Design of Takagi-Sugeno fuzzy controller for automatic stabilization system of missiles with blended aerodynamic and lateral impulsive reaction-jet

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***Abstract*—In this paper, the design of a fuzzy-based controller for automatically stabilizing missiles with blended control system of aerodynamic and lateral impulsive reaction-jet is presented. The controller uses a Takagi-Sugeno fuzzy system that allows the parallel control systems to operate together at a moment to improve performance of the system and minimize energy usage of lateral impulsive reaction-jet. The designed system is objectively tested through simulation experiments on the computer.**

Keywords—Takagi-Sugeno fuzzy controller, automatic stabilization system, aerodynamic fin, lateral impulsive reaction-jet, blended aero-fin and reaction-jet control

I. INTRODUCTION

There is a serious fact that, in the flight path, missiles might become unstable because of many changing factors, e.g. wind forces, air pressures, etc. To deal with the problem of stabilization, the aerodynamic features of the missile, such as wings and fins, must be carefully considered for building automatic stabilization systems.

Technically, missiles use aerodynamic or thrusters to maneuver and control their attitude. However, some of the others, which can maneuver highly, take advantage of a combination of these control systems [1].

It is proven that a reaction-jet control system (RCS) uses thrusters to provide additional forces and moments [2]. For this reason, a large number of solutions for reaction-jet control systems have been proposed to improve the maneuverability of missiles. In comparison with traditional models of aerodynamic fin control, it is more difficult to design an automatic stabilization system for models of blended control combining aerodynamic and reaction-jet. According to [1], the model of missile with RCS is nonlinear, coupling and uncertain, which makes gain–scheduling method difficult to attain control objectives. In addition, it is necessary to deal with many difficult questions such as how reaction-jet forces and aerodynamic forces are distributed? what kind of blending strategy should be chosen?

Automatic stabilization system (ASS) is an indispensable part of the missile. To automatically stabilize the motions of the missile, many researchers have proposed solutions of combining aerodynamic and lateral impulsive reaction-jet by switching control [2-4], and parallel control [5].

In the solutions of switching control, the control channels work sequentially. For example, [2] uses a switch scheme for combining two channels of reaction-jet control and aerodynamic control. It means that in the first stage of attacking, the reaction-jet control channel is activated and the aerodynamic control channel is disconnected; otherwise, in the last stage of attacking, the first channel is disconnected and the second one is activated. The drawback of these solutions is the allowance of only one channel operating at a moment.

To defeat the above drawback, the solutions of parallel control allows many channels to work together at a moment. For example, [5] uses a weighting function to combine two control channels of aerodynamic fin and side thrusters. It means that the weighting functions of the control channels are changeable after a set of critical values. The critical values are automatically computed by the missile system, but they are discontinuous. The discontinuity makes the control unsmooth.

Fuzzy logic control has been intensely investigated as a paradigm of rule-based control systems for intelligent missiles. It is proven that IF-THEN rules help fuzzy controllers improve “the tolerance for imprecision needed in situations” [6]. Moreover, according to [7], fuzzy-based control has proven to be a successful control approach to many complex nonlinear systems or even non-analytic systems.

Takagi-Sugeno (TS) fuzzy systems also use IF-THEN rules to represent local input-output relations of nonlinear systems. The main feature of a TS fuzzy system is the expression of the local dynamics for fuzzy implications/rules by linear system models. For this reason, the overall TS fuzzy-based system is achieved by blending of the linear system models. According to [8], TS fuzzy systems are universal approximators of any smooth nonlinear system.

To overwhelm the unsmooth property of the above solutions of parallel control, in this paper, the design of a TS fuzzy controller for automatic stabilization system of missile

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with blended aero-fin and lateral impulsive thruster is presented. The TS fuzzy controller allows the parallel control channels (aerodynamic and RCS channel) to work together at a moment. This makes the automatic stabilization system more smooth and effective. The performance of the designed system will be verified through simulation experiments. The results of the simulation experiments are analyzed at the end of the paper.

II. Proposal of TS-Fuzzy Model for Nonlinear Model of Missiles

A. Takagi-Sugeno Fuzzy Model

The fuzzy model proposed by Takagi-Sugeno uses fuzzy IF-THEN rules to represent local input-output relations of a nonlinear system [10]. The main idea of a TS fuzzy model is the expression of fuzzy rules by linear system model. A plant can be represented by fuzzy rules as follows:

R^i: IF $x_1(t)$ is M_1^i and ... and $x_n(t)$ is M_n^i THEN

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) \end{cases}, \quad i = 1,2,...,r$$

where M_j^i (j = 1,2,...,n) denote input fuzzy sets, r is number of fuzzy rules, $\mathbf{x}(t) = \left[x_1(t), x_2(t), \cdots, x_n(t)\right]^T$ denotes the state vector, $\mathbf{y}(t)$ and $\mathbf{u}(t)$ are output and input vectors; ($\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$) are coefficient matrices of linearization system model with respect to fuzzy rule *i*th. For more details, it is possible to reference to [10].

B. Proposal of TS-Fuzzy Model for Nonlinear Model of Missiles

The nonlinear longitudinal model of missiles using aerodynamic and reaction-jet control systems is introduced in [9] and illustrated in Fig. 1.

Mathematically, the nonlinear longitudinal system are represented by the following forms:

$$\dot{\alpha} = \omega_{Z_1} - \frac{1}{mV}\left(C_y^\alpha qS\alpha + P\sin(\alpha) - C_y^\delta qS\delta_y + P_{IM}\cos(\alpha)n\right) \tag{1}$$

$$\dot{\omega}_{Z_1} = \frac{1}{I_{Z_1}}\left\{C_y^\alpha qS\cos(\alpha)x_{F\alpha}\alpha + m_{Z_1}^{\omega_{Z_1}}\frac{qSL_\Phi^2}{V}\omega_{Z_1} + C_y^\delta qS\cos(\alpha)x_{F\delta}\delta_y + P_{IM}x_{GD}n\right\} \tag{2}$$

$$\mathrm{W} = V(\omega_{Z_1} - \dot{\alpha}) \tag{3}$$

where:

- α is angle of attack (AOA);
- ω_{Z_1} is pitch rate;
- V and W are velocity and normal acceleration, respectively;
- C_y^α and C_y^δ are normal force coefficient derivatives with AOA and aero-fin angle, respectively;
- S is reference area;
- P and P_{IM} are forces of jet engine propulsion and impulsive reaction jet, respectively;
- q is velocity pressure;
- n is number of impulses;
- m is mass;
- $m_{Z_1}^{\omega_{Z_1}}$ is pitching moment coefficient derivative with pitch rate;
- I_{Z_1} is moment of inertia;
- L_Φ is the length of missile;
- δ_y is aero-fin angle;
- $x_{F\alpha}, x_{F\delta}, x_{GD}$ are center of pressure of missile, center of pressure on aero fins, and center of gravity of lateral impulsive thruster.

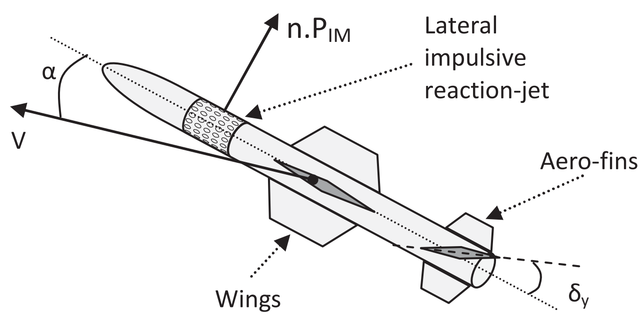

Fig. 1. Model of the missile using aero-fins and RCS

In order to segment nonlinear system dynamics into locally linearized sub-systems, it is possible to apply TS fuzzy inference systems. By this way, a complex nonlinear behavior of the system can be handled by fuzzy-based blending of the linearized sub-systems. Three reasonable approaches for building the TS fuzzy-based model are usually undertaken including sector nonlinearity, local approximation, and hybrid of them [10].

In the first researches, the sector nonlinearity approaches take a lot of consideration from the researchers because these approaches allow exact construction of fuzzy-based models. However, it is demonstrated that it is not easy to determine global sectors for general nonlinear systems. This difficulty can be dealt with by applying the local sector nonlinearity approaches with the notice that variables of physical systems must always be bounded.

To overcome the above drawback, it is better to apply the local approximation approaches because they enable to approximate nonlinear terms by judiciously chosen linear terms. Additionally, they allow reduction of the number of rules for fuzzy models.

For this reason, the approach of local approximation has been adopted for building the TS fuzzy-based nonlinear model of the missile.

Let us define state variables for output data and control signals as follows:

$$\begin{cases} x_1(t) = \alpha(t) \\ x_2(t) = \omega_{Z_1}(t) \end{cases}, \quad \begin{cases} y_1(t) = \mathrm{W}(t) \\ y_2(t) = \omega_{Z_1}(t) \\ y_3(t) = \alpha(t) \end{cases}, \quad \begin{cases} u_1(t) = \delta_y(t) \\ u_2(t) = n(t) \end{cases}$$

Now, we produce a TS fuzzy-based model for equations (1), (2), and (3) with linearized models at the values of angle of attacks α_i ($0 < |\alpha_i| \le \alpha_{max}$) as follows:

IF $x_1(t)$ is about α_i ($0 < |\alpha_i| \le \alpha_{max}$) ***THEN***

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) + \mathbf{D}_i\mathbf{u}(t) \end{cases}, \quad i = 1, 2, ..., r \tag{4}$$

where,

- $\mathbf{x}(t) = [x_1(t), x_2(t)]^T$ is a state vector;
- $\mathbf{u}(t) = [u_1(t), u_2(t)]^T$ is a control vector;
- $\mathbf{y}(t) = [y_1(t), y_2(t), y_3(t)]^T$ is an output vector;
- r is the number of fuzzy rules;
- $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$ and $\mathbf{D}_i$ are coefficient matrices of linearization system model with respect to fuzzy rule ith.

$$\mathbf{A}_i = \begin{bmatrix} -\dfrac{C_y^{\alpha_i} qS + P\dfrac{\sin(\alpha_i)}{\alpha_i}}{mV} & 1 \\ \dfrac{C_y^{\alpha_i} qS\cos(\alpha_i)x_{F\alpha}}{I_{Z_1}} & \dfrac{m_{Z_1}^{\omega_{Z_1}} qSL_{\Phi}^2}{I_{Z_1}V} \end{bmatrix},$$

$$\mathbf{B}_i = \begin{bmatrix} \dfrac{C_y^{\delta} qS}{mV} & -\dfrac{P_{IM}\cos(\alpha_i)}{mV} \\ \dfrac{C_y^{\delta} qS\cos(\alpha_i)x_{F\delta}}{I_{Z_1}} & \dfrac{P_{IM}x_{GD}}{I_{Z_1}} \end{bmatrix},$$

$$\mathbf{C}_i = \begin{bmatrix} \dfrac{C_y^{\alpha_i} qS + P\dfrac{\sin(\alpha_i)}{\alpha_i}}{m} & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{D}_i = \begin{bmatrix} -\dfrac{C_y^{\delta} qS}{m} & \dfrac{P_{IM}\cos(\alpha_i)}{m} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By applying the center-of-gravity method for defuzzification introduced in [10], we can represent the TS fuzzy-based model as follows:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{r} h_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)) \tag{5}$$

$$\mathbf{y}(t) = \sum_{i=1}^{r} h_i(\mathbf{x}(t))(\mathbf{C}_i\mathbf{x}(t) + \mathbf{D}_i\mathbf{u}(t)) \tag{6}$$

where, $h_i(\mathbf{x}(t))$ is weight of the rule ith and computed by the form:

$$\begin{cases} h_i(\mathbf{x}(t)) \ge 0 \\ \sum_{i=1}^{r} h_i(\mathbf{x}(t)) = 1 \end{cases} \tag{7}$$

III. Fuzzy Controller for the ASS with Blended Aerodynamic and Lateral Impulsive Reaction-Jet

The diagram of the ASS of missile control system with blended aerodynamic and lateral impulsive reaction-jet is illustrated in Fig. 2. In this diagram, modules of aerodynamic fins and lateral impulsive thruster operate parrallel and indepenently from each other.

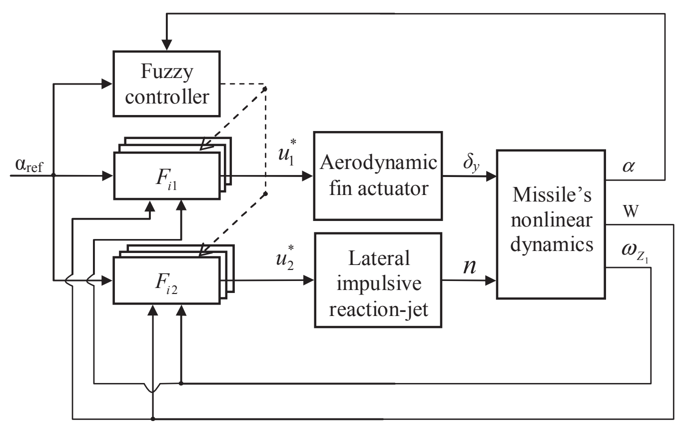

Fig. 2. Diagram of the automatic stabilization system using fuzzy controller

The advantage of aerodynamic fins is to provide stability of missile control during flight under suitable conditions concerning speed and altitude of the missile. Their disadvantage, however, involves in generating force with slow dynamic variation and reducing efficiency of missile control at high altitude because of low air density.

Otherwise, the solution of lateral impulsive reaction-jet offers high efficiency of missile control despite the fact that the missile flights at high altitude or with slow speed. However, the disadvantage of this solution is the limitation of fuel in the lateral impulsive reaction-jet. For this reason, it is very difficult to stabilize the control system in a long flight.

To improve the advantages and reduce disadvantages of the two solutions, it is possible to combine them but their roles are changed after the flight conditions. Therefore, it is necessary to implement a blender to combine them.

A. Module of TS Fuzzy Controller

It is obvious that the restriction of aerodynamic fin control and frequency of aerodynamic fin actuator make the missile difficult to immediately speed up the normal acceleration in order to quickly change the flight path. Meanwhile, lateral impulsive reaction-jet allows the missile to reach a huge normal acceleration in very short time. For this reason, a fuzzy controller is designed to combine two kinds of missile control.

In our work, the cooperation is executed by a TS fuzzy controller. The TS fuzzy controller is controlled by the estimated angle of attack α and reference angle of attack α_{ref}.

This fuzzy controller uses a threshold value of angle of attack to activate the lateral impulsive reaction-jet and utilizes the offset $\Delta\alpha$ between the reference angle of attack and the estimated one, $\Delta\alpha = \alpha_{\text{ref}} - \alpha$, to adjust the operation of the lateral impulsive reaction-jet. In other words, if $\Delta\alpha$ is big, the lateral impulsive thruster channel will be activated; if $\Delta\alpha$ is small, this channel will be paused.

The designed model uses fuzzy rules to approximate equations (1), (2), and (3). The membership functions of the fuzzy controller are simply formed as the illustrations in Fig. 3

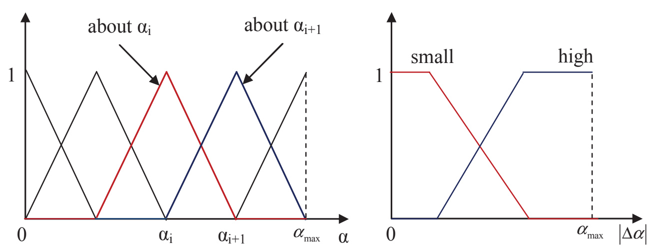

Fig. 3. Membership functions of α and |Δα|

Let $\mathbf{F}_{i1}$ and $\mathbf{F}_{i2}$ stand for functions of the linear feedback controller of aerodynamic and lateral impulsive reaction-jet in the fuzzy rule ith của of the TS fuzzy-based model formed as equation (5) and (6).

Moreover, assume that there is no interference from thrust of lateral impulsive reaction-jet onto the aero fins. So, the fuzzy logic rules for the cooperation are the following:

IF $x_1(t)$ is about α_i ($0 < |\alpha_i| \le \alpha_{\max}$),
THEN $u_1^*(t) = -\mathbf{F}_{i1}\mathbf{x}(t)$

IF $x_1(t)$ is about α_i ($0 < |\alpha_i| \le \alpha_{\max}$) AND $|\Delta\alpha|$ is big,
THEN $u_2^*(t) = -\mathbf{F}_{i2}\mathbf{x}(t)$

Generally, the TS fuzzy controller is formed as the following equation:

$$\mathbf{u}^*(t) = -\sum_{i=1}^{r} h_i(\mathbf{x}(t))\mathbf{F}_i\mathbf{x}(t) \tag{8}$$

where,

- $\mathbf{u}^*(t) = \left[u_1^*(t),\, u_2^*(t)\right]^T$;
- $\mathbf{F}_i = \mu_1\mathbf{F}_{i1} + \mu_2\mathbf{F}_{i2}$;
- μ_1 and μ_2 are control coefficients of channels of aerodynamic fins and lateral impulsive thrust, $\mu_1 \ge 0,\ \mu_2 \ge 0$ and $\mu_1 + \mu_2 = 1$.

The parameters of the control functions $\mathbf{F}_{i1}, \mathbf{F}_{i2}$ are able to be calculated by linear feedback approaches, such as PID (Proportional Integral Derivative), pole placement, LQG (Linear Quadratic Gaussian), CDM (Coefficient Diagram Method) to deal with the limitations of the parameters and the performance of the ASS.

B. Optimal Regulation of Membership Function

The requirements for optimal regulation of membership functions are minimum steady-state error, minimum number of the activated implusive reaction-jets, and minimum response time. The optimal regulation is mathematically illustrated by a function of performance as follows:

$$J = \int_0^{\tau_{td}} k_1|e|\,dt + k_2\sum n + k_3\tau_{qd} \tag{9}$$

where,

- k_1, k_2 and k_3 are performance coefficients of error, number of the lateral impulsive reaction-jets, and response time of the system. These coefficients are selected under the design requirement.
- τ_{td} is the time of self-contained guidance of the missile.
- τ_{qd} is the rise time.

The membership functions of the fuzzy controller are regulated to minimize value of J.

IV. Simulation Experiment

A. Simulation Experiment Setup

The simulation experiments have been performed on the computer via simulink models, Matlab program, and fuzzy toolbox, illustrated in Fig. 4.

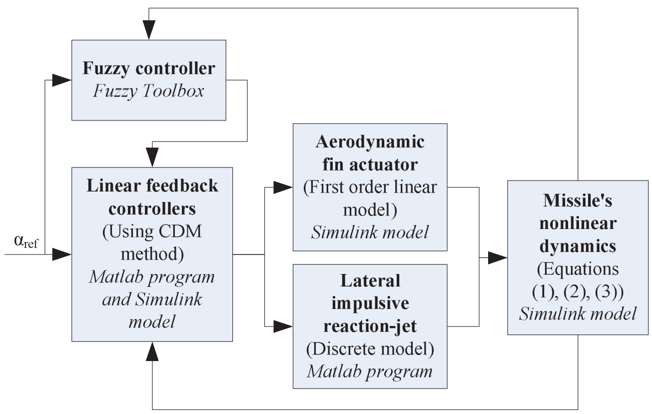

Fig. 4. Software structure for simulation experiments

Additionally, the geometric parameters of the missile are used in all of the simulation experiments are shown in Fig. 5.

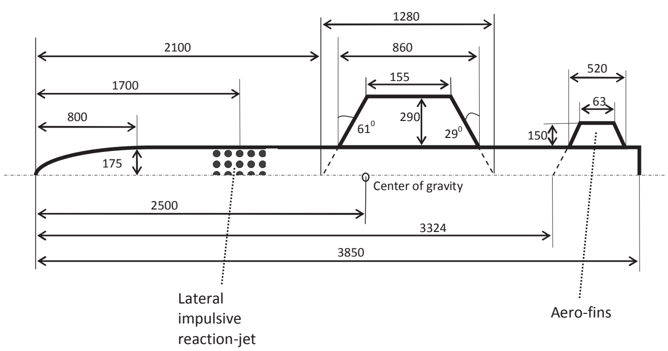

Fig. 5. Geometric parameters of the missile

The experiments of the ASS on the designed model of the missile have simulated under the following conditions: flight altitude is 10000m and missile speed is 1200m/s, moment of inertia I_{Z_1} = 180kg·m^2, maximum aero-fin angle is 25^0, the ignition period of impulsive thruster is 0.032s, and thrust of each impulsive reaction-jet is 3000N.

B. Synthesis Algorithm

The synthesis process of the fuzzy controller of the ASS is listed as follows:

- Constructing a structure diagram of the closed-loop control system of the ASS;
- Defining working points of α_i;
- Computing a linearized model of the missile at α_i by equation (4);
- Computing/selecting parameters of the controllers with linearized feedback for the linearized model;
- Checking the stability of the closed-loop control system. In the case of instability, the parameters of the feedback-linearized controller must be recomputed;
- Adjusting the membership functions in order to minimize the rise time of the ASS, numbers of used impulsive thruster, and steady-state error.

C. Experiment Results

The aerodynamic force and moment coefficients of the missile are calculated by Digital Datcom [11]. These coefficients are organized as lookup tables with respect to the angle of attack α, aero-fin angle δ_y, and Mach number. After that, they have been curve-fitted from the values of the lookup tables by using Curve Fitting Toolbox. Resulting from the serial processes, these functions of coefficients have a polynomial form in order to design a simulation study.

The step response of the ASS is illustrated in Fig. 6. After the tracking control command was given, the angle of attack α quickly increased to the reference angle of attack α_{ref}. There are three graphs with respect to three control modes: (1) fuzzy-based control of aerodynamic fins and lateral impulsive reaction-jet, (2) control of aerodynamic fins, and (3) control of lateral impulsive reaction-jet.

The simulation results prove that the response time of the ASS with blended aerodynamic and lateral impulsive reaction-jet is smaller than that of the ASS with aerodynamic fins.

Additionally, the stabilization state of the ASS with blended aerodynamic and lateral impulsive reaction-jet is smoother than that of the ASS with lateral impulsive reaction-jet. The designed ASS with TS fuzzy controller accurately tracks step commands with a rise time about 0.1 sec and does not have overshoot during tracking the step command.

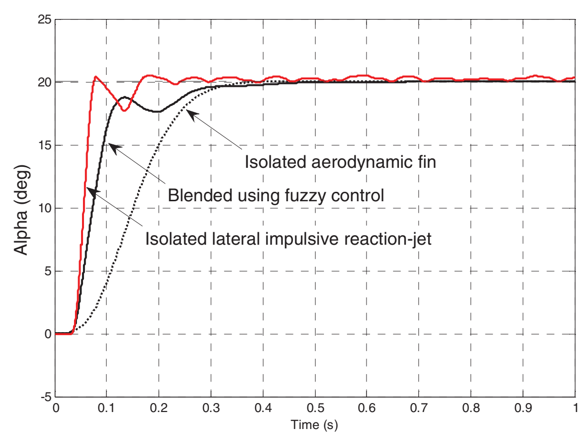

Fig. 6. Step response of the ASS with reference angle of attack input 20^0

The response of the parameters of the ASS using the switch-based controller under the control command with reference angle of attack 20° is exhibited in Fig. 7. On the other hand, the response of the parameters of the ASS using the fuzzy-based controller under the control command with reference angle of attack 20° is illustrated in Fig. 8.

To compare perfomance of the switch-based controller with that of the fuzzy-based controller, it is necessary to execute simulation experiments under the same conditions. The conditions are set with reference angle of attack 20°.

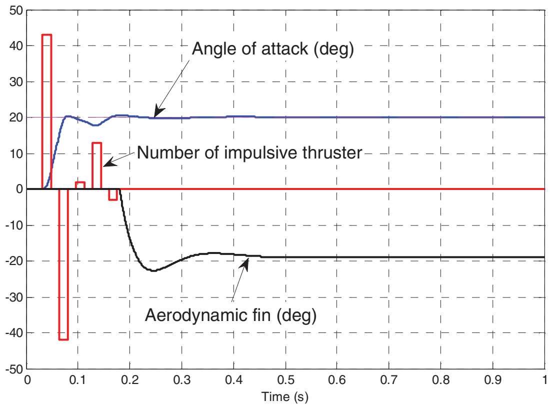

Fig. 7. Response of the parameters of the ASS using the switch-based controller under a control command with reference angle of attack 20°

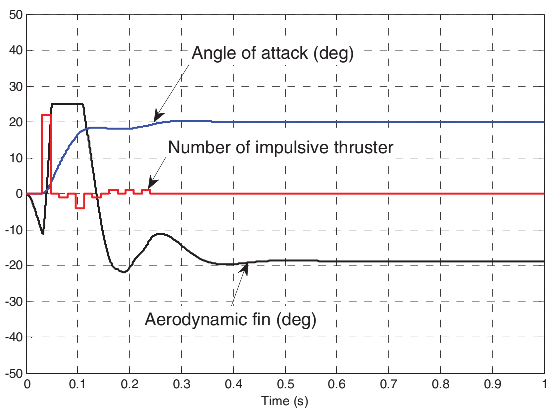

Fig. 8. Response of the parameters of the ASS using the fuzzy-based controller under a control command with reference angle of attack 20°

Comparing the responses of the ASS using the switch-based controller described in [4] with those of the ASS using the fuzzy-based controller, it is easy to recognize some advantages of the fuzzy-based ASS as follows:

- The required number of impulsive thruster is significantly reduced;
- It is not necessary to determine the switching time for control channels;
- It is possible to maintain the big angle of attack in the setting state.

To validate performance of the ASS using the fuzzy-based controllers under different conditions, we executed simulation experiments with reference angle of attack 30°. The response of the parameters of the ASS is shown in Fig. 9.

Comparing the results in Fig. 8 with those in Fig. 9, it is possible to find out that the aerodynamic fin in Fig. 9 reached to the limited angle value (25°). The reason of this problem is that the aerodynamic fin are not able to satisfy requirement of making force control with such a big reference angle of attack. For this reason, the impulsive reaction-jets were ignited. The aerodynamic fin deflection is chattering during implusive thruster igniting.

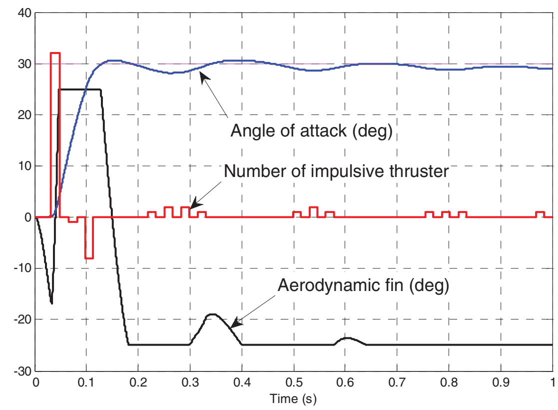

Fig. 9. Response of the parameters of the ASS using the fuzzy-based controller under a control command with reference angle of attack 30°

Additionally, the number of impulsive thrusters illustrated in Fig. 9 presents two phases of control: the first one is to reduce the rise time, and the second one is to maintain angle of attack in steady state.

V. Conclusion

This paper has presented the application of TS fuzzy logic to design the automatic stabilization system of missiles with blended aerodynamic and lateral impulsive reaction-jet. The TS fuzzy-based blender allows parallel control systems to effectively work together and makes the missile maneuver faster and more stable. The TS fuzzy-based control system is successfully tested through simulation experiments.

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