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Research to identify mathematical model of quadrotor

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Abstract

This research presents recognition algorithm for mathematical model, whose the object is Quadrotor and demonstrates the properness and feasibility of this algorithm by stimulating the comparison of the recognition algorithm with the pattern model.

Keywords: mathematical model, identification, quadrotor, quadrotor dynamic model.

Introduction

Recently, research for manufacturing Unmanned Aerial Vehicle’s equipment (UAV) has become popular and developed extensively, a series of UAVs have been widely applied not only in the civil of field but also in the military that brings high efficiency in utilization. Among these, the UAV type which owns high stability of dynamic characteristic, easy control, and maneuverability in complex terrain is Quadrotor. With such outstanding features, Quadrotor has been increasingly used, together with new features, more modern and smarter, they can fly according to a determined program, perform precisely movements in small spaces, has the ability to automatically follow targets moving on the ground, at the same time, carry out other complex military missions. In Vietnam, the research and development of Quadrotor has begun to increase. However, the Quadrotor made in Vietnam has been limited by manually controlling without modern automatic control system, so that application range of Quadrotor is still narrow and it is unable to applied in important tasks with high practice. Thus, the research to solve problems of building modern automatic control system for Quadrotor is essential today, such as an adaptive control system, optimal control system, etc., and to build such modern system for Quadrotor, firstly we have to find its mathematical modern, that is very important for creating high quality automatic control system for Quadrotor.

1. Introduction of Quadrotor

Quadrotor is an unmanned flying object with vertical takeoff and landing. It has a special design with 4 propellers, which are arranged symmetrically left - right and front – behind. About the structure, it has quite a simple mechanical structure. The dynamics of Quadrotor are far different with conventional aircraft.

Figure 1: Model of a Quadrotor

Quadrotor has small and tidy structure, the surface of frame has small area, so when it flies, the forces impact on its surface is very small. These forces and their moments are ignored. As the result, there are only forces and moments which create by propellers of Quadrotor and its mass, impacting on the Quadrotor [2],[6] .

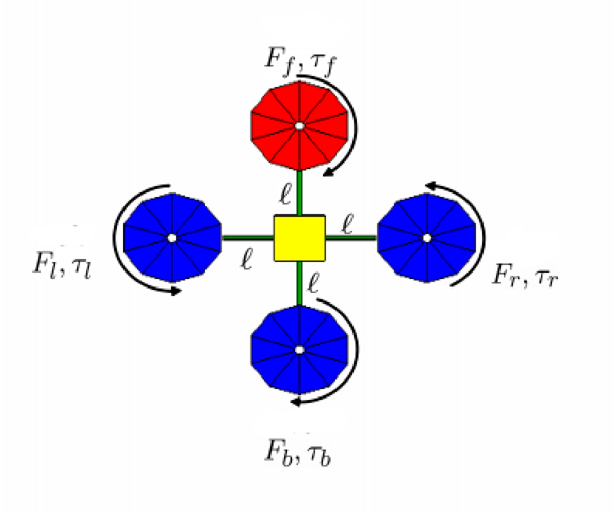

Figure 2: Forces and moments impacting on Quadrotor

From the picture we see that the forces and moments impacting on Quadrotor are:

F_f, F_b,F_l,F_r: force creating by front, back, left and right propellers, respectively.

τ_f, τ_b, τ_l, τ_r: moment creating by front, back, left and right propellers, respectively.

Principle control the Quadrotor is: Quadrotor has a structure of 4 propellers, each of them connect to an engine through a gearbox. Axis of four propellers are stuck parallel each other. The speed of each propeller could control independently through Electronic speed controllers (ESC). The position of Quadrotor is control by changing speed of each propellers.

2. The importance of using identified algorithm for Quadrotor

As far as we know, for flying object, to determine the mathematical model we have some basic methods such as using a small sample device placed in the wind tunnel then determine the kinematical parameters and calculate the corresponding analysis for real flying device. The second method is theoretical analysis, this method relies on the law which is available on the internal physical relation and external environment of the object. This method is mainly based on structural indicators and aerodynamic calculations for numerical kinematical parameters. However, due to the complex aerodynamic structures of the device and the participation of many structures of control systems with different features so the accuracy in determining the kinematic parameters is not high. The third approach is to conduct testing experiments for a flying object, which has completed the structural system, then tested in various flight conditions to determine the kinematical parameters and correcting transmission ratio control system. This method is based on the second one and could apply to determine the simple mathematical model and synthesis the simple control laws for the flying object which has high static stability. The fourth approach is the identifying method of algorithmic mathematical model, this approach is based on optimization algorithms to process input and output information of the system, then the evaluation mathematical model of control object. By applying the method, the controller object can be anything if we can get the input and its output, [1], [4], [5], [9].

Quadrotor is a vertical take-off and landing aircraft with the aerodynamic structure different from the fixed wing aircraft and helicopter. Hence, determining the mathematical model of a Quadrotor by the first method is not feasible, by the second and third method is difficult and complex. The new control system stops only at the synthesis of the regulator, and cannot respond the building control system with more advanced features like adaptive or optimized. With special structures such kinetic method should determine the most appropriate costing model and to be a method for identifying Quadrotor. With this method the mathematical model can determine fairly accurately for each flight conditions, i.e. allow the application to build better algorithms control for control systems.

To identify the mathematical model for a control object people can use many different methods such as instrumental variables method, method of forecasting error, the method of the least squares error, extreme reasonable methods [1], [4], [6], ... Method identification used in this paper is method of least squares error based on the following: Firstly, Quadrotor has structurally complex kinematics therefore it is difficult to accurately describe the form of kinetic equations by the modeling method, and thereby the determining channel transfer function under the control of quadrotor by this way is troubled; The second is determining the mathematical model of control object recognition method has been developed very strongly in recent times and it is a method that allows determining the mathematical model object very effectively, which the control is based on the data of signal and output signals to the controller object.

Furthermore, in fact Quadrotor was designed and could hand-operated thus identifing input and output through the onboard sensors to determine the mathematical model will form more realistic, more accurately determining the mathematical model by modeling method; Besides, the algorithm of least squares error method is easy to understand, easy programming and it enables to calculate parameters and thus determine the mathematical model of a controlled object fairly accurate.

Indeed, the algorithm of the least squares error method is as follows:

Assuming that the transfer function of the system with the unknown kinematical parameters has a common form:

$$W(p) = \frac{Y(p)}{u(p)} = \frac{b_0 p + b_1}{p^3 + a_1 p^2 + a_2 p + a_3} \quad (1)$$

When change the transfer function into discrete form with period T_0:

$$\begin{aligned} Y(k) &= (3 - a_1 T_0)Y(k-1) + (2a_1 T_0 - a_2 T_0^2 - 3)Y(k-2) + \\ &(1 - a_1 T_0 + a_2 T_0^2 - a_3 T_0^3)Y(k-3) + b_0 T_0^2 u(k-2) + (b_1 T_0^3 - b_0 T_0^2)u(k-3) \end{aligned} \quad (2)$$

Signs Y(k) as Y(k)/(k-1), the equation (2) is written as follows:

$$Y(k)/(k-1) = \alpha_1 Y(k-1) + \alpha_2 Y(k-2) + \alpha_3 Y(k-3) + \alpha_4 u(k-2) + \alpha_5 u(k-3), \quad (3)$$

Where:

$$\alpha_1 = (3 - a_1 T_0); \alpha_2 = 2a_1 T_0 - a_2 T_0^2 - 3; \alpha_3 = 1 - a_1 T_0 + a_2 T_0^2 - a_3 T_0^3;$$
$$\alpha_4 = b_0 T_0^2; \alpha_5 = b_1 T_0^3 - b_0 T_0^2.$$

From the equation (3) with the expression Y(k)/(k-1), is calculated:

$$Y(k)/(k-1) = \Psi^T \hat{\alpha}(k-1) \quad (4)$$

Where:

Ψ^T : is a vector data which is received from the system, is determined:

$\Psi^T = [Y(k-1)\ \ Y(k-2)\ \ Y(k-3)\ \ u(k-2)\ \ u(k-3)]$;

$[Y(k-1)\ \ Y(k-2)\ \ Y(k-3)]$: is a vector data n the output which corresponds to the time (k-1), (k-2), (k-3);

$[u(k-2)\ \ u(k-3)]$: is a input vector at time (k-2),(k-3)

$\hat{\alpha} = [\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5]$: is evaluated vector parameters.

Expression of error is formed:

$$e(k) = Y(k) - Y(k)/(k-1) \quad (5)$$

Where:

$Y(k)$ -is a new measured value;

$Y(k)/(k-1)$ the value calculated by the transfer function.

Assuming that the process of measuring the input signal and the output of the system at the time of k=1,2,…, (N+4). Then from (4), (5) getting equation

$$Y(k) = \Psi^T(k-1)\hat{\alpha}(k-1) + e(k) \quad (6)$$

Với $Y^T(k-1) = [Y(4)Y(5).....Y(N+4)]$ - the output vector data obtained

$$\Psi^T(k-1) = \begin{bmatrix} -Y(3) & -Y(2) & -Y(1) & u(2) & u(1) \\ -Y(4) & -Y(3) & -Y(2) & u(3) & u(2) \\ ..... & ...... & ...... & .... & ..... \\ -Y(N+3) & -Y(N+2) & -Y(N+1) & u(N+2) & u(N+1) \end{bmatrix} \quad (7)$$

Deviation $e^T(N+4) = [e(4)e(5).....e(N+4)]$ - is the vector deviation of assessed value by input signal and the measured value.

Function squared deviations were identified:

$$\mathrm{F} = \mathrm{e}^{\mathrm{T}}(\mathrm{N}+4)\mathrm{e}(\mathrm{N}+4) = \sum_{\mathrm{k}=4}^{\mathrm{N}+4} \mathrm{e}^2(\mathrm{k}), \left.\frac{\partial \mathrm{F}}{\partial \alpha}\right|_{\alpha=\hat{\alpha}} = 0 \quad (8)$$

Assuming that N>>6 , put:

$$\mathrm{P}(\mathrm{N}+4) = [\Psi^{\mathrm{T}}(\mathrm{N}+4)\Psi(\mathrm{N}+4)]^{-1} \quad (9)$$

The target function F in expression (8) reaches the smallest value when:

$$\hat{\alpha}(N+4) = P(N+4)\Psi^T(N+4)Y(N+4) \quad (10)$$

Thus, to determine the parameters assessed by reference model $\hat{\alpha}(\mathrm{N}+4)$, it is necessary to measure the input vector data and vector data output from the time of k = 1 ... N + 4. The expression (10) is become is recognition algorithm according to the principle of the least squares error.

he algorithm identifies the principle of the least squared error obtained by addition (10) a component $\hat{\alpha}(\mathrm{k}+1)$:

$$\hat{\alpha}(\mathrm{k}+1) = \hat{\alpha}(\mathrm{k}) + \gamma(\mathrm{k})[\mathrm{Y}(\mathrm{k}+1) - \Psi^{\mathrm{T}}(\mathrm{k}+1)\hat{\alpha}(\mathrm{k})] \quad (11)$$

Values of adjustment parameter determined by the expression:

$$\gamma(\mathrm{k}) = \mathrm{P}(\mathrm{k}-1)\Psi(\mathrm{k}) / \Psi^{\mathrm{T}}(\mathrm{k})\mathrm{P}(\mathrm{k}-1)\Psi(\mathrm{k}) + 1 \quad (12)$$

Value of vector P(k+1) is:

$$\mathrm{P}(\mathrm{k}+1) = [1 - \gamma(\mathrm{k})\Psi^{\mathrm{T}}(\mathrm{k}+1)]\mathrm{P}(\mathrm{k})$$

Thus after n measuring the input values and the output we get the input signal vector and the output u (k), Y (k), k = 1 ... N + 4, respectively.

Using these two input vector data and outputs into the recognition algorithm following by the least squared error, we get the evaluated parameters. The parameters assessed by recognition algorithm. These parameters depend on the sampling period T_0 in the system. Therefore, to determine the transfer function of a similar system, we need to know T_0.

3. Analyzing the feasibility of identification algorithm and simulation evaluation results

Based on the recognition algorithm which has been presented above we have examined the feasibility of identification algorithm which were applied to a Quadrotor. The Quadrotor was determined the accurate mathematical model. Logic of the problem is confirmed the feasibility of the method of identification, or otherwise evaluate the accuracy of the algorithm by comparing the mathematical model of pattern. Specifically as follows: Mathematical model performs in simulation, then the data is collected in order to carry out the identification, the identifying results are then evaluated by comparing the coincidence of system reactions with the input signal format different to form mathematical model and mathematical model which has been identified.

The authors simulated and evaluated the results with different form of input signals :pulsed step input with noise; sinusoidal input signal with noise and signal noise input signal combines pulsed step and sinusoidal signal with noise. Simulation results are shown in the following figures:

- ***Following by roll channel:***

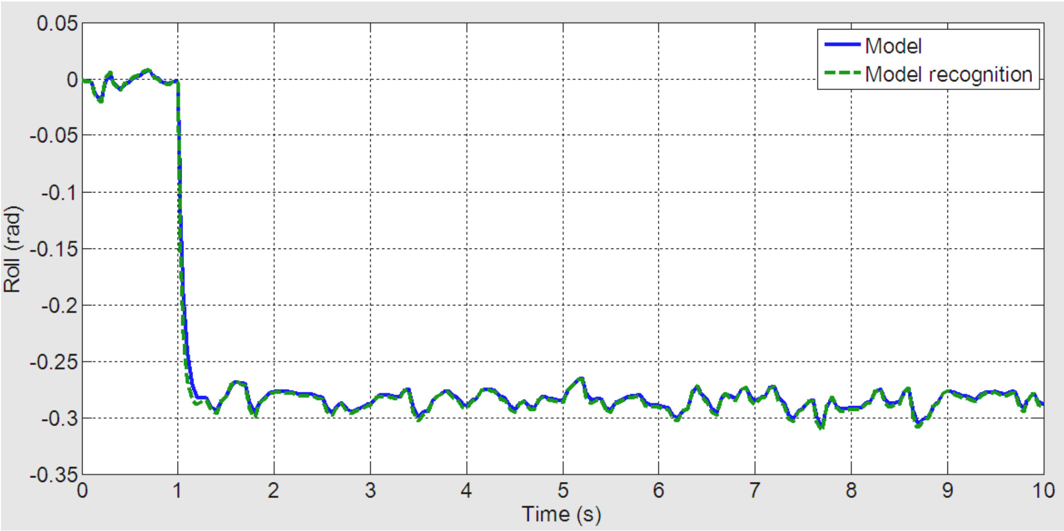

Figure 3. Simulation results with step input which has pulsed interference effects

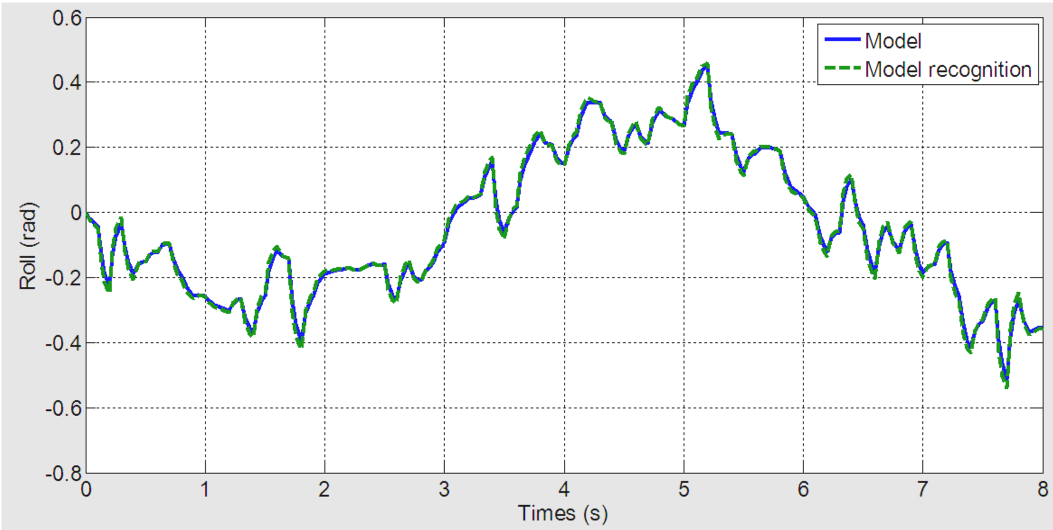

Figure 4. Simulation results with the input signal with sinusoidal interference effects

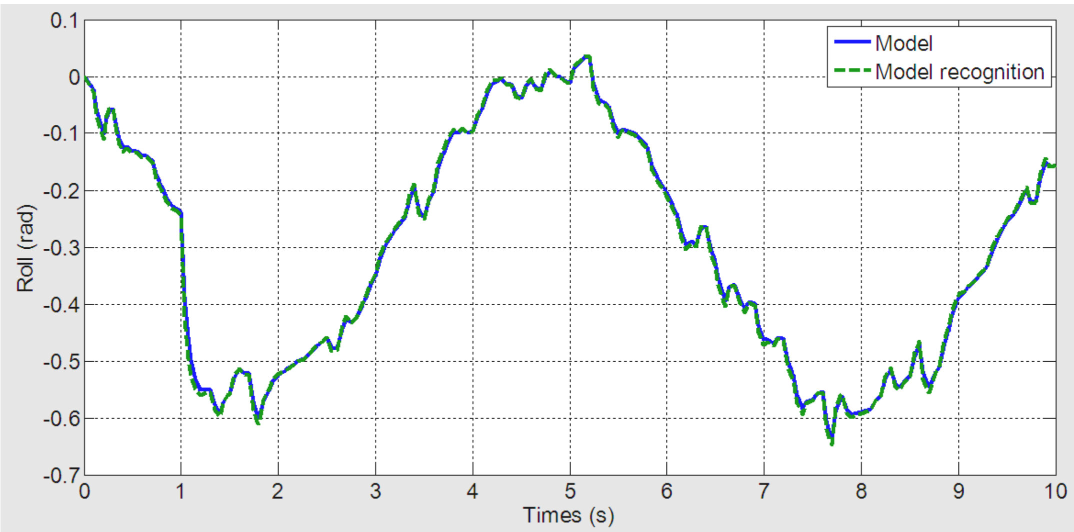

Figure 5. The simulation results with input signals mixed sinusoidal and step pulse interference effects

- Following by pitch channel:

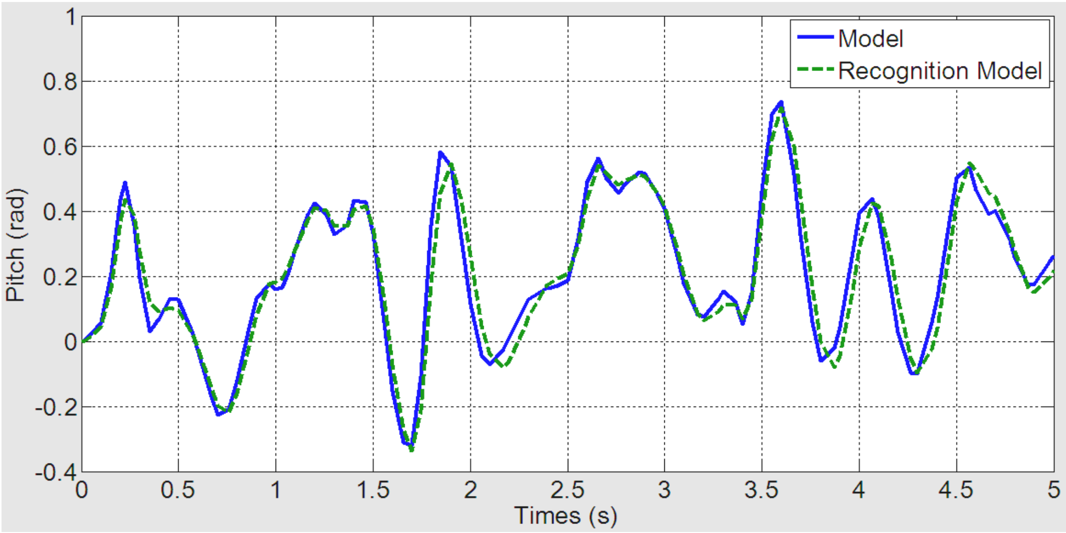

Figure 6. The simulation results with the step input which has pulsed interference effects

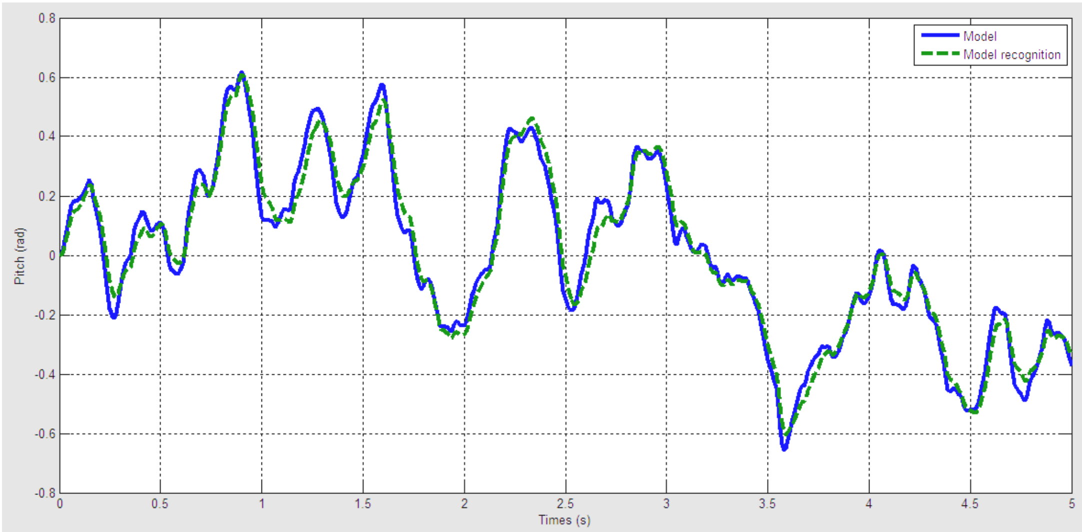

Figure 7 Simulation results with the input signal with sinusoidal interference effects

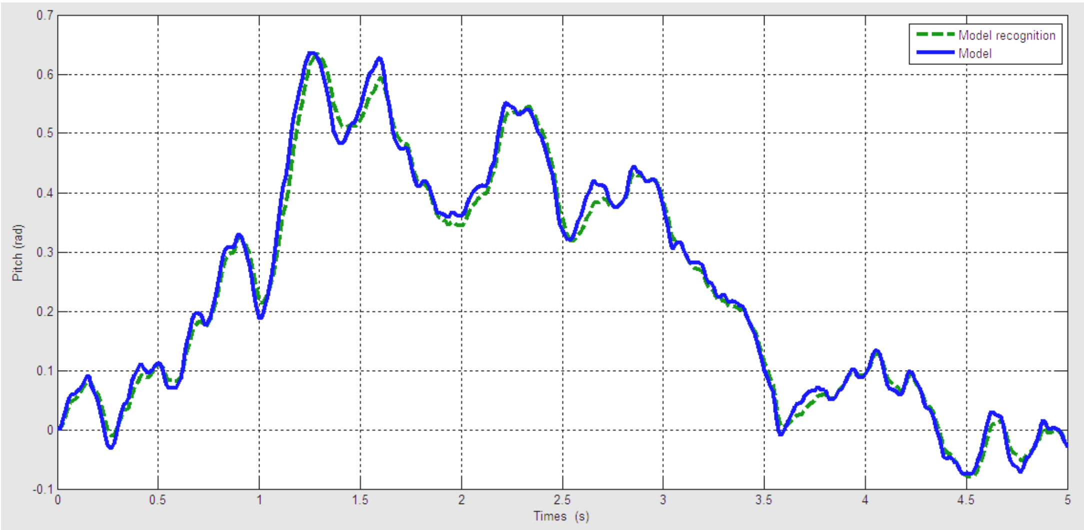

Figure 8. The simulation results with input signals mixed sinusoidal and step pulse interference effects

- ***Following by yaw channel:***

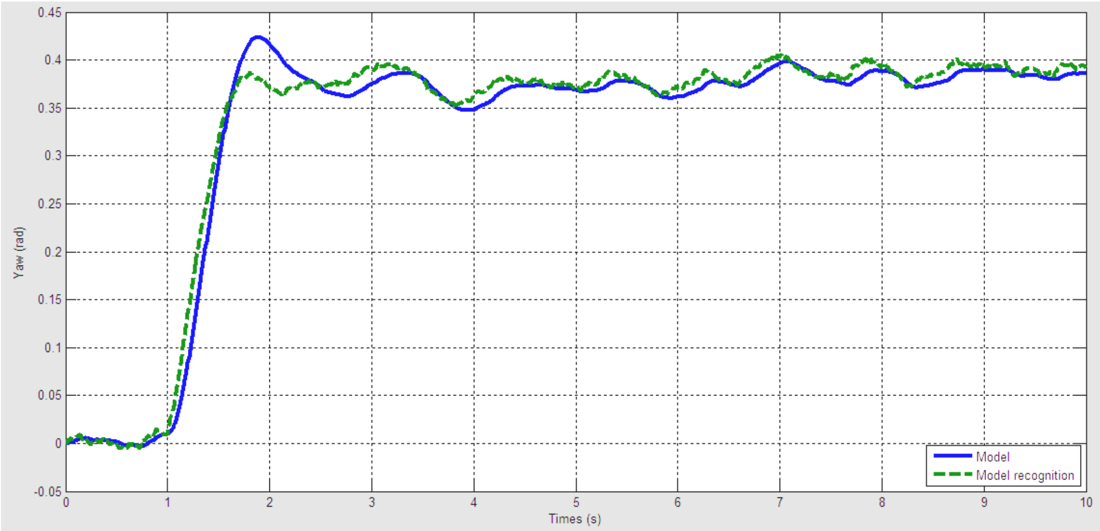

Figure 9. The simulation results with the step input which has pulsed interference effects

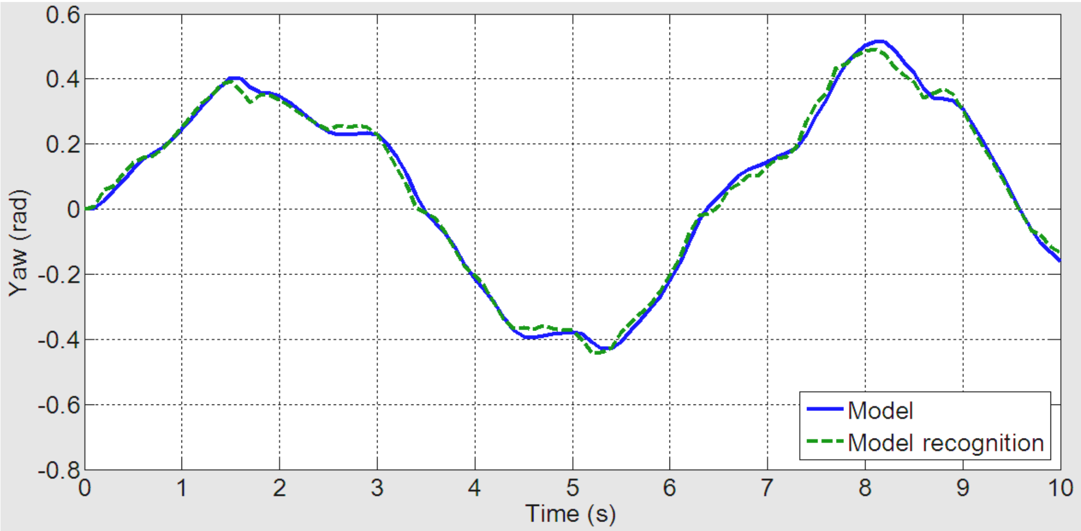

Figure 10. Simulation results with the input signal with sinusoidal interference effects

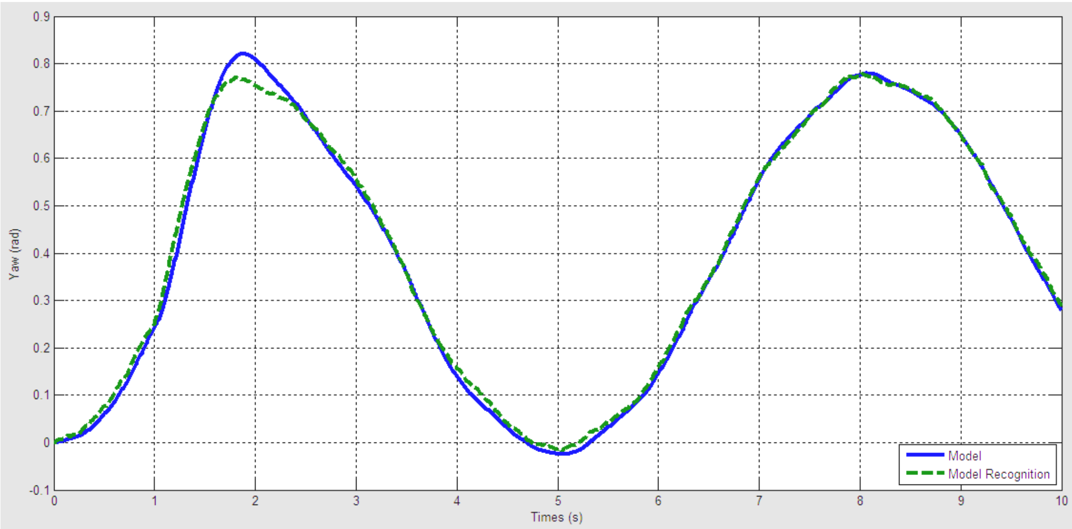

Figure 11. The simulation results with input signals mixed sinusoidal and step pulse interference effects

4. Conclusion

By simulating the output signal of the model and the identified models according to the Quadrotor channel shows that although the input signal varies with different signal types under the impact of noise but output signals of the pattern recognition are always abreast with the output signal of the model (similar to the model's signals), i.e. mathematical model can recognize quite accurately and describes the motion of Quadrotor according to the channel.

With the analysis above we can absolutely confirm that the selected recognition algorithms could well apply to the Quadrotor. So after building basic control system, allowing collection of the control signal input and output signals of the system, we can fully carry out the mathematical model identification. Identifing process is carried out several times in each flight condition, then perform calibration parameters. Identified Results of the mathematical model can be used to synthesize control systems with better quality.

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