PAPER *Special Section on Information and Communication Systems for Safe and Secure Life*

Design of Two-Way Relay Network Using Space-Time Block Coded Network Coding with Relay Selection*

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SUMMARY This paper presents the design of an ad hoc two-way two-hop relay network using physical-layer network coding (PNC) in which multiple antennas are used at all nodes. In the considered network, the Alamouti's space-time block code (STBC) is used for transmission while linear detection is used for signal recovery. In order to facilitate linear estimation, we develop an equivalent multiuser STBC model for the proposed network and design the sum-and-difference matrix which allows convenient combination of the transmitted symbols from the end nodes. In addition, a simple relay selection method based on minimum mean square error (MSE) is proposed for performance improvement. Simulation results show that the proposed network achieves diversity order 2 while requiring only polynomial complexity. Moreover, it is possible to achieve significant bit error rate (BER) performance improvement when the proposed relay selection algorithm is used.

key words: *two-way relay, network coding, relay selection, MIMO, STBC, MMSE, ad hoc network*

1. Introduction

Nowadays, wireless ad hoc networks are becoming more popular. It is easy to find their deployment in various applications such as distributed computing, search-and-rescue supports, inter-vehicle communications, as well as military deployment. A most important feature of wireless ad hoc networks is capability to exchange information among network nodes with the help of intermediate nodes without requiring existing infrastructure. Communications in such relay networks are done in a two-way, also referred to as bidirectional, fashion. In order to improve network throughput as well as transmission reliability, large research efforts were paid on two-way relay networks [1]–[11]. A typical example of such a two-way relay network is the simple two-hop three-node network, where the two end nodes exchange their data with each other via a relay. For this network, physical-layer network coding (PNC) is known as an effective scheme to improve network throughput by exploit the broadcast nature of wireless channels [1]–[3], [5], [7]–[11]. Different from the conventional two-way relaying or network coding (NC), by using PNC the two end nodes can transmit simultaneously to the relay during the multiple access (MA) phase. The relay will extract the network coded symbols directly at the physical layer and transmit them to the end nodes during the broadcast (BC) phase. Since it requires only two transmission phases compared with three phases in the conventional two-way relaying, PNC clearly helps to increase the throughput by one third. Moreover, as the network coded symbols are directly estimated at the physical layer, PNC can avoid error propagation and thus has improved bit error rate (BER) performance over the conventional two-way relaying [1].

Meanwhile, multiple-input multiple-output (MIMO) transmission techniques are also known as an effective method to achieve improved channel capacity and signal reception quality [12]–[14]. Thus, a suitable combination of MIMO and PNC will promise further improvement in network throughput as well as system performance. Various combined MIMO and PNC (MIMO-PNC) schemes for two-way relaying channels can be found in the literature. These schemes can be roughly classified into 2 groups, namely distributed MIMO-PNC and full MIMO-PNC. In the distributed MIMO-PNC systems, it was assumed that the end nodes have two antennas while the relay has only single [4] or both end nodes have only single antenna while the relay has two antennas [5], [9], [10]. In order to differentiate the distributed with the original MIMO systems we will call the former systems MISO-PNC** and the latter SIMO-PNC***. In the full MIMO-PNC system multiple, commonly two, antennas are employed at all nodes [6]–[8], [11]. The full MIMO-PNC system will be referred to simply as MIMO-PNC in the current paper.

Related to the MIMO-PNC systems, there have been some related works in the literature [6]–[8], [11]. The work in [6] proposed a system where all nodes use the Alamouti space-time block code (STBC) for transmission and maximum likelihood (ML) detection is used for signal estimation. At the relay signal estimation is done for each individual symbol from the end nodes before XOR is used to obtain the network-coded symbols for transmission. Thus the proposed system should be considered MIMO-NC rather than MIMO-PNC where the network-coded symbols are es-

Manuscript received November 25, 2014.
Manuscript revised March 27, 2015.

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*Part of this paper was presented at The 2nd International Workshop on 5G Mobile and Wireless Communication System for 2020 and Beyond (MWC2020), the 2014 IEEE 79th Vehicular Technology Conference, Spring, Seoul, Korea.

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DOI: 10.1587/transfun.E98.A.1657

**Multiple Input Single Output
***Single Input Multiple Output

timated at the relay. In the system in [7] channel coding was combined with PNC for the Alamouti STBC two-way relay system. In order to avoid high-complexity ML detection, precoding is designed for the two end nodes to facilitate multiuser detection. This precoding MIMO-PNC system, however, requires channel state information (CSI) feedback from the relay to the two end nodes. This requirement is sometimes difficult to fulfill in practical systems, such as those using frequency division duplexing (FDD), as the frequency separation between the forward and reverse link is comparatively large. The MIMO-PNC system in [8] also combined STBC with PNC but for a multi-hop relay network. The late MIMO-PNC system in [11] did not use STBC but spatial division multiplexing (SDM) for transmission in order to achieve multiplexing gain.

In this paper, we reconsider the problem of designing a two-way relay MIMO-PNC system using the Alamouti STBC. Our objective is to achieve improved system performance with low complexity at all nodes. Specifically, we propose two low-complexity MIMO linear processing schemes at the relay and the two end nodes. While the linear minimum mean square error (MMSE) estimation at the relay during the MA phase is developed from that for the SIMO-PNC in [5], the simple fading compensation scheme at the end nodes during the BC phase is of our originality. Compared with the previous works, the proposed schemes help to save computational complexity over the ML estimation in [6] and do not require CSI feedback as in [8]. Next, based on the estimated mean square error (MSE) we propose a simple distributed relay selection scheme which helps to achieve significant BER performance improvement.

The remainder of the paper is organized as follows. Sect. 2 presents a general model of MIMO-PNC for two-way relay channels. The proposed design of two-way relay MIMO communication network using STBC-PNC is described in Sect. 3. The proposal of the relay selection based on MMSE criterion is given Sect. 4. Performance evaluation is presented in Sect. 5 and, finally, conclusions are drawn in Sect. 6.

2. General System Model of PNC for Two-Way Relay MIMO Channels

In this section, we present a general system model of PNC for two-way relay MIMO channels. In the system, independent data streams are assumed to be transmitted from antennas. This model will be applied to the equivalent model of our proposed two-way relay MIMO STBC-PNC system in the next section.

We consider a network exchanging data over a two-way relay MIMO channel as illustrated in Fig. 1. The network consists of two end nodes, denoted by N_1 and N_2, communicating with each other via the help of a relay (relaying node) R in semi-duplex mode. In a general model, all nodes are assumed to be equipped with multiple antennas. However, for simplicity, we assume further that the number of antennas of each node is the same and equal to 2, $N = 2$. The transmit

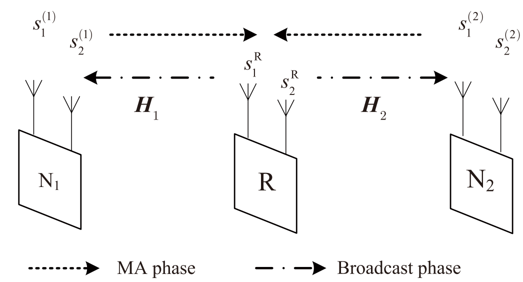

Fig. 1 System model of a two-way relay MIMO-PNC network.

vectors from each end node N_k denoted by $\boldsymbol{s}_k = [s_1^{(k)},\ s_2^{(k)}]^T$, where $k = 1, 2$, contain complex modulated symbols. For ease of presentation, we assume that binary phase shift keying (BPSK) is used for modulation. The model can be certainly applied for the M-ary modulation with $M = 4$ such as quadrature phase shift keying (QPSK) or 4-ary quadrature amplitude modulation (4-QAM) in which each in-phase and quadrature branch can be treated as a BPSK stream. The MIMO channels between each end node and the relay, $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, are flat and undergo independent and identically distributed (i.i.d.) Rayleigh fading. Each element $h_{ji}^{(k)}$ of $\boldsymbol{H}_k$, denoting the channel between the i-th antenna of node N_k and j-th antenna of the relay R, is modelled using a complex Gaussian variable with zero mean and unit variance, i.e., $h_{ji}^{(k)} \sim \mathcal{N}_c(0, 1)$. The noise vector induced at each node receiver is assumed i.i.d. complex Gaussian distributed with mean zero and the same variance σ_z^2.

The exchange of data between the two end nodes N_1 and N_2 is performed over two phases, i.e., MA and BC, using PNC. It is worth noting that different from a relaying multiple access (RMA) channel, the MA phase in PNC is non-orthogonal. As a result, the relay R will rely on an efficient signal detection scheme in order to recover the transmitted symbols from the two end nodes before encoding them using PNC. The received signal vector at the relay R during the MA phase is given by

$$\boldsymbol{y} = \boldsymbol{H}_1\boldsymbol{s}_1 + \boldsymbol{H}_2\boldsymbol{s}_2 + \boldsymbol{z}_{\mathrm{R}} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{z}_{\mathrm{R}}, \qquad (1)$$

where $\boldsymbol{H} = [\boldsymbol{H}_1,\ \boldsymbol{H}_2] \in \mathbb{C}^{2\times4}$ and $\boldsymbol{s} = [\boldsymbol{s}_1^T,\ \boldsymbol{s}_2^T]^T \in \mathbb{C}^{4\times1}$ and $\boldsymbol{z}_{\mathrm{R}} = [z_1,\ z_2]^T \in \mathbb{C}^{2\times1}$. Here, the superscript ^{T} denotes the transpose of matrix.

Upon receiving $\boldsymbol{y}$, the relay R would use a detector to estimate each transmitted symbol $s_1^{(1)}$, $s_2^{(1)}$ and $s_1^{(2)}$, $s_2^{(2)}$. However, in the MIMO-PNC channel instead of estimating each symbol independently the detector can efficiently detect the combined version of transmitted symbols: $s_1^{(1)} + s_1^{(2)}$, $s_1^{(1)} - s_1^{(2)}$ and $s_2^{(1)} + s_2^{(2)}$, $s_2^{(1)} - s_2^{(2)}$. These estimated symbols are mapped to the PNC symbols as follows [11]:

$$\left.\begin{array}{l} s_1^{(1)} + s_1^{(2)} \\ s_1^{(1)} - s_1^{(2)} \end{array}\right\} \rightarrow s_1^{(1)} \oplus s_1^{(2)}, \qquad (2)$$

$$\left.\begin{array}{l} s_2^{(1)} + s_2^{(2)} \\ s_2^{(1)} - s_2^{(2)} \end{array}\right\} \rightarrow s_2^{(1)} \oplus s_2^{(2)}, \qquad (3)$$

where $\oplus$ denotes the symbol XOR operator. For BPSK modulation, using the mapping in [1] the symbol XOR operation can be simply expressed as: $s_1^{(1)} \oplus s_1^{(2)} = s_1^{(1)} s_1^{(2)}$ and $s_2^{(1)} \oplus s_2^{(2)} = s_2^{(1)} s_2^{(2)}$.

During the next BC phase, these PNC symbols will be broadcast over the relay two antennas. The first antenna will transmit $s_1^{\mathrm{R}} \triangleq s_1^{(1)} \oplus s_1^{(2)}$ and the second $s_2^{\mathrm{R}} \triangleq s_2^{(1)} \oplus s_2^{(2)}$. The transmit vector of the relay is defined as $\boldsymbol{s}_{\mathrm{R}} \triangleq [s_1^{\mathrm{R}}, \ \ s_2^{\mathrm{R}}]^T$. Assume that the channels between each end node and the relay are reciprocal, the received signal vectors at the two end nodes can be expressed respectively as

$$\boldsymbol{y}_1 = \boldsymbol{H}_1^T \boldsymbol{s}_{\mathrm{R}} + z_1, \tag{4}$$

$$\boldsymbol{y}_2 = \boldsymbol{H}_2^T \boldsymbol{s}_{\mathrm{R}} + z_2. \tag{5}$$

where $z_k = [z_1^{(k)}, \ z_2^{(k)}]^T$. Both end nodes N_1 and N_2 will then estimate the transmitted PNC vector $\bar{\boldsymbol{s}}_{\mathrm{R}}$ of $\boldsymbol{s}_{\mathrm{R}}$. Under the assumption that the estimated vector is correct, the end nodes will simply perform symbol XOR operation of the estimated vector with its own transmitted vector $\boldsymbol{s}_k$ to obtain the transmitted vector from its partner. Specifically, we have $\boldsymbol{s}_2 = \boldsymbol{s}_{\mathrm{R}} \oplus \boldsymbol{s}_1$ at N_1 and $\boldsymbol{s}_1 = \boldsymbol{s}_{\mathrm{R}} \oplus \boldsymbol{s}_2$ at N_2. For BPSK modulation, this is equivalent to $\boldsymbol{s}_2 = \boldsymbol{s}_{\mathrm{R}} \odot \boldsymbol{s}_1$ and $\boldsymbol{s}_1 = \boldsymbol{s}_{\mathrm{R}} \odot \boldsymbol{s}_2$, where $\odot$ denotes the element-wise vector multiplication operator. For example, assume that node N_1 has transmitted $\boldsymbol{s}_1 = [-1, \ -1]^T$ and that it has estimated the vector of PNC symbols $\boldsymbol{s}_{\mathrm{R}} = [1, \ -1]^T$ from the relay. Node N_1 then can easily obtain the transmitted vector from node N_2 as $\boldsymbol{s}_2 = \boldsymbol{s}_1 \odot \boldsymbol{s}_{\mathrm{R}} = [-1, \ -1]^T \odot [1, \ -1]^T = [-1, \ 1]^T$.

3. Proposed Two-Way Relay MIMO Communication Network Using STBC-PNC

In this section, we present the model of our proposed two-way relay MIMO-STBC-PNC system based on the previous general model of the MIMO-SDM-PNC system. We will first convert the system model of MIMO-STBC-PNC into the equivalent MIMO-SDM-PNC and then apply similar network coding operation at the relay and signal estimation at the end nodes. In order to make it consistent with the previous model, the same notations for transmit signal vector $\boldsymbol{s}$, noise vectors z_R, z_k, and channel matrices $\boldsymbol{H}_k$, $\boldsymbol{H}$ will be adopted in our model. However, please note that the definitions of these vectors/matrices may be different from the previous section.

3.1 Multiple Access Phase

The system model of the MIMO-STBC-PNC under consideration is shown in Fig. 2. During the MA phase transmission is divided into time slots. The two end nodes N_1 and N_2 use the Alamouti's STBC to encode the transmit symbols over two consecutive time slots. The objective of using STBC instead of SDM as in [11] is to achieve diversity gain rather than multiplexing gain. In order to enable the Alamouti's STBC transmission the channel in this case is further assumed to be quasi-static over at least two consecutive time slots.

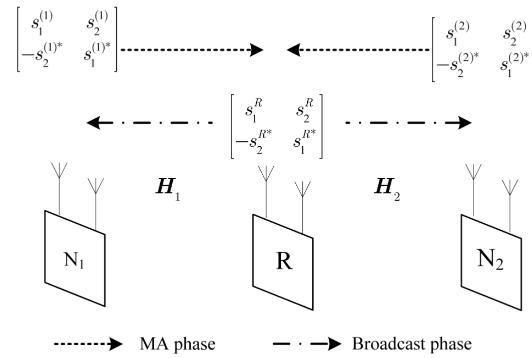

Fig. 2 System model of the proposed two-way relay MIMO-STBC-PNC network.

The received signals at the relay R during the first time slot is given by

$$y_{1,1} = \sqrt{\frac{\gamma_{sr}}{2E_s}} \left(h_{11}^{(1)} s_1^{(1)} + h_{12}^{(1)} s_2^{(1)} + h_{11}^{(2)} s_1^{(2)} + h_{12}^{(2)} s_2^{(2)} \right) + z_{1,1}, \tag{6}$$

$$y_{2,1} = \sqrt{\frac{\gamma_{sr}}{2E_s}} \left(h_{21}^{(1)} s_1^{(1)} + h_{22}^{(1)} s_2^{(1)} + h_{21}^{(2)} s_1^{(2)} + h_{22}^{(2)} s_2^{(2)} \right) + z_{2,1}, \tag{7}$$

and during the second time slot

$$y_{1,2} = \sqrt{\frac{\gamma_{sr}}{2E_s}} \left(h_{12}^{(1)} s_1^{(1)*} - h_{11}^{(1)} s_2^{(1)*} + h_{12}^{(2)} s_1^{(2)*} - h_{11}^{(2)} s_2^{(2)*} \right) + z_{1,2}, \tag{8}$$

$$y_{2,2} = \sqrt{\frac{\gamma_{sr}}{2E_s}} \left(h_{22}^{(1)} s_1^{(1)*} - h_{21}^{(1)} s_2^{(1)*} + h_{22}^{(2)} s_1^{(2)*} - h_{21}^{(2)} s_2^{(2)*} \right) + z_{2,2}, \tag{9}$$

where $y_{j,t}$, $j = 1, 2, t = 1, 2$, denotes the received signal at the j-th antenna of the relay during the t-th time slot; $h_{ji}^{(k)}$ the channel between the i-th antenna of end node N_k and the j-th antenna of the relay; $s_1^{(k)}$ and $s_2^{(k)}$ the two transmit symbols from user k; $z_{j,t}$ the AWGN noise component induced at the j-th antenna of the relay during the t-th time slot. In the above equations, γ_{sr} denotes the received signal-to-noise ratio at the relay, $E_s = \mathrm{E}\left\{\left|s_i^{(k)}\right|^2\right\}$ is the transmit signal energy, and the fraction $\frac{1}{\sqrt{2}}$ is the power normalization factor. For ease of presentation, we assume that γ_{sr} is the same for both the end nodes.

During the MA phase, the relay needs to estimate the PNC symbols $s_1^{\mathrm{R}} = s_1^{(1)} \oplus s_1^{(2)}$ and $s_2^{\mathrm{R}} = s_2^{(1)} \oplus s_2^{(2)}$ directly at the physical layer. In order to do so, we use the same MMSE-based approach as proposed in [5]. We first derive a linear MMSE detector to estimate the sum and difference combined symbols transmitted from the two end nodes: $s_1^+ \triangleq s_1^{(1)} + s_1^{(2)}$, $s_1^- \triangleq s_1^{(1)} - s_1^{(2)}$, $s_2^+ \triangleq s_2^{(1)} + s_2^{(2)}$, and $s_2^- \triangleq s_2^{(1)} - s_2^{(2)}$. These combined symbols will be then mapped into the PNC symbols as in (2) and (3) using likelihood ratio (LR) or selective combining (SC) estimation. The advantage of these combined MMSE and LR (MMSE-LR) and MMSE and SC (MMSE-SC) estimation over the simple MMSE estimation is the improved BER performance [5]. Moreover, thanks to simple LR and SC operations, the combined MMSE-LR or MMSE-SC estimation approach does not require significantly increased complexity.

3.1.1 Estimation of Sum and Difference Combined Symbols at Relay

The process of detecting the transmit symbols from the two end nodes is similar to the multiuser detection of STBC system. Therefore, we can apply the linear multiuser detection of STBC system in [15] to the current system. Taking complex conjugation over the received signal during the second time slot and using the method presented in [15], we can convert equations (6)–(9) into a convenient matrix form of the system equation as follows

$$\boldsymbol{y} = \sqrt{\frac{\gamma_{sr}}{2E_s}}\boldsymbol{H}\boldsymbol{s} + \boldsymbol{z}_R, \tag{10}$$

where $\boldsymbol{s} \triangleq [\boldsymbol{s}_1^T, \boldsymbol{s}_2^T]^T = [s_1^{(1)}, s_2^{(1)}, s_1^{(2)}, s_2^{(2)}]^T$, $\boldsymbol{s}_k \triangleq [s_1^{(k)}, s_2^{(k)}]^T$, $z_{\mathrm{R}} \triangleq [z_{1,1}, z_{1,2}^*, z_{2,1}, z_{2,2}^*]^T$, $\boldsymbol{y} \triangleq [y_{1,1}, y_{1,2}^*, y_{2,1}, y_{2,2}^*]^T$ and

$$\boldsymbol{H} \triangleq \begin{bmatrix} h_{11}^{(1)} & h_{12}^{(1)} & h_{11}^{(2)} & h_{12}^{(2)} \\ h_{12}^{(1)*} & -h_{11}^{(1)*} & h_{12}^{(2)*} & -h_{11}^{(2)*} \\ h_{21}^{(1)} & h_{22}^{(1)} & h_{21}^{(2)} & h_{22}^{(2)} \\ h_{22}^{(1)*} & -h_{21}^{(1)*} & h_{22}^{(2)*} & -h_{21}^{(2)*} \end{bmatrix}. \tag{11}$$

Note that the system model in (10) is similar to that for the MIMO-SDM-PNC in (1). Thus signal estimation and PNC mapping for MIMO-SDM-PNC can certainly be applied for the proposed MIMO-STBC-SDM. Now as explained in Sect. 2 the objective of PNC is to estimate the following combined symbols $s_1^+ = s_1^{(1)} + s_1^{(2)}$, $s_1^- = s_1^{(1)} - s_1^{(2)}$, $s_2^+ = s_2^{(1)} + s_2^{(2)}$, and $s_2^- = s_2^{(1)} - s_2^{(2)}$ and then map them to the PNC symbols. Linear estimation can be a good choice for the sake of simple optimization and complexity minimization. The principle of the linear detection is to use a combing matrix $\boldsymbol{W}$ to estimate the decision statistics of the transmitted symbols. Bearing in mind that we need to estimate the addition combinations and subtraction combinations of the transmitted symbols rather than individual symbols, we will convert the transmit vector $\boldsymbol{s}$ into an equivalent vector $\hat{\boldsymbol{s}}$ using a sum-difference matrix similar to [5]. For our system, we propose the sum-difference matrix as follows

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{I}_2 \\ \boldsymbol{I}_2 & -\boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}. \tag{12}$$

For $N > 2$ this matrix can be easily generalized with $\boldsymbol{I}_N$. Using the proposed matrix, we can convert the transmitted vector into the following desired vector

$$\hat{\boldsymbol{s}} \triangleq \boldsymbol{D}\boldsymbol{s} = [s_1^+,\ s_2^+,\ s_1^-,\ s_2^-]^T. \tag{13}$$

The system equation in (10) can be now expressed as follows

$$\boldsymbol{y} = \sqrt{\frac{\gamma_{sr}}{2E_s}}(\boldsymbol{H}\boldsymbol{D}^{-1})(\boldsymbol{D}\boldsymbol{s}) + z_{\mathrm{R}} = \rho\bar{\boldsymbol{H}}\hat{\boldsymbol{s}} + z_{\mathrm{R}} = \hat{\boldsymbol{H}}\hat{\boldsymbol{s}} + z_{\mathrm{R}}, \tag{14}$$

where $\rho \triangleq \sqrt{\frac{\gamma_{sr}}{2E_s}}$, $\bar{\boldsymbol{H}} \triangleq \boldsymbol{H}\boldsymbol{D}^{-1}$ and $\hat{\boldsymbol{H}} \triangleq \rho\bar{\boldsymbol{H}}$.

When the linear MMSE estimation is used the weight combining matrix can be given by solving the following cost function

$$\boldsymbol{W} = \arg\min_{\boldsymbol{W}} \mathrm{E}\{\|\Delta_{\hat{s}}\|_2^2\} = \arg\min_{\boldsymbol{W}} \mathrm{E}\left\{\|\hat{\boldsymbol{s}} - \boldsymbol{W}\boldsymbol{y}\|_2^2\right\}. \tag{15}$$

In order to find the weight matrix that minimizes the MSE we note that

$$\mathrm{E}\{\|\Delta_{\hat{s}}\|_2^2\} = \mathrm{E}\{\mathrm{trace}(\boldsymbol{R}_{\Delta_{\hat{s}}}) = \mathrm{trace}\left(\mathrm{E}\left\{\boldsymbol{R}_{\Delta_{\hat{s}}}\right\}\right), \tag{16}$$

where $\boldsymbol{R}_{\Delta_{\hat{s}}} = [\hat{\boldsymbol{s}} - \boldsymbol{W}\boldsymbol{y}]\,[\hat{\boldsymbol{s}} - \boldsymbol{W}\boldsymbol{y}]^H = \hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^H - \boldsymbol{W}\boldsymbol{y}\hat{\boldsymbol{s}}^H - \hat{\boldsymbol{s}}\boldsymbol{y}^{\mathrm{H}}\mathbf{W}^H + \boldsymbol{W}\boldsymbol{y}\boldsymbol{y}^H\boldsymbol{W}^H$, and the superscript ^{H} denotes the conjugate transpose of matrix. The error covariance matrix is given by

$$\begin{aligned}\mathrm{E}\left\{\boldsymbol{R}_{\Delta_{\hat{s}}}\right\} &= \mathrm{E}\left\{\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^H\right\} - \mathbf{W}\mathrm{E}\left\{\boldsymbol{y}\hat{\boldsymbol{s}}^H\right\} - \mathrm{E}\left\{\hat{\boldsymbol{s}}\boldsymbol{y}^H\right\}\boldsymbol{W}^H \\ &\quad + \boldsymbol{W}\mathrm{E}\left\{\boldsymbol{y}\boldsymbol{y}^H\right\}\boldsymbol{W}^H. \end{aligned}\tag{17}$$

Note that since the two end nodes N_1 and N_2 are assumed to transmit at the same energy, so we have $\mathrm{E}\{\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^{\mathrm{H}}\} = 2E_s\boldsymbol{I}_4$. Further simple calculation gives us $\mathrm{E}\{\boldsymbol{y}\hat{\boldsymbol{s}}^{\mathrm{H}}\} = \hat{\boldsymbol{H}}\mathrm{E}\{\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^H\} = 2E_s\hat{\boldsymbol{H}}$, $\mathrm{E}\{\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}\} = \hat{\boldsymbol{H}}\mathrm{E}\{\hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^H\}\hat{\boldsymbol{H}}^H + \mathrm{E}\{z_{\mathrm{R}}z_{\mathrm{R}}^H\} = 2E_s\hat{\boldsymbol{H}}\hat{\boldsymbol{H}}^{\mathrm{H}} + \boldsymbol{I}_4$. The error covariance matrix is thus given by

$$\begin{aligned}\mathrm{E}\left\{\boldsymbol{R}_{\Delta_{\hat{s}}}\right\} &= 2E_s\boldsymbol{I}_4 - 2E_s\boldsymbol{W}\hat{\boldsymbol{H}} - 2E_s\hat{\boldsymbol{H}}^H\boldsymbol{W}^H \\ &\quad + \boldsymbol{W}(2E_s\hat{\boldsymbol{H}}\hat{\boldsymbol{H}}^H + \boldsymbol{I}_4)\boldsymbol{W}^H. \end{aligned}\tag{18}$$

As a result we have the MSE

$$\begin{aligned}\mathrm{E}\{\|\Delta_{\hat{s}}\|_2^2\} &= \mathrm{trace}\Big(2E_s\boldsymbol{I}_4 - 2E_s\boldsymbol{W}\hat{\boldsymbol{H}} \\ &\quad - 2E_s\hat{\boldsymbol{H}}^H\boldsymbol{W}^H + \boldsymbol{W}(2E_s\hat{\boldsymbol{H}}\hat{\boldsymbol{H}}^H + \boldsymbol{I}_4)\boldsymbol{W}^H\Big). \end{aligned}\tag{19}$$

Taking derivative of the trace of $\mathrm{E}\{\|\Delta_{\hat{s}}\|^2\}$ with respect to $\boldsymbol{W}$ and set it to zero we obtain the combining matrix as follows

$$\boldsymbol{W} = 2E_s\hat{\boldsymbol{H}}^H\left(2E_s\hat{\boldsymbol{H}}\hat{\boldsymbol{H}}^H + \boldsymbol{I}_4\right)^{-1}. \tag{20}$$

Given the combining matrix $\boldsymbol{W}$ the decision statistics at the output of the MMSE combiner are given by $\check{\boldsymbol{s}} = \boldsymbol{W}\boldsymbol{y}$, where the estimated vector can be express as $\check{\boldsymbol{s}} = [\check{s}_1^+,\ \check{s}_2^+,\ \check{s}_1^-,\ \check{s}_2^-]^T$.

3.1.2 Estimation of Network Coded Symbols Using Likelihood Ratio (LR)

First we note that the entries of $\check{\boldsymbol{s}}$ can be expressed as

$\breve{s}_1^+ = \breve{s}_1^{(1)} + \breve{s}_1^{(2)} + \breve{z}_{1,1}$, $\breve{s}_2^+ = \breve{s}_2^{(1)} + \breve{s}_2^{(2)} + \breve{z}_{1,2}$, $\breve{s}_1^- = \breve{s}_1^{(1)} - \breve{s}_1^{(2)} + \breve{z}_{2,1}$, $\breve{s}_2^- = \breve{s}_2^{(1)} - \breve{s}_2^{(2)} + \breve{z}_{2,2}$, where $\breve{z}_{j,t}$ are the residual noise at the outputs of the combiner with variances $\sigma^2_{\breve{z}_{j,t}} = \sigma_z^2(\boldsymbol{W}\boldsymbol{W}^H)_{2(j-1)+t,2(j-1)+t}$. For ease of presentation, let $\ell \triangleq 2(j-1)+t$ so that we can express $\breve{z}_\ell = \breve{z}_{j,t}$ and $\sigma_\ell^2 = \sigma_z^2(\boldsymbol{W}\boldsymbol{W}^H)_{\ell,\ell}$. Here $\boldsymbol{A}_{\ell,\ell}$ denotes the ℓ-th diagonal entry of $\boldsymbol{A}$. The probability density function (pdf) of the residual noise $\breve{z}_\ell$ is given by

$$p(\breve{z}_\ell) = \frac{1}{\sqrt{2\pi}\sigma_\ell} e^{-\frac{\breve{z}_\ell^2}{2\sigma_\ell^2}}. \tag{21}$$

From the estimated combined symbols given above our objective is to map them into the network coded symbols s_1^{R} and s_2^{R} using LR estimation as follows: $\left\{\breve{s}_1^+, \breve{s}_1^-\right\} \xrightarrow{\mathrm{LR}} s_1^{(1)} \oplus s_1^{(2)}$ and $\left\{\breve{s}_2^+, \breve{s}_2^-\right\} \xrightarrow{\mathrm{LR}} s_2^{(1)} \oplus s_2^{(2)}$. Since the two mappings are similar, we will present detailed derivation for the first one. The second can be easily obtained in the same manner. The LR mapping involves maximum a posteriori (MAP) estimation of $s_1^{\mathrm{R}} = s_1^{(1)} \oplus s_1^{(2)}$ using both $\breve{s}_1^+$ and $\breve{s}_1^-$. In order to make it easy to follow we will start by two cases of estimating s_1^{R} using each $\breve{s}_1^+$ and $\breve{s}_1^-$. The obtained results will be then used for the case of using both $\breve{s}_1^+$ and $\breve{s}_1^-$.

A. Estimation of s_1^{R}

Case 1: Estimation of s_1^{R} from $\breve{s}_1^+$. For BPSK modulation, $s_1^{\mathrm{R}} = s_1^{(1)} \oplus s_1^{(2)}$ takes on 1 when $s_1^{(1)} + s_1^{(2)} = \pm 2$ and -1 when $s_1^{(1)} + s_1^{(2)} = 0$. The MAP rule to estimate s_1^{R} is given by

$$\hat{s}_1^{\mathrm{R}} = \arg\max_{s=\pm 1}\ p\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = s\right) P\left(s_1^{\mathrm{R}} = s\right), \tag{22}$$

where $p(\breve{s}_1^+ | s_1^{\mathrm{R}} = s)$ is the conditional pdf of the observed $\breve{s}_1^+$ given s_1^{R} and is given by

$$p\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = s\right) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(\breve{s}_1^+)^2}{2\sigma_1^2}} & \text{for } s = -1 \\ \frac{1}{2\sqrt{2\pi}\sigma_1}\left(e^{-\frac{(\breve{s}_1^+-2)^2}{2\sigma_1^2}} + e^{-\frac{(\breve{s}_1^++2)^2}{2\sigma_1^2}}\right) & \text{for } s = 1. \end{cases} \tag{23}$$

Now for ease of presentation, let us denote

$$P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = -1\right) \triangleq \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(\breve{s}_1^+)^2}{2\sigma_1^2}} \tag{24}$$

$$P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = 1\right) \triangleq \frac{1}{2\sqrt{2\pi}\sigma_1}\left(e^{-\frac{(\breve{s}_1^+-2)^2}{2\sigma_1^2}} + e^{-\frac{(\breve{s}_1^++2)^2}{2\sigma_1^2}}\right). \tag{25}$$

Note that since the transmitted bits are equally probable we have $P(s_1^{\mathrm{R}} = -1) = P(s_1^{\mathrm{R}} = 1) = \frac{1}{2}$. The PNC mapping for the network coded symbols given $\breve{s}_1^+$ now becomes

$$\hat{s}_{\mathrm{R}}^{(1)} = \begin{cases} -1 & \text{if } P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = -1\right) \geq P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = 1\right) \\ 1 & \text{if } P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = -1\right) < P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = 1\right) \end{cases}. \tag{26}$$

Case 2: Estimation of s_1^{R} from $\breve{s}_1^-$. Using the similar derivation for $\breve{s}_1^+$ we have the PNC mapping given $\breve{s}_1^-$ as

$$\hat{s}_1^{\mathrm{R}} = \begin{cases} -1 & \text{if } P\left(\breve{s}_1^- \middle| s_1^{\mathrm{R}} = -1\right) \geq P\left(\breve{s}_1^- \middle| s_1^{\mathrm{R}} = 1\right) \\ 1 & \text{if } P\left(\breve{s}_1^- \middle| s_1^{\mathrm{R}} = -1\right) < P\left(\breve{s}_1^- \middle| s_1^{\mathrm{R}} = 1\right) \end{cases}, \tag{27}$$

where

$$P\left(\breve{s}_1^- \middle| s_1^{\mathrm{R}} = -1\right) \triangleq \frac{1}{2\sqrt{2\pi}\sigma_3}\left(e^{-\frac{(\breve{s}_1^--2)^2}{2\sigma_3^2}} + e^{-\frac{(\breve{s}_1^-+2)^2}{2\sigma_3^2}}\right) \tag{28}$$

$$P\left(\breve{s}_1^- \middle| s_1^{\mathrm{R}} = 1\right) \triangleq \frac{1}{\sqrt{2\pi}\sigma_3} e^{-\frac{(\breve{s}_1^-)^2}{2\sigma_3^2}}. \tag{29}$$

Case 3: Estimation of s_1^{R} from both $\breve{s}_1^+$ and $\breve{s}_1^-$. Given both $\breve{s}_1^+$ and $\breve{s}_1^-$ we have

$$P\left(\breve{s}_1^+\breve{s}_1^- \middle| s_1^{\mathrm{R}} = -1\right) \triangleq P\left(\breve{s}_1^+ \middle| s_1^+ = 0\right)\left[P\left(\breve{s}_1^- \middle| s_1^- = 2\right) + P\left(\breve{s}_1^- \middle| s_1^- = -2\right)\right], \tag{30}$$

$$P\left(\breve{s}_1^+\breve{s}_1^- \middle| s_1^{\mathrm{R}} = 1\right) \triangleq P\left(\breve{s}_1^- \middle| s_1^- = 0\right)\left[P\left(\breve{s}_1^+ \middle| s_1^+ = 2\right) + P\left(\breve{s}_1^+ \middle| s_1^+ = -2\right)\right]. \tag{31}$$

The LR of s_1^{R} can be expressed as

$$\mathcal{L}\left(s_1^{\mathrm{R}} \middle| \breve{s}_1^+\breve{s}_1^-\right) = \frac{P\left(\breve{s}_1^+\breve{s}_1^- \middle| s_1^{\mathrm{R}} = -1\right)}{P(\breve{s}_1^+\breve{s}_1^- | s_1^{\mathrm{R}} = 1)} = \frac{P\left(\breve{s}_1^+ \middle| s_1^+ = 0\right)\left[P\left(\breve{s}_1^- \middle| s_1^- = 2\right) + P\left(\breve{s}_1^- \middle| s_1^- = -2\right)\right]}{P\left(\breve{s}_1^- \middle| s_1^- = 0\right)\left[P\left(\breve{s}_1^+ \middle| s_1^+ = 2\right) + P\left(\breve{s}_1^+ \middle| s_1^+ = -2\right)\right]}. \tag{32}$$

Note that since we have

$$P\left(\breve{s}_1^+ | s_1^+ = 0\right) = P\left(\breve{s}_1^+ | s_1^{\mathrm{R}} = -1\right), \tag{33}$$

$$P\left(\breve{s}_1^- | s_1^- = 2\right) + P\left(\breve{s}_1^- | s_1^- = -2\right) = P\left(\breve{s}_1^- | s_1^{\mathrm{R}} = -1\right), \tag{34}$$

$$P\left(\breve{s}_1^- | s_1^- = 0\right) = P\left(\breve{s}_1^- | s_1^{\mathrm{R}} = 1\right), \tag{35}$$

$$P\left(\breve{s}_1^+ | s_1^+ = 2\right) + P\left(\breve{s}_1^+ | s_1^+ = -2\right) = P\left(\breve{s}_1^+ | s_1^{\mathrm{R}} = 1\right), \tag{36}$$

then (32) becomes

$$\mathcal{L}\left(s_1^{\mathrm{R}} \middle| \breve{s}_1^+\breve{s}_1^-\right) = \frac{P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = -1\right) P(\breve{s}_1^- | s_1^{\mathrm{R}} = -1)}{P\left(\breve{s}_1^- \middle| s_1^{\mathrm{R}} = 1\right) P\left(\breve{s}_1^+ \middle| s_1^{\mathrm{R}} = 1\right)}. \tag{37}$$

Now replace (24), (25), (28), (29) into (32) we obtain

$$\mathcal{L}\left(s_1^{\mathrm{R}} \middle| \breve{s}_1^+\breve{s}_1^-\right) = \frac{e^{-\frac{(\breve{s}_1^+)^2}{2\sigma_1^2}}\left(e^{-\frac{(\breve{s}_1^--2)^2}{2\sigma_3^2}} + e^{-\frac{(\breve{s}_1^-+2)^2}{2\sigma_3^2}}\right)}{e^{-\frac{(\breve{s}_1^-)^2}{2\sigma_3^2}}\left(e^{-\frac{(\breve{s}_1^+-2)^2}{2\sigma_1^2}} + e^{-\frac{(\breve{s}_1^++2)^2}{2\sigma_1^2}}\right)} = \frac{\cosh\left(\frac{2\breve{s}_1^-}{\sigma_3^2}\right)}{\cosh\left(\frac{2\breve{s}_1^+}{\sigma_1^2}\right)} e^{\left(\frac{2}{\sigma_1^2} - \frac{2}{\sigma_3^2}\right)}. \tag{38}$$

Based on the obtained LR, the decisions are then made for the network coded symbols as follows:

$$s_1^{\mathrm{R}} = \begin{cases} -1 & \text{if } \mathcal{L}\left(s_1^{\mathrm{R}} \middle| \breve{s}_1^+ \breve{s}_1^-\right) \geq 1 \\ 1 & \text{if } \mathcal{L}\left(s_1^{\mathrm{R}} \middle| \breve{s}_1^+ \breve{s}_1^-\right) < 1 \end{cases}. \tag{39}$$

B. Estimation of s_2^{R}

From the estimated combined symbols $\breve{s}_2^+$ and $\breve{s}_2^-$ following the similar derivation for s_1^{R} we have the LR for estimating s_2^{R} given by

$$\mathcal{L}\left(s_2^{\mathrm{R}} \middle| \breve{s}_2^+ \breve{s}_2^-\right) = \frac{e^{-\frac{(\breve{s}_2^+)^2}{2\sigma_2^2}}\left(e^{-\frac{(\breve{s}_2^- -2)^2}{2\sigma_4^2}} + e^{-\frac{(\breve{s}_2^- +2)^2}{2\sigma_4^2}}\right)}{e^{-\frac{(\breve{s}_2^-)^2}{2\sigma_4^2}}\left(e^{-\frac{(\breve{s}_2^+ -2)^2}{2\sigma_2^2}} + e^{-\frac{(\breve{s}_2^+ +2)^2}{2\sigma_2^2}}\right)} = \frac{\cosh\left(\frac{2\breve{s}_2^-}{\sigma_4^2}\right)}{\cosh\left(\frac{2\breve{s}_2^+}{\sigma_2^2}\right)} e^{\left(\frac{2}{\sigma_2^2} - \frac{2}{\sigma_4^2}\right)}. \tag{40}$$

The decision rule for the network coded symbols s_2^{R} as follows

$$s_2^{\mathrm{R}} = \begin{cases} -1 & \text{if } \mathcal{L}\left(s_2^{\mathrm{R}} \middle| \breve{s}_2^+ \breve{s}_2^-\right) \geq 1 \\ 1 & \text{if } \mathcal{L}\left(s_2^{\mathrm{R}} \middle| \breve{s}_2^+ \breve{s}_2^-\right) < 1 \end{cases}. \tag{41}$$

3.1.3 Estimation of Network Coded Symbols Using Selective Combining

The method of estimating the network coded symbols using LR promises good BER performance. However, it requires the knowledge of the residual noise variance at the outputs of the combiner. When it is difficult to estimate the noise variance, selective combining (SC) as proposed in [5] can be employed at the cost of performance degradation. Similar to LR estimation, upon having estimated $\breve{\boldsymbol{s}} = [\breve{s}_1^+, \breve{s}_2^+, \breve{s}_1^-, \breve{s}_2^-]^T$ using the MMSE estimation, the principle of SC is to map these estimated combined symbols into the network coded symbols s_1^{R} and s_2^{R} as follows: $\left\{\breve{s}_1^+, \breve{s}_1^-\right\} \xrightarrow{\mathrm{SC}} s_1^{(1)} \oplus s_1^{(2)}$ and $\left\{\breve{s}_2^+, \breve{s}_2^-\right\} \xrightarrow{\mathrm{SC}} s_2^{(1)} \oplus s_2^{(2)}$. Using SC the network coded symbols are decided as follows

$$s_1^{\mathrm{R}} = \begin{cases} \mathrm{sign}\left\{|\breve{s}_1^+| - \gamma\right\} & \text{if } (\boldsymbol{W}\boldsymbol{W}^H)_{1,1} < (\boldsymbol{W}\boldsymbol{W}^H)_{3,3} \\ \mathrm{sign}\left\{\gamma - |\breve{s}_1^-|\right\} & \text{otherwise} \end{cases} \tag{42}$$

$$s_2^{\mathrm{R}} = \begin{cases} \mathrm{sign}\left\{|\breve{s}_2^+| - \gamma\right\} & \text{if } (\boldsymbol{W}\boldsymbol{W}^H)_{2,2} < (\boldsymbol{W}\boldsymbol{W}^H)_{4,4} \\ \mathrm{sign}\left\{\gamma - |\breve{s}_2^-|\right\} & \text{otherwise} \end{cases} \tag{43}$$

where $\mathrm{sign}(\cdot)$ denotes the signum function and γ the combining threshold. Note that decisions in the above equations depend on the selected threshold and the residual noise at the combiner outputs. Therefore, the threshold should be selected carefully for a particular system. This can be done by simulation as shown in the next section.

3.2 Broadcast Phase

Upon obtaining the network coded symbols s_1^{R} and s_2^{R}, the relay R will broadcast these symbols to the two end nodes N_1 and N_2. This broadcast transmission can be done using either spatial multiplexing as in [11] or STBC as the two end-nodes do. In this paper, we use the STBC as illustrated in Fig. 2 in order to achieve diversity gain and reduce the computational complexity of the detectors at the end nodes. Using the Alamouti's STBC these symbols will be encoded and sent through its two antennas. Each end node will perform linear processing on the received vector and detect the transmitted vector $\boldsymbol{s}^{\mathrm{R}} = \left[s_1^{\mathrm{R}}, s_2^{\mathrm{R}}\right]^T$ from the relay and then try to recover the transmitted vector from the other end node by performing XOR of the estimated vector with its transmitted vector. Using the equivalent model in (10), we can express the received signal vector at the two end nodes after processing over two time slots as follows

$$\underbrace{\begin{bmatrix} u_{1,1}^{(k)} \\ u_{1,2}^{(k)*} \\ u_{2,1}^{(k)} \\ u_{2,2}^{(k)*} \end{bmatrix}}_{\boldsymbol{u}_k} = \sqrt{\frac{\gamma_{rd}}{2E_s}} \underbrace{\begin{bmatrix} \hat{h}_{11}^{(k)} & \hat{h}_{12}^{(k)} \\ \hat{h}_{12}^{(k)*} & -\hat{h}_{11}^{(k)*} \\ \hat{h}_{21}^{(k)} & \hat{h}_{22}^{(k)} \\ \hat{h}_{22}^{(k)*} & -\hat{h}_{21}^{(k)*} \end{bmatrix}}_{\hat{\boldsymbol{H}}_k} \underbrace{\begin{bmatrix} s_1^{\mathrm{R}} \\ s_2^{\mathrm{R}} \end{bmatrix}}_{\boldsymbol{s}^{\mathrm{R}}} + \underbrace{\begin{bmatrix} z_{1,1}^{(k)} \\ z_{1,2}^{(k)*} \\ z_{2,1}^{(k)} \\ z_{2,2}^{(k)*} \end{bmatrix}}_{\boldsymbol{z}_k} \tag{44}$$

where $u_{i,t}^{(k)}$ and $z_{i,t}^{(k)}$ denote respectively the received signal and induced noise at the i-th antenna of node N_k at the t-th time slot; γ_{rd} is the received SNR at the end nodes; $\hat{h}_{ij}^{(k)}$ denotes the channel from the j-th antenna of of node N_k to the i-th antenna of the relay. Using appropriate notations we can simply express (44) as

$$\boldsymbol{u}_k = \sqrt{\frac{\gamma_{rd}}{2E_s}} \hat{\boldsymbol{H}}_k \boldsymbol{s}^{\mathrm{R}} + \boldsymbol{z}_k. \tag{45}$$

It is noted that since $\hat{\boldsymbol{H}}_k^H \hat{\boldsymbol{H}}_k = \sum_i \sum_j |\hat{h}_{ij}^{(k)}|^2 \boldsymbol{I}_2$ then if the end nodes know the CSI of the reverse links from the relay to the end nodes they can easily estimate $\boldsymbol{s}^{\mathrm{R}}$ from the received vector $\boldsymbol{u}_k$ by equalizing the fading effect using an equalization matrix $\hat{\boldsymbol{H}}_k^H$ as $\hat{\boldsymbol{s}}_k^{\mathrm{R}} = \hat{\boldsymbol{H}}_k^H \boldsymbol{u}_k = \left[\hat{s}_1^{\mathrm{R}}, \hat{s}_2^{\mathrm{R}}\right]^T$. Different from the precoding approach in [7] where CSI feedback is necessary, the requirement of the CSI in our system is possible in practical systems since the end nodes can estimate the reverse channels by extracting information during the control packet† exchange process [16]. Moreover, in the precoding approach the CSI is needed for joint transceiver optimization which requires more complexity at both the end nodes and the relay. The CSI in our system is, however, only used for channel equalization at the receiver of the end nodes.

After the channel equalization, a quantization function will be used to make decision on the transmitted network coded symbols as $\bar{\boldsymbol{s}}_k^{\mathrm{R}} = Q(\hat{\boldsymbol{s}}_k^{\mathrm{R}})$, where each component is independently quantized. The last operation is to perform

†Eg. Request to Send (RTS) and Clear to Send (CTS)

XOR of the quantized vector $\bar{s}_k^{\mathrm{R}}$ with its own transmitted vector s_k to recover the transmitted vector from the other side.

4. Proposed Relay Selection Based on MSE

In this section, we propose a relay selection algorithm which aims at minimizing the estimation MSE. Now using (18), (20) and after some manipulations we obtain

$$\begin{aligned} \mathrm{E}\{\boldsymbol{R}_{\Delta_{\hat{s}}}\} &= 2E_s\left(\boldsymbol{I}_4 - \boldsymbol{W}\hat{\boldsymbol{H}}\right) \\ &= 2E_s\left(\boldsymbol{I}_4 - \hat{\boldsymbol{H}}^H\left(\hat{\boldsymbol{H}}\hat{\boldsymbol{H}}^H + \frac{1}{2E_s}\boldsymbol{I}_4\right)^{-1}\hat{\boldsymbol{H}}\right). \end{aligned} \quad (46)$$

Using the matrix inversion lemma $(\boldsymbol{A}+\boldsymbol{BCD})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}\left(\boldsymbol{DA}^{-1}\boldsymbol{B}+\boldsymbol{C}^{-1}\right)^{-1}\boldsymbol{DA}^{-1}$ and matching the right side of the lemma equation with that of (46) we can obtain the error covariance matrix in the following short form $\mathrm{E}\{\boldsymbol{R}_{\Delta_{\hat{s}}}\} = 2E_s\left(\boldsymbol{I}_4 + 2E_s\hat{\boldsymbol{H}}^H\hat{\boldsymbol{H}}\right)^{-1}$. The average MSE is then given by

$$\begin{aligned} \overline{\mathrm{MSE}} &= \frac{1}{4}\mathrm{trace}\left(2E_s\left(\boldsymbol{I}_4 + 2E_s\hat{\boldsymbol{H}}^H\hat{\boldsymbol{H}}\right)^{-1}\right) \\ &= \frac{E_s}{2}\mathrm{trace}\left(\boldsymbol{I}_4 + 2E_s\hat{\boldsymbol{H}}^H\hat{\boldsymbol{H}}\right)^{-1}. \end{aligned} \quad (47)$$

Now assume that there are L intermediate nodes which are capable of serving as the relay. The network needs to select the best one to act as the relay. In this paper we propose to use the MSE as a criterion to select the best relay. It is clear that the intermediate node with the minimum average MSE will provide the best error performance. Thus the relay selection algorithm needs to select one out of L intermediate nodes with minimum MSE.

Given an intermediate node $l = 1, 2, ..., L$ the MSE associated with it is given from (19) as

$$\overline{\mathrm{MSE}}_l = \frac{E_s}{2}\mathrm{trace}\left(\boldsymbol{I}_4 + 2E_s\hat{\boldsymbol{H}}_l^H\hat{\boldsymbol{H}}_l\right)^{-1} \quad (48)$$

where $\hat{\boldsymbol{H}}_l$ denotes the channel matrix via node N_l. Note that since E_s takes on positive value and is independent of l while $\boldsymbol{I}_4$ is an identity matrix, the MSE based relay select algorithm can be expressed in a more compact form as follows

$$\hat{l} = \arg\min_l \mathrm{trace}\left(\hat{\boldsymbol{H}}_l^H\hat{\boldsymbol{H}}_l\right)^{-1}. \quad (49)$$

As the channel matrix $\hat{\boldsymbol{H}}$ contains both the channel $\boldsymbol{H}_l^{(1)}$ from the end node 1 to intermediate l and $\boldsymbol{H}_l^{(2)}$ from node 2 to l, a distributed relay selection algorithm such as proposed in [16] can be used at the intermediate nodes to determine if one can be the relay.

5. Performance Evaluation

In this section we present our simulation results to evaluate the BER performance of the proposed network. In our simulations, slowly varying uncorrelated flat Rayleigh fading is assumed and BPSK is used for modulation.

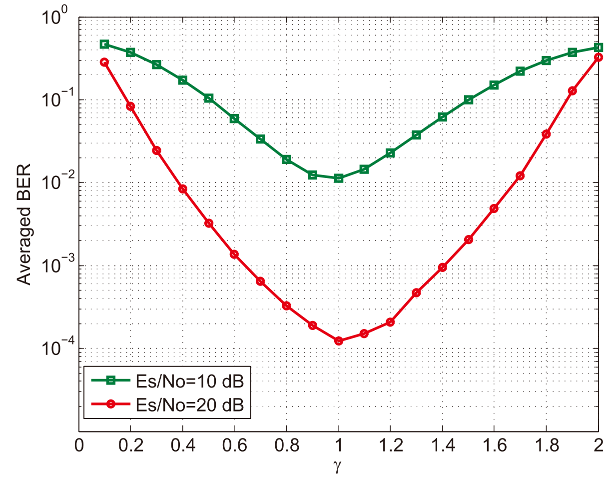

Fig. 3 Threshold selection in MIMO-STBC-PNC system.

In the first investigation, we analyse the effect of choosing the threshold γ for the case of selective combining on the BER performance. The threshold was varied from 0 to 2 for 2 typical values of E_s/N_0, namely, 10 dB and 20 dB. Figure 3 shows the simulated averaged BER versus E_s/N_0. It can be seen clearly from the figure that the best value is $\gamma = 1$, particularly at the high E_s/N_0 region. We will use this value in our following simulations for the case of selective combining. We would like to note that for the case zero forcing (ZF) estimation is used the best threshold value is $\gamma = 1.2$. The simulated results however are not shown here due to limited space.

Figure 4 shows the end-to-end averaged BER performance of the proposed MIMO-STBC-PNC system for the case without using relay selection, i.e. $L = 1$. In order to compare with the proposed system in [5], BER performance of SIMO-PNC is also shown in the figures for reference. It is shown clearly that the proposed MIMO-STBC-PNC system significantly outperforms the SIMO-PNC system due to having higher diversity order. We can see that our proposed system can double diversity order compared to the SIMO-PNC system in [5]. Apart from achieving the same diversity order of 2, Fig. 4 also indicates the E_s/N_0 gain of about 1 dB over the 2×1 Alamouti STBC. Comparing BER curves of the case using LR with SC we can see a gap of about 2.5 dB in E_s/N_0. The LR decision is thus more superior to the SC and clearly a good choice if noise variance estimation is possible.

In order to highlight more about the effectiveness of our proposed system, we have also included in Fig. 4 the BER performance of the MIMO-STBC network coding (MIMO-STBC-NC) system. In this MIMO-STBC-NC system the MMSE detection is used to estimate transmitted symbols from N_1 and N_2 separately followed by applying XOR to obtain the PNC symbols. As it was known from [5] this NC is inferior to the PNC scheme using combined linear estimation and LR/selective combination for both cases of

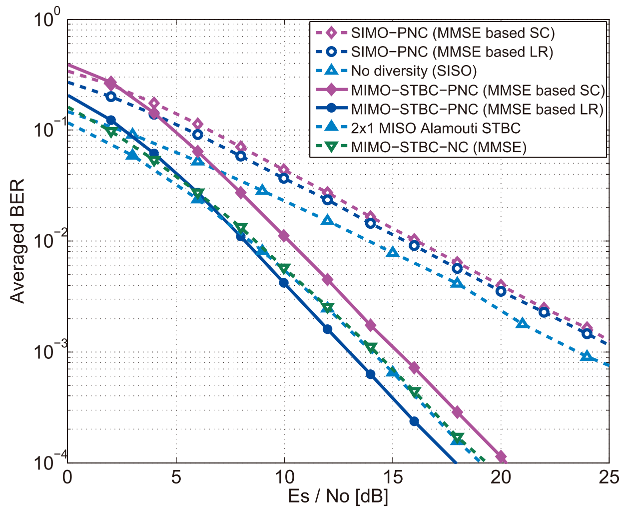

Fig. 4 BER performance of the proposed MIMO-STBC-PNC system.

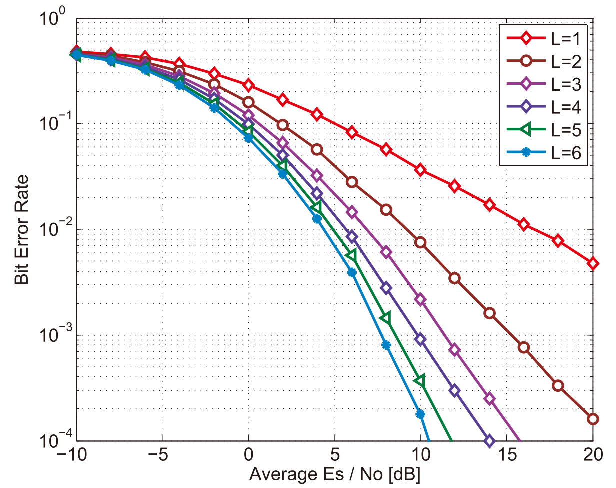

Fig. 5 BER performance of the proposed MIMO-STBC-PNC system with relay selection.

using ZF or MMSE. It can be seen from the figure that the BER performance of the MIMO-STBC-NC scheme is better than that of the proposed system using SC about 1 dB. However, compared with the proposed system using LR it has similar performance at low E_s/N_0 region while loosing about 1 dB gain at high E_s/N_0 region. Although the MIMO-STBC-NC system performs better than our proposed system using SC, we would like to note that performing NC implies that the received signals during the MA phase need to be detected to the symbol level followed by applying XOR to achieve PNC symbols. Meanwhile, PNC symbols can be estimated directly from the output of the MMSE estimator in the proposed system. It can also be noted that the BER performance of the MIMO-STBC-NC coincides with that of the 2×1 MISO STBC system for $E_s/N_0 > 6$ dB. However, at when $E_s/N_0 < 6$ dB, the MIMO-STBC-NC performs worse than the STBC due to error propagation in the BC phase.

In order to evaluate the efficacy of the proposed relay selection method, we have set up a more realistic scenario where intermediate nodes are uniformly distributed between the two end nodes. In order to simulate such scenario the local links N_1–R and N_2–R are still generated using i.i.d. complex random variables with mean zero and unit variance to account for the Rayleigh fading. However, the average SNRs γ_{sr1} and γ_{sr2} of each link is randomly distributed about an average SNR value, denoted by $\mathrm{SNR_{ave}}$, to account for different path loss associated with relay positions. This means that given a value of $\mathrm{SNR_{ave}}$ the instantaneous γ_{sr1} and γ_{sr2} can be generated by $\gamma_{srk} = 2 \times \mathrm{SNR_{ave}} \times \mathsf{rand}$, where rand represents a uniformly distributed variable taking on values over the range from 0 to 1. The simulated BER performance is shown in Fig. 5. It can be clearly seen from the figure the proposed relay selection method allows us to achieve significant BER performance improvement. The more intermediate nodes exist, the deeper slope of BER curve can be achieved.

In order to compare the complexity of our proposed MIMO-STBC-PNC network with the previous SIMO-PNC in [5], we note that the increase in computational complexity at the two end nodes is not significant since only linear processing is used for both transmission and reception. The complexity at the relay is, however, increased since the linear detector needs to invert a larger (double size) matrix to compute the combining weight $\boldsymbol{W}$. Since the matrix to be inverted in the weight matrix is a Hermitian matrix, the complexity order of our proposed system will be $C_{\mathrm{Proposed}} \sim O(4N^3)$. The complexity in the system of [5] is $C_{\mathrm{Ref.}\,[5]} \sim O(N^3)$. Thus our system requires quadruple complexity of that in [5]. Clearly that for small N the increase in complexity is acceptable.

When comparing with the counterpart MIMO-STBC-NC in [6], we note that both networks use a same system model. The network in [6], however, uses ML detection at all nodes N_1, N_2, and R. As a result, its complexity at all nodes can be expressed as $C_{\mathrm{Ref.}\,[6]} \sim O(M^{2N})$. This complexity is an exponential function of the modulation order M and the number of antenna N. Clearly it will become significantly large when higher order modulation $M \geq 4$ is used as well as if the network is extended to the case with more antennas $N > 2$.

Table 1 compares the estimated complexity of our proposed system with those of related systems based on the complexity order. It can be seen from the table that for small number of antennas the complexity of our system is comparable with that of the SIMO-PNC in [5]. Comparing with the MIMO-STBC-NC in [6] our system requires double complexity for the case $N = 2$ and $M = 2$. However, for

Table 1 Computational complexity comparison.

N	2	4	8
$C_{\mathrm{ref.}[5]} = N^3$	8	16	512
$C_{\mathrm{proposed}} = 4N^3$	32	64	2048
$C_{\mathrm{ref.}[6]} = M^{2N}, M = 2$	16	256	65,536
$C_{\mathrm{ref.}[6]} = M^{2N}, M = 4$	256	65,536	$\approx 4.3 \cdot 10^9$

large M or N, the computational complexity of the MIMO-STBC-NC increases significantly over that of our system.

6. Conclusions

In this paper we have proposed a design of a two-way relay network using the Alamouti's space-time block coded physical layer network coding. Thanks to the proper design of the sum-difference matrix to separate the multiuser signals the proposed network can use only linear processing at all nodes to estimate signal and save the computational complexity. Using computer simulations, we have demonstrated that the network achieves diversity order 2 while requiring only polynomial complexity. The proposed network can achieve more BER performance improvement when employing the proposed MSE-based simple relay selection method.

Acknowledgement

This work was supported by the National Foundation for Science and Technology Development (NAFOSTED) of Vietnam under project number 102.03.2012.18.

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