# Asymmetrical Radiation Pattern Synthesis of a Linear Array Antenna by a Least Mean Square Method 

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#### Abstract

In mobile base station antennas, linear array antennas are employing and asymmetrical radiation patterns are achieved. For multi frequency use, unequally spaced array configurations are preferable because of reducing grating lobes in higher frequency operations. Previously, radiation pattern synthesis methods were developed for equally spaced array configurations. In radiation pattern synthesis method, the least mean square (LMS) method was very conveniently used in mobile base station designing. The convenience of this method is obtaining array excitation coefficients by simply giving the objective radiation pattern. Moreover, the merit of this method is having the function of emphasizing design importance in the radiation angles that is called the weighting matrix.


In this paper, a radiation pattern synthesis program based on LMS scheme is developed for unequally spaced array configuration. As an initial step, the accuracies of a developed program are examined. Through designing actual asymmetric radiation pattern with very low side lobe levels, the accuracies of program and effects of weighting matrix are ensured.

Keywords- linear array antenna, radiation pattern synthesis, least mean square method, weighting matrix

## I. Introduction

Mobile base station antennas are requested variety of characteristics such as asymmetrical radiation pattern, low side lobe level, electrical beam tilt [1] and multi-frequency use [2]. The mobile base station antennas are configured by linear array antennas of equally spaced array elements. In the case of designing radiation patterns, radiation pattern synthesis method of the least mean square (LMS) is employed. Details of theoretical explanations and achieving LMS program are shown in the ref. [3]. According to that paper, objective radiation pattern and weighting matrix are employed in the radiation pattern synthesis. And the weighting matrix plays the important part in achieving the given objective pattern accurately. In multi frequency operations, unequally spaced array configuration is preferable in order to reduce grating lobes at high frequency operations.

In this paper, a LMS program applicable for unequally spaced array configuration is developed. In order to ensure the accuracies of radiation pattern synthesis abilities, asymmetrical
low side lobe radiation pattern is designed. And, the performances of weighting matrix are examined.

## II. Antenna Configuration

An array antenna configuration and an objective radiation pattern are shown in Fig.1. Radiation elements number is $q$. Excitation coefficients are expressed by a column vector [V] of q components. Antenna spacing is determined by the next expression.

$$
\begin{equation*}
\mathrm{d}_{\mathrm{n}}=\mathrm{d}+\mathrm{n} \Delta \mathrm{~d} \quad(\mathrm{n}=1,2, \ldots, \mathrm{q}-1) \tag{1}
\end{equation*}
$$

Here, $\Delta \mathrm{d}$ expresses the degree of unequally spacing. The objective radiation pattern is expressed by a column vector [A] of $p$ components.

## III. Design Method

## A. Fundamental equations

Radiation pattern ([F]) of p components is given by the next expression.

$$
\begin{equation*}
[\mathrm{F}]=[\mathrm{B}][\mathrm{V}] \tag{2}
\end{equation*}
$$

Here, $[\mathrm{B}]$ is a $\mathrm{q} \times \mathrm{p}$ matrix which expresses contributions of [V] to each direction of $\theta_{\mathrm{i}}(\mathrm{i}=1,2, \ldots, \mathrm{p})$. Components of the matrix [B] are given by the next expression.

$$
\begin{align*}
& \mathrm{b}_{\mathrm{in}}=\mathrm{v}_{\mathrm{n}} \exp \left[\mathrm{j} 2 \pi / \lambda\left(\mathrm{d}_{1}+\mathrm{d}_{2}+\ldots+\mathrm{d}_{\mathrm{n}}\right) \sin \theta_{\mathrm{i}}\right]  \tag{3}\\
& (\mathrm{i}=1,2, \ldots, \mathrm{p}),(\mathrm{n}=1,2, \ldots, \mathrm{q})
\end{align*}
$$

According to the conventional LMS method, the radiation pattern difference ( $\varepsilon$ ) between the objective [A] and [F] is given by the next expression.

$$
\begin{equation*}
\varepsilon=([\mathrm{T}][\mathrm{B}][\mathrm{V}]-[\mathrm{T}][\mathrm{A}])^{\mathrm{H}} \times([\mathrm{T}][\mathrm{B}][\mathrm{V}]-[\mathrm{T}][\mathrm{A}]) \tag{4}
\end{equation*}
$$

Here, $H$ indicates complex conjugate and transpose of a matrix. [T] is a weighting matrix, which has only diagonal components $\left(\mathrm{t}_{\mathrm{i}}\right)$. In the case of $\mathrm{t}_{\mathrm{i}}=1(\mathrm{i}=1,2, \ldots, \mathrm{p})$, all radiation levels of $\theta_{i} \quad(i=1,2, \ldots, \mathrm{p})$ have equal weighting. However, giving large number to $\mathrm{t}_{\mathrm{j}} \quad(\mathrm{j}=\mathrm{k}, \ldots, \mathrm{k}+\mathrm{m})$, radiation corresponding to the angular region from $\theta_{\mathrm{k}}$ to $\theta_{\mathrm{k}+\mathrm{m}}$ are emphasized in pattern synthesis.

Eq. (4) can be transformed to Eq. (5) by completing the square [4].
$\varepsilon=\left\{[\mathrm{V}]^{\mathrm{H}}-[\mathrm{A}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{B}]\left([\mathrm{B}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{B}]\right)^{-1}\right\} \times\left([\mathrm{B}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{B}]\right)$
$\times\left\{[\mathrm{V}]-\left([\mathrm{B}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{B}]\right)^{-1}[\mathrm{~B}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{A}]\right\}+[\mathrm{A}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{A}]$
$-[\mathrm{A}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{B}]\left([\mathrm{B}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{B}]\right)^{-1}[\mathrm{~B}]^{\mathrm{H}}[\mathrm{T}]^{\mathrm{H}}[\mathrm{T}][\mathrm{A}]$

The minimum value of Eq. (5) is achieved when [V] is given by the next expression. Here, $[T]^{\mathrm{H}}[\mathrm{T}]$ is converted to [ $\mathrm{T}_{0}$ ].

$$
\begin{equation*}
\left[\mathrm{V}_{0}\right]=\left([\mathrm{B}]^{\mathrm{H}}\left[\mathrm{~T}_{0}\right][\mathrm{B}]\right)^{-1}[\mathrm{~B}]^{\mathrm{H}}\left[\mathrm{~T}_{0}\right][\mathrm{A}] \tag{6}
\end{equation*}
$$

By applying the condition of Eq. (6) to Eq. (5), the normalized aberration of $\varepsilon /[\mathrm{A}]^{\mathrm{H}}\left[\mathrm{T}_{0}\right][\mathrm{A}]$ is expressed as follows.

$$
\begin{align*}
\varepsilon & =\left\{[\mathrm{A}]^{\mathrm{H}}\left[\mathrm{~T}_{0}\right][\mathrm{A}]-[\mathrm{A}]^{\mathrm{H}}\left[\mathrm{~T}_{0}\right][\mathrm{B}]\left[\mathrm{V}_{0}\right]\right\} /[\mathrm{A}]^{\mathrm{H}}\left[\mathrm{~T}_{0}\right][\mathrm{A}] \\
& =\left\{[\mathrm{A}]^{\mathrm{H}}\left[\mathrm{~T}_{0}\right]\left([\mathrm{A}]-[\mathrm{B}]\left[\mathrm{V}_{0}\right]\right)\right\} /[\mathrm{A}]^{\mathrm{H}}\left[\mathrm{~T}_{0}\right][\mathrm{A}] \tag{7}
\end{align*}
$$

## B. MATLAB program

The radiation pattern synthesis program is constructed on a MATLAB software. Content of the program is shown in Fig.2. First, array antenna parameters such as calculation frequency (f), element number ( N ) and array spacing $\left(\mathrm{d}_{\mathrm{i}}\right)$ are input. Next, design objectives of radiation pattern ([A]) and angular information $\left(\theta_{\mathrm{i}}\right)$ are input. And the weighting matrix ([T]) is set. After these inputs, array excitation coefficient ([V]) is calculated. Moreover, a radiation pattern is calculated by using obtained [V] value in order to ensure the design adequateness. The process from design objective setting to the radiation result is repeated until satisfactory radiation pattern is obtained.


Fig.2. Flow chart of MATLAB program.

## IV. Calculation Process

## A. Initial values

Initial values used in calculation are shown in Table 1. As for array parameters, element number of $23(\mathrm{~N}=23)$, equal element spacing of 0.5 wavelength ( $\mathrm{d}=0.5 \lambda$ ) and calculation frequency of 1 GHz are used. As for the objective radiation pattern ([A]), angular positions from -90 deg. to 90 deg. in every two degrees are selected. The objective values are divided into three areas such as low side lobe region of -30 dB , the main beam region and low side lobe region of -20 dB for the simplicity of determining object values. For main lobe region, magnitude of 1 is assigned. For -30 dB and -20 dB , magnitudes of 0.001 and 0.01 are assigned, respectively. This object pattern means that all radiated field intensities in side lobe regions should have uniformly 0.001 or 0.01 values not only in the peak levels. This request doesn't permit nulls in side lobe regions. For the simplicity of setting [A] values, these simple values are selected. However, this request is very severe in radiation pattern synthesis.

## B. Calculated results

The obtained radiation pattern by the array synthesis program of Fig. 2 is shown in Fig. 3. Low side lobes of -30 dB and -20 dB are achieved in peak levels of all side lobes. However, many ripples are observed. Most null depths are maintained within 20 dB . The request of the object values that all levels in the side lobe region should be the same is working in maintaining the null depth in 20 dB . The constant levels of 30 and -20 dB in side lobe regions cannot be achieved.

In order to achieve radiation pattern of Fig. 3, adjustment of weighting matrix ([T]) is very important. Values of elements of [A] are shown in Fig. 4. The maximum value of 4,500 is assigned at the $T_{11}$ angular region. This large weighting value means that the side lobe in the $\mathrm{T}_{11}$ angle is very difficult to suppress into -30 dB . Other T values are less than 300 . In low side lobe region of -30 dB , rather large T values are given. In side lobe region of -20 dB , rather small T values are given. As a result of radiation synthesis, if the objective values are simple, the weighting matrix becomes complicated. In the process of pattern synthesis, if the objective values set suitable for achieved pattern, the weighting matrix becomes simple.

Calculated results of excitation amplitude and phase are shown in Fig. 5 and Fig. 6, respectively. As for the excitation amplitudes, rather simple and common distribution is obtained, although small fluctuations are observed. And amplitude deviation is small such as from 1 to 3 . As for the excitation phase, relatively large deviations are observed at elements in array end. However, the maximum phase deviation is about 60 degrees, this is not so large. In the other elements, almost the same phase is achieved. As a result, the excitation amplitude and phase are quite common. So, very common feeding network is applicable and this array can be easily fabricated.

## V. Conclusions

Radiation pattern synthesis program for unequally spaced array configurations is achieved on MATLAB software. As the initial step, accuracies of radiation pattern synthesis function
are ensured through designing asymmetric radiation pattern. And effects of weighting matrix are ensured. As the next step, radiation pattern synthesis for unequally spaced array configuration should be expected.

## References

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Fig.1. Array configuration.

Table. 1 Initial values of the program.
\% Radiation pattern synthesis by Least Mean Square Method
\% BY Ngo Thi HUONG at 30 May 2014
\% Assignment angles of the objective radiation pattern
sit=[-90;-88;-86;-84;-82;-80;-78;-76;-74;-72;-70;-68;-66;-64;-62;-60;-58;-56;-54;-52;-50;-48;-46;-44;-42;-40;-38;$36 ;-34 ;-32 ;-30 ;-28 ;-26 ;-24 ;-22 ;-20 ;-18 ;-16 ;-14 ;-12 ;-10 ;-8 ;-6 ;-4 ;-$
2;0;2;4;6;8;10;12;14;16;18;20;22;24;26;28;30;32;34;36;38;40;42;44;46;48;50;52;54;56;58;60;62;64;66;68;70;72;74; 76;78;80;82;84;86;88;90];
\% Assignment of strength of radiation pattern
an $=[0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.00$
$1 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0$ $.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 0.001 ; 1 ; 1 ; 1 ; 1 ; 1 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.0$
$1 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.0$
$1 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01 ; 0.01]$;
\% Assignment of initial values
$\mathrm{N}=23 ; \%$ Number of array elements
Freq $=1 ; \%$ Frequency $(\mathrm{GHz})$
Lamd=30/Freq;\% Wave length (cm)
$\mathrm{d}=$ Lamd $* 0.5 ; \%$ Element spacing (cm)


Fig.3. Asymmetrical radiation pattern.


Fig.4. Weighting values.


Fig.5. Amplitude of array element.


Fig.6. Phase of array element.

