\ fuzzy\ set$, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i. e. ,

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \qquad (1)$$

or

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u))/(x,u), J_x \subseteq [0,1] \qquad (2)$$

in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$, see [16].
At each value of x, say $x = x'$, the 2-D plane whose axes are u and $\mu_{\tilde{A}}(x',u)$ is called a $vertical\ slice$ of $\mu_{\tilde{A}}(x,u)$. A

secondary membership function is a vertical slice of $\mu_{\tilde{A}}(x,u)$. It is $\mu_{\tilde{A}}(x=x',u)$ for $x \in X$ and $\forall u \in J_{x'} \subseteq [0,1]$, i. e.

$$\mu_{\tilde{A}}(x=x',u) \equiv \mu_{\tilde{A}}(x') = \int_{u \in J_{x'}} f_{x'}(u)/u, J_{x'} \subseteq [0,1] \quad (3)$$

in which $0 \leq f_{x'}(u) \leq 1$.

Type-2 fuzzy sets are called an interval type-2 fuzzy sets [17] if the secondary membership function $f_{x'}(u) = 1\ \forall u \in J_x$ i. e. a type-2 fuzzy set are defined as follows:

Definition 2.2: An *interval type-2 fuzzy set* $\tilde{A}$ is characterized by an interval type-2 membership function $\mu_{\tilde{A}}(x,u) = 1$ where $x \in X$ and $u \in J_x \subseteq [0,1]$, i. e. ,

$$\tilde{A} = \{((x,u),1) | \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \quad (4)$$

Uncertainty of $\tilde{A}$, denoted FOU, is union of primary functions i. e. $FOU(\tilde{A}) = \bigcup_{x \in X} J_x$. Upper/lower bounds of membership function (UMF/LMF), denoted $\overline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$, of $\tilde{A}$ are two type-1 membership function and bounds of FOU which is limited by two membership functions of an type-1 fuzzy set are UMF and LMF.

B. Fuzzy Co-Clustering

Fuzzy co-clustering (FCC) is a clustering method that performed in simultaneous fuzzy clustering of objects and features. The results of co-clusters which reflect the interrelations between object clusters and feature clusters. FCC have been studied, improvements and applications in [5], [6], [9], [18], [19], [20], [21], [22]. FCC provides two membership functions, the one is for the partition or data points and the one is for the components of the data points. The objective function $J_{FCC}(U,V,P)$ is described as follows:

$$J_{FCC}(U,V,P) = \sum_{c=1}^{C}\sum_{i=1}^{N}\sum_{j=1}^{K} u_{ci} v_{cj} D_{cij} + T_U \sum_{c=1}^{C}\sum_{i=1}^{N} u_{ci} \log u_{ci} + T_V \sum_{c=1}^{C}\sum_{j=1}^{K} v_{cj} \log v_{cj} + \sum_{i=1}^{N} \lambda_i (\sum_{c=1}^{C} u_{ci} - 1) + \sum_{c=1}^{C} \gamma_c (\sum_{j=1}^{K} v_{cj} - 1) \quad (5)$$

The components of the objective function (5) is determined by the following formulas: u_{ci} is membership grade of data points, calculated as follows:

$$u_{ci} = \frac{e^{(-\sum_{j=1}^{K} \frac{v_{cj} D_{cij}}{T_U})}}{\sum_{f=1}^{C} e^{(-\sum_{j=1}^{K} \frac{v_{fj} D_{cij}}{T_U})}} \quad (6)$$

v_{cj} is membership grade of features/components of each data point, calculated as follows:

$$v_{cj} = \frac{e^{(-\sum_{i=1}^{N} \frac{u_{ci} D_{cij}}{T_V})}}{\sum_{q=1}^{K} e^{(-\sum_{i=1}^{N} \frac{u_{ci} D_{ciq}}{T_V})}} \quad (7)$$

III. Interval Type-2 Fuzzy Co-Clustering in Color Image Segmentation

The idea of the Interval Type-2 Fuzzy Co-Clustering algorithm (IT2FCC) derived from the need to determine I image which obtained from a camera coincide with P image in the sample photo gallery. One of the methods that have been studied and conducted clustering both images I and P, then from clustering results to determine I and P coincide or not. As above mention, there have been numerous studies involving the fuzzy clustering model, fuzzy co-clustering and type-2 fuzzy clustering. In this section, we present the proposed algorithm, called Interval Type-2 Fuzzy Co-Clustering.

The first, we represent color image I with size $W \times H = N$ as a set X of N pixels and

$$X = \{x_1, x_2, ..., x_i, ..., x_N\} \in R^K$$

K is the number of features of each pixel ($K = 3$ in RGB color space). Let x_{ij} denote the j^{th} feature of the i^{th} data pixel, p_{cj} be the prototype of clusters and D_{cij}=Dist(x_{ij}, p_{cj}) be the square of Euclidean distance between feature data pixel x_{ij} and the feature centroid p_{cj}, given by:

$$D_{cij} = d^2(x_{ij}, p_{cjj}) = (x_{ij} - p_{cj})^2 \quad (8)$$

Let c_{ci} denote the object membership grade of the i^{th} data pixel to c^{th} cluster, $U = \{u_{ci}\}$ be the $C \times N$ object membership grade matrix of image I, v_{cj} denotes the feature membership grade defined as the membership of feature j^{th} to the c^{th} cluster and $V = \{v_{cj}\}$ be the corresponding $C \times K$ feature membership matrix for image I.

The objective function of Type-2 Fuzzy Co-Clustering algorithm $J_{T2FCC}(U,V,P)$ is described as follows:

$$J_{T2FCC}(U,V,P) = \sum_{c=1}^{C}\sum_{i=1}^{N}\sum_{j=1}^{K} u_{ci}^m v_{cj}^m D_{cij} + T_U \sum_{c=1}^{C}\sum_{i=1}^{N} u_{ci}^m \log u_{ci}^m + T_V \sum_{c=1}^{C}\sum_{j=1}^{K} v_{cj}^m \log v_{cj}^m + \sum_{i=1}^{N} \lambda_i (\sum_{c=1}^{C} u_{ci}^m - 1) + \sum_{c=1}^{C} \gamma_c (\sum_{j=1}^{K} v_{cj}^m - 1) \quad (9)$$

To get optimal clustering results, the (9) reaches a minimum and J_{T2FCC} is minimized subject to the following constraints:

$$\sum_{c=1}^{C} u_{ci} = 1, u_{ci} \in [0,1], \forall i = \overline{1,N}$$
$$\sum_{j=1}^{K} v_{cj} = 1, v_{cj} \in [0,1], \forall c = \overline{1,C} \quad (10)$$

Where λ_i, γ_c are Lagrange coefficients, T_u and T_v are weights that indicate fuzziness. Increasing T_U and T_V will increase the opacity of the cluster. x_{ij} is j^{th} element of i^{th} data point, $P = \{p_{cj}\}$ be the set of feature centroids.

To determine the value of the objective function (9), we need to find formulas of the membership matrices u_{ci}, v_{cj} and distance D_{cij}.

Firstly, taking the partial derivative of $J(U,V,P)$ in (9) with respect to U and setting the gradient to zero, we have,

$$\frac{\partial J}{\partial U}=\sum_{j=1}^{K}v_{cj}^{m}D_{cij}+T_U(m\log u_{ci}+1)+\lambda_i=0 \tag{11}$$

Subjecting u_{ci} derived from (11) to the constraint in (10), we obtain the formula u_{ci} as follows:

$$u_{ci}=\frac{e^{(-\sum_{j=1}^{K}\frac{v_{cj}^{m}D_{cij}}{mT_U})}}{\sum_{c=1}^{C}e^{(-\sum_{j=1}^{K}\frac{v_{cj}^{m}D_{cij}}{mT_U})}} \tag{12}$$

In a similar manner, taking the partial derivative of $J(U,V,P)$ with respect to V and setting the gradient to zero we have,

$$\frac{\partial J}{\partial V}=\sum_{i=1}^{N}u_{ci}^{m}D_{cij}+T_V(m\log v_{cj}+1)+\gamma_c=0 \tag{13}$$

Applying the constraint (10) to v_{cj} derived from (13) , we obtain the formula for the feature membership function v_{cj} as follows:

$$v_{cj}=\frac{e^{(-\sum_{i=1}^{N}\frac{u_{ci}^{m}D_{cij}}{mT_V})}}{\sum_{j=1}^{K}e^{(-\sum_{i=1}^{N}\frac{u_{ci}^{m}D_{cij}}{mT_V})}} \tag{14}$$

Take the partial derivative of $J(U,V,P)$ with respect to P and setting the gradient to zero, we have,

$$\frac{\partial J}{\partial P}=v_{cj}^{m}\sum_{i=1}^{N}u_{ci}^{m}x_{ij}-v_{cj}^{m}p_{cj}\sum_{i=1}^{N}u_{ci}^{m}=0 \tag{15}$$

and finally, from (15), we have,

$$p_{cj}=\frac{\sum_{i=1}^{N}u_{ci}^{m}x_{ij}}{\sum_{i=1}^{N}u_{ci}^{m}} \tag{16}$$

IT2FCC is extension of fuzzy co-clustering by using two fuzziness parameters m_1, m_2 to make a FOU, corresponding to upper and lower values of fuzzy co-clustering. The use of fuzzifiers gives two objective functions to be minimized as follows:

$$\begin{aligned}J_{m_1}(U,V,P)=&\sum_{c=1}^{C}\sum_{i=1}^{N}\sum_{j=1}^{K}u_{ci}^{m_1}v_{cj}^{m_1}D_{cij}+\\ T_U\sum_{c=1}^{C}\sum_{i=1}^{N}u_{ci}^{m_1}\log u_{ci}^{m_1}&+T_V\sum_{c=1}^{C}\sum_{j=1}^{K}v_{cj}^{m_1}\log v_{cj}^{m_1}\\ +\sum_{i=1}^{N}\lambda_i(\sum_{c=1}^{C}u_{ci}^{m_1}-1)&+\sum_{c=1}^{C}\gamma_c(\sum_{j=1}^{K}v_{cj}^{m_1}-1)\end{aligned} \tag{17}$$

and

$$\begin{aligned}J_{m_2}(U,V,P)=&\sum_{c=1}^{C}\sum_{i=1}^{N}\sum_{j=1}^{K}u_{ci}^{m_2}v_{cj}^{m_2}D_{cij}+\\ T_U\sum_{c=1}^{C}\sum_{i=1}^{N}u_{ci}^{m_2}\log u_{ci}^{m_2}&+T_V\sum_{c=1}^{C}\sum_{j=1}^{K}v_{cj}^{m_2}\log v_{cj}^{m_2}\\ +\sum_{i=1}^{N}\lambda_i(\sum_{c=1}^{C}u_{ci}^{m_2}-1)&+\sum_{c=1}^{C}\gamma_c(\sum_{j=1}^{K}v_{cj}^{m_2}-1)\end{aligned} \tag{18}$$

Upper/lower degrees of membership $\overline{u}\,ci$ and $\underline{u}_{ci}$ are determined as follows:

$$\overline{u}_{ci}=\frac{e^{(-\sum_{j=1}^{K}\frac{v_{cj}^{m1}D_{cij}}{m_1T_U})}}{\sum_{f=1}^{C}e^{(-\sum_{j=1}^{K}\frac{v_{fj}^{m1}D_{fij}}{m_1T_U})}}$$

and

$$\underline{u}_{ci}=\frac{e^{(-\sum_{j=1}^{K}\frac{v_{cj}^{m2}D_{cij}}{m_2T_U})}}{\sum_{f=1}^{C}e^{(-\sum_{j=1}^{K}\frac{v_{fj}^{m2}D_{fij}}{m_2T_U})}}$$

Meanwhile, we conducted defuzzification and obtained u_{ci} as following:

$$u_{ci}=\frac{\overline{u}_{ci}+\underline{u}_{ci}}{2} \tag{19}$$

In the same way, upper/lower degrees of membership, and are determined as follows:

$$\overline{v}_{cj}=\frac{e^{(-\sum_{i=1}^{N}\frac{u_{ci}^{m1}D_{cij}}{m_1T_V})}}{\sum_{j=1}^{K}e^{(-\sum_{i=1}^{N}\frac{u_{ci}^{m1}D_{cij}}{m_1T_V})}}$$

and

$$\underline{v}_{cj}=\frac{e^{(-\sum_{i=1}^{N}\frac{u_{ci}^{m2}D_{cij}}{m_2T_V})}}{\sum_{j=1}^{K}e^{(-\sum_{i=1}^{N}\frac{u_{ci}^{m2}D_{cij}}{m_2T_V})}}$$

v_{cj} is defined as follows:

$$v_{cj}=\frac{\overline{v}_{cj}+\underline{v}_{cj}}{2} \tag{20}$$

With $\overline{p}_{cj}$ and $\underline{p}_{cj}$, we have,

$$\overline{p}_{cj}=\frac{\sum_{i=1}^{N}u_{ci}^{m1}x_{ij}}{\sum_{i=1}^{N}u_{ci}^{m1}}$$

and

$$\underline{p}_{cj}=\frac{\sum_{i=1}^{N}u_{ci}^{m2}x_{ij}}{\sum_{i=1}^{N}u_{ci}^{m2}}$$

and

$$p_{cj} = \frac{\overline{p}_{cj} + \underline{p}_{cj}}{2} \tag{21}$$

IT2FCC algorithm perform the following steps:

1. Initialize the parameters T_u, T_v, m_1, m_2, maximum error limit and maximum number of iterations $_{max}$.
2. Set iteration number $\tau = 1$.
3. Initialize u_{ci} such that $0 \le u_{ci} \le 1$.
4. DO
5. Calculate p_{cj} using (21)
6. Calculate D_{cij} using (8).
7. Calculate v_{cj} using (20).
8. Calculate u_{ci} using (19).
9. Increase $\tau = \tau + 1$.
10. WHILE $(max(|u_{ci}[\tau+1] - u_{ci}[\tau]|) \le \varepsilon$ or $\tau = \tau_{max})$
11. Output result.

IV. Experimental Results

A. Validity indexes

Fuzzy clustering in general, type-2 fuzzy co-clustering in particular are unsupervised learning process, the data objects are not labeled and anticipated structure.

Xie and Beni [7] proposed the validity function S of the clusters as follows:

$$S(c) = \frac{\sigma/N}{d_{\min}} \tag{22}$$

Where d_{min} is evaluated from

$$d_{\min} = \min_{\forall c}\{\sum_{j=1}^{K}(p_{(c+1)j} - p_{cj})^2\} \tag{23}$$

where d_{min} is the minimum distance between the centroids p_{cj} for cluster $c = 1, ..., C$ and feature j and σ is the maximum variance among all the clusters C, given by

$$\sigma = \max_{\forall c}\{\sum_{i=1}^{N} u_{ci}^2 \sum_{j=1}^{K}(x_{\mathrm{ij}} - p_{cj})^2\} \tag{24}$$

H.L Shieh [23] defined cluster validity measure the separation between clusters and the compactness in each cluster to indicate the optimal cluster number obtained by fuzzy clustering algorithms as follows,

$$CS(c) = Com(c)xSE(c) \tag{25}$$

where, $Com(c)$ is measure compactness in each cluster, defined as follows,

$$Com(c) = (\frac{1}{N}\sum_{c=1}^{C}\sum_{i=1}^{N} u_{ci}\left\|x_i - p_c\right\| \mathrm{w}_{ci})^2$$

, and

$$\mathrm{w}_{ci} = \exp(-\frac{\|x_i - p_c\|^2}{2\sigma}) \tag{26}$$

and $SE(c)$ is measure the separation between clusters, defined as follows

$$SE(c) = \frac{1}{\frac{\sqrt{\sum_{i=1}^{C}\sum_{k=1}^{C}\|p_i - p_c\|}}{\frac{c(c-1)}{2}}\sqrt{\min_{i\neq j}(\|p_i - p_j\|)}} \tag{27}$$

The better algorithms exhibit smaller values of S and CS. Therefore, we can determine the optimal number of clusters by searching the minimal value of $S(c^*)$ or $CS(c^*)$ and c^* is the optimal number of clusters.

To assess the quality of clustering, we need to use a set of appropriate indicators of the parameters related to the clustering algorithm respectively. There have been many studies around the development of clustering assessment criteria, but studies are limited to the fuzzy clustering algorithm, and fuzzy co-clustering. Bezdek [4] proposed two coefficients, that are Partition Coefficient (PC) and Partition Entropy (PE):

$$PC = \frac{1}{N}\sum_{c=1}^{C}\sum_{i=1}^{N} u_{ci}^2 \tag{28}$$

and

$$PE = -\frac{1}{C}\sum_{c=1}^{C}\sum_{i=1}^{N} u_{ci}\log u_{ci}$$

The better algorithms exhibit smaller values of PE and the larger value of PC. So, the optimal partition can be the one having larger PC (or smaller PE). In fuzzy co-clusstering, the validity indexes are not only object membership grades u_{ci} but also feature membership grades v_{cj}. Meanwwhile, Honda [5] also adjusted the indexes PC and PE becoming four components PCu, PCw, PEu and PEw:

$$PC_u = \frac{1}{N}\sum_{c=1}^{C}\sum_{i=1}^{N} u_{ci}^2, PC_v = \frac{1}{C}\sum_{c=1}^{C}\sum_{j=1}^{K} \mathrm{v}_{cj}^2 \tag{29}$$

$$PE_u = -\frac{1}{N}\sum_{c=1}^{C}\sum_{i=1}^{N} u_{ci}\log u_{ci}, PE_v = -\frac{1}{C}\sum_{c=1}^{C}\sum_{i=1}^{K} \mathrm{v}_{cj}\log \mathrm{v}_{cj}$$

To assess the quality of image segmentation, we use two indexes, Mean Squared Error (MSE)[25] and Image Quality Index (IQI)[26]:

$$MSE(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i) \tag{30}$$

$$IQI = \frac{4\sigma_{xy}\overline{xy}}{(\sigma_x^2 + \sigma_y^2)(\overline{x}^2 + \overline{y}^2)} \tag{31}$$

with $\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i, \overline{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$, $\sigma_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})$, $\sigma_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \overline{y})$ and $\sigma_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})(y_i - \overline{y})$ Where, $x = \{x_i\} = \{x_1, x_2, ..., x_N\}$ and $y = \{y_i\} = \{y_1, y_2, ..., y_N\}$ corresponding to the original image and segmented image.

Thus, the better algorithms exhibit the smaller value of PE, MSE and the larger value of PC, IQI.

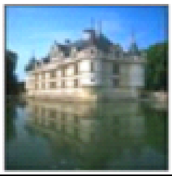
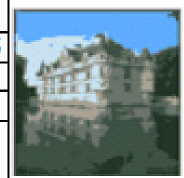
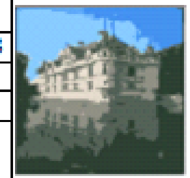
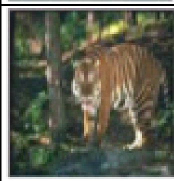
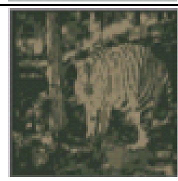
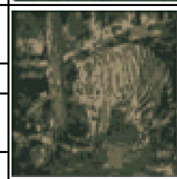
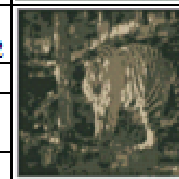
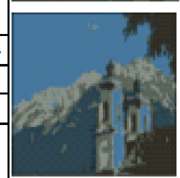
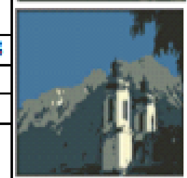
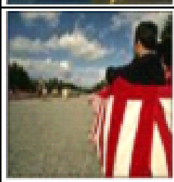
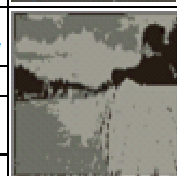
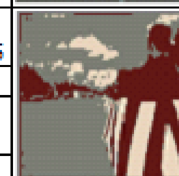
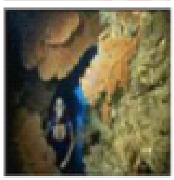
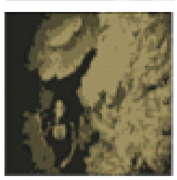
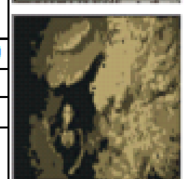
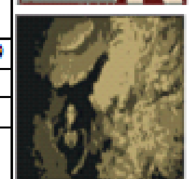
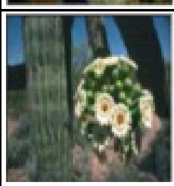
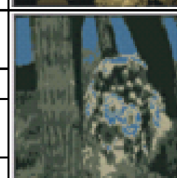
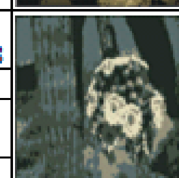
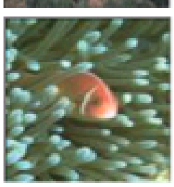
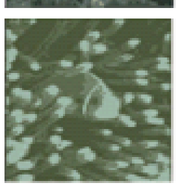
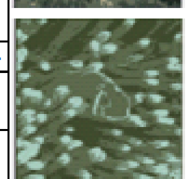
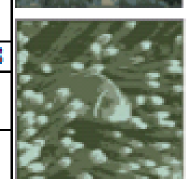
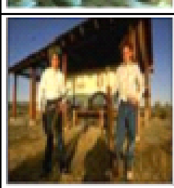
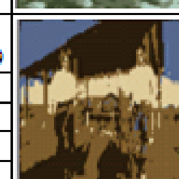
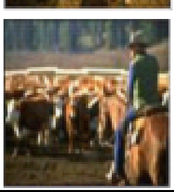
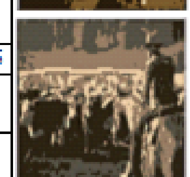
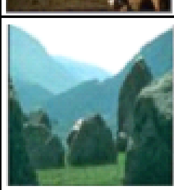
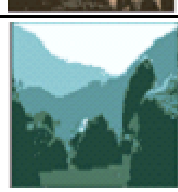
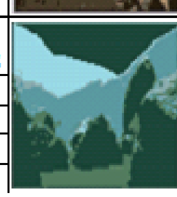
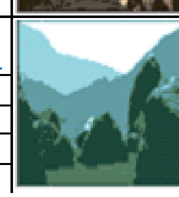
No.	Original image	Results of FCM		Results of FCC		Results of IT2FCC	
1			PC_u: 0.5257 PE_u: 0.42436 MSE: 36.45 IQI: 0.9354		PC_u: 0.9859 PE_u: 0.00143 MSE: 28.75 IQI: 0.9796		PC_u: 0.9887 PE_u: 0.00860 MSE: 26.80 IQI: 0.9767
2			PC_u: 0.5984 PE_u: 0.32641 MSE: 23.07 IQI: 0.8549		PC_u: 0.9760 PE_u: 0.00212 MSE: 25.08 IQI: 0.8587 S: 0.0907 CS: 0.787		PC_u: 0.9788 PE_u: 0.01490 MSE: 21.84 IQI: 0.9324
3			PC_u: 0.4592 PE_u: 0.45904 MSE: 31.40 IQI: 0.7513		PC_u: 0.9650 PE_u: 0.00133 MSE: 24.48 IQI: 0.8708		PC_u: 0.9872 PE_u: 0.00991 MSE: 22.80 IQI: 0.9628
4			PC_u: 0.5618 PE_u: 0.32807 MSE: 57.64 IQI: 0.7337		PC_u: 0.7655 PE_u: 0.00026 MSE: 64.43 IQI: 0.5628		PC_u: 0.9984 PE_u: 0.00118 MSE: 50.12 IQI: 0.9414
5			PC_u: 0.6438 PE_u: 0.29420 MSE: 27.06 IQI: 0.9355		PC_u: 0.9872 PE_u: 0.00109 MSE: 25.90 IQI: 0.9556		PC_u: 0.9879 PE_u: 0.00834 MSE: 25.54 IQI: 0.9618
6			PC_u: 0.5680 PE_u: 0.37121 MSE: 25.36 IQI: 0.9016		PC_u: 0.9680 PE_u: 0.00148 MSE: 26.71 IQI: 0.7963		PC_u: 0.9827 PE_u: 0.01335 MSE: 22.33 IQI: 0.9605
7			PC_u: 0.6198 PE_u: 0.30734 MSE: 24.65 IQI: 0.9128		PC_u: 0.9761 PE_u: 0.00128 MSE: 25.34 IQI: 0.9304		PC_u: 0.9838 PE_u: 0.01250 MSE: 23.25 IQI: 0.9536
8			PC_u: 0.5242 PE_u: 0.38783 MSE: 45.63 IQI: 0.8636		PC_u: 0.8693 PE_u: 0.00039 MSE: 42.62 IQI: 0.8007		PC_u: 0.9962 PE_u: 0.00298 MSE: 34.36 IQI: 0.9577
9			PC_u: 0.5881 PE_u: 0.33718 MSE: 34.50 IQI: 0.8808		PC_u: 0.9471 PE_u: 0.00105 MSE: 35.99 IQI: 0.8106		PC_u: 0.9899 PE_u: 0.00771 MSE: 30.89 IQI: 0.9571
10			PC_u: 0.7841 PE_u: 0.19678 MSE: 20.97 IQI: 0.9872		PC_u: 0.8369 PE_u: 0.00031 MSE: 36.34 IQI: 0.9269		PC_u: 0.9964 PE_u: 0.00281 MSE: 18.55 IQI: 0.9903

Fig. 1. Image segmentation results on 10 sampple images using FCM, FCC and IT2FCC

TABLE I
COLOR IMAGE SEGMENTATION RESULTS USING IT2FCC WITH DIFFERENT NUMBER OF CLUSTERS

No	Type	C	K	PCu	PEu	MSE	IQI	S	CS
1	FCM	3	15	0.7332	0.20602	39.44	0.9344	0.1631	0.010
	FCC	3	12	0.9399	0.00018	44.07	0.8754	0.0610	0.035
	IT2FCC	3	35	0.9972	0.00219	36.95	0.9569	0.0649	0.057
2	**FCM**	**4**	**26**	**0.6949**	**0.25638**	**32.72**	**0.9524**	**0.1052**	**0.044**
	FCC	**4**	**32**	**0.9704**	**0.00037**	**32.61**	**0.9505**	**0.0474**	**0.030**
	IT2FCC	**4**	**34**	**0.9952**	**0.00379**	**30.67**	**0.9736**	**0.0392**	**0.008**
3	FCM	5	27	0.6225	0.32337	29.96	0.9643	0.3307	0.070
	FCC	5	35	0.9851	0.00089	28.68	0.9582	0.0736	0.108
	IT2FCC	5	43	0.9916	0.00647	27.14	0.9811	0.0598	0.102
4	FCM	6	28	0.5892	0.36517	27.12	0.9709	0.1989	0.076
	FCC	6	60	0.9987	0.00096	22.70	0.9875	0.0524	0.074
	IT2FCC	6	39	0.9883	0.00898	22.71	0.9875	0.0498	0.186
5	FCM	7	26	0.5431	0.41882	26.61	0.9738	0.1338	0.172
	FCC	7	57	0.9978	0.00147	20.86	0.9898	0.0723	0.357
	IT2FCC	7	27	0.9841	0.01231	20.74	0.9899	0.0487	0.479
6	FCM	8	24	0.5162	0.45083	24.26	0.9770	0.8675	0.319
	FCC	8	35	0.9977	0.00165	19.63	0.9909	0.0556	0.302
	IT2FCC	8	44	0.9818	0.01381	19.64	0.9909	0.0529	0.237
7	FCM	9	26	0.4868	0.48969	23.39	0.9797	23.0918	0.191
	FCC	9	34	0.9972	0.00199	18.66	0.9919	0.1039	0.275
	IT2FCC	9	36	0.9745	0.01976	18.67	0.9920	0.0925	0.258
8	FCM	10	27	0.4629	0.52590	22.76	0.9814	6.8071	0.196
	FCC	10	31	0.9970	0.00219	18.17	0.9925	0.0877	0.263
	IT2FCC	10	38	0.9711	0.02231	18.01	0.9923	0.0859	0.182

B. *Experiments*

To conduct experiments, we have implemented the proposed algorithm using 100 color images in UC Berkeley photo library [1]. In experiment, the results are compared with the previous clustering method to assess the quality of the proposed algorithm.

Fig. 1 shows the results of experiments on 10 sample images with the number of cluster and fuzzy parameters were identified, using three clustering algorithms FCM, FCC and IT2FCC. From the result in Fig. 1, validity indexes of IT2FCC are better than FCM and FCC. Clustering method achieves the higher value of PC_u and IQI or the lower value of PE_u and MSE then the algorithm exhibits the better clustering quality.

Table I lists the results of clustering experiments on a sample image ”147091.jpg” with number of clusters from 3 to 10, using three difference clustering methods FCM, FCC and IT2FCC. According to results shown in the table, the validity indexes S and CS of three considered clustering algorithms reaches minimal value at the number of clusters is 4.

Fig. 2 shows the results of clustering experiments on sample image ”147091.jpg” as color images, with the number of clusters from 2 to 10, using clustering method IT2FCC, the resulting pixels in the same cluster is filled with a specified color.

[1]http://www.eecs.berkeley.edu/ Research/ Projects/ CS/ vision/ bsds/BSDS300/html/dataset/images.html, downloaded in 2014

V. CONCLUSION

In this paper, Interval Type-2 Fuzzy Co-Clustering algorithm has presented as a combination fuzzy co-clustering and interval type-2 fuzzy sets. This extension is to use the fuzzifier parameters in the objective function of fuzzy co-clustering with two fuzzifiers to make a FOU. Experiment was conducted on the UC Berkeley image data sets in comparison with FCM and FCC algorithms to show advance of the proposed algorithm.

The next goal is to extend to the proposed algorithm for other application such as text classification or multi-spectral image classification.

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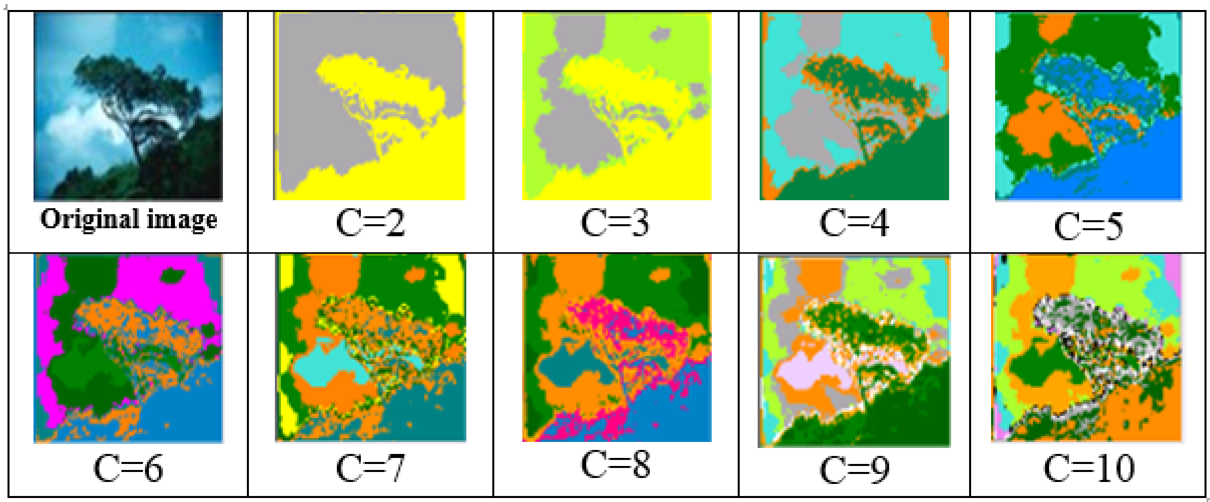

Fig. 2. Image segmentation results with different number of clusters

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