

# Study of Phase Behavior of Carbon Dioxide as the Power Gas for Gas Guns

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**Abstract**—The article is focused on the examination of the phase behavior of carbon dioxide (CO<sub>2</sub>) utilized as the source of energy to propel the projectile in gas guns technology. The objective is to develop an equilibrium discharge mathematical model for carbon dioxide tanks in order to assume thermodynamic equilibrium states at every point in time throughout discharging under ambient temperature conditions. The speed of sound in a two-phase saturation is taken into consideration for the calculation algorithm. The presented model is solved using the MATLAB environment. An experimental setup has been established for the purpose of verifying the results of solution.

**Keywords**—two-phase flow; carbon dioxide; gas guns; airsoft; paintball; discharge; pressure tank; speed of sound

## I. INTRODUCTION

Carbon dioxide is widely used as the power gas in the gas guns community due to its ease of handling, storability at room temperature, and high vapor pressure depending only upon temperature, not tank size, as long as some liquid carbon dioxide remains in the tank. This high vapor pressure can be used as the pressurant, making it what is referred to as a self-pressurizing propellant. A positive advantage gained from using CO<sub>2</sub> as a propellant is that a small volume of liquid can convert to a large volume of pressurized gas, thus, the carbon dioxide tanks tend to be smaller and lighter compared to high pressure air tanks while yielding the same or more shots per fill. Carbon dioxide was the first propellant used in paintball and airsoft technology and due to its low cost carbon dioxide has been being widely utilized as a power gas for gas guns.

Besides, CO<sub>2</sub> can also be used for simulation the sound effect of RPG-7 anti-tank grenade launcher for shooting training [11] or as for using of the liquid CO<sub>2</sub> as the emergency power gas generator for pneumatic systems of aircraft and military vehicles.

On the other hand, as a two-phase substance carbon dioxide does have its drawbacks: (1) vaporization of liquefied CO<sub>2</sub> inside a tank when shooting rapidly or a lot causes the tank to get cool resulting in pressure fluctuating that makes the guns performance and accuracy worse, (2) solid carbon dioxide that

is also known as dry ice can appear on the output valve of the tank while shooting and it can cause damage or slow the gun's performance down if it works its way into some control components, including the barrel of the gun. Hence, it is crucial to obtain a scientific understanding of carbon dioxide behavior and further the discharge characteristics of a wide range of pressure-tank configurations. For the purpose of satisfying this goal, a comprehensive discharge mathematical model for carbon dioxide tank dynamics is required.

In this paper, the thermodynamic properties of carbon dioxide are examined and the most significant characteristics of them are presented. A novel equilibrium discharge mathematical model for CO<sub>2</sub> tanks is formulated. In order to verify the model an experimental apparatus is also established.

## II. THERMODYNAMIC PROPERTIES OF CARBON DIOXIDE

In recent thirty years a great deal of attention has been paid to develop a properties database of carbon dioxide that could be used for both industrial and scientific applications. In this work, the thermodynamic properties of saturated carbon dioxide are required for the two-phase flow calculation. Such database of properties is available in [1-3]. The most significant physical constants of carbon dioxide are summarized in Tab. 1.

TABLE I. PHYSICAL CONSTANTS OF CARBON DIOXIDE

Properties	Values	
	U.S. Units	SI Units
Molar mass	44.01 g/mol	44.01 g/mol
Critical point	87.9°F at 1070.6 psia	31.1°C at 7381.8 kPa
Critical density	29.2 lb/ft <sup>3</sup>	468 kg/m <sup>3</sup>
Triple point	-69.9°F at 75.1 psia	-56.6°C at 518 kPa

Data obtained from [1]

Carbon dioxide can exist as a solid, liquid, gas or supercritical fluid depending upon conditions of temperature and pressure in the range of common working conditions.

Carbon dioxide at its triple point exists simultaneously as a liquid, gas, and solid at  $-56.6\text{ }^\circ\text{C}$  and  $518\text{ kPa}$ . Any change in pressure or temperature causes carbon dioxide to revert to a two-phase condition (see Fig. 1). Carbon dioxide at its critical point exists simultaneously as a liquid, gas and supercritical fluid at  $31.1\text{ }^\circ\text{C}$  and  $7381.8\text{ kPa}$ . At pressures and temperatures greater than the critical point, carbon dioxide exists only as a supercritical fluid [1].

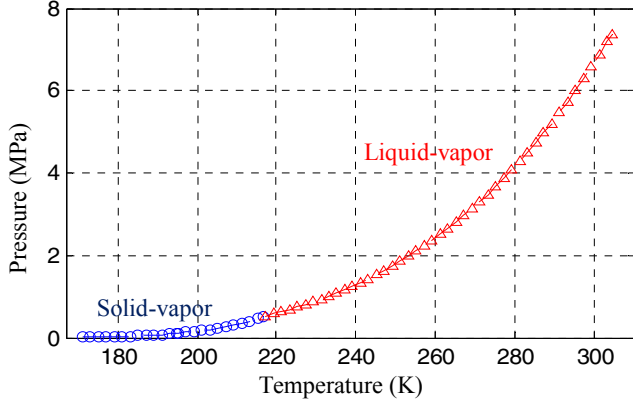


Figure 1. Pressure-temperature phase diagram of carbon dioxide

Fig. 2 shows values of specific enthalpy on both saturated curves (i.e. red and green lines) and in the single phase region (i.e. black lines). It is seen that there is a gap in the specific enthalpy at the triple point, as indicated by the horizontal arrow. Along the saturated curves, the specific enthalpy is only a function of temperature or pressure. In the two-phase region the saturation specific enthalpy depends on the specific enthalpy of individual saturated phases and the mixture quality, i.e. the ration of the mass of vapor phase to the total mass of the mixture. In the single-phase region, the specific enthalpy depends upon both the fluid temperature and pressure.

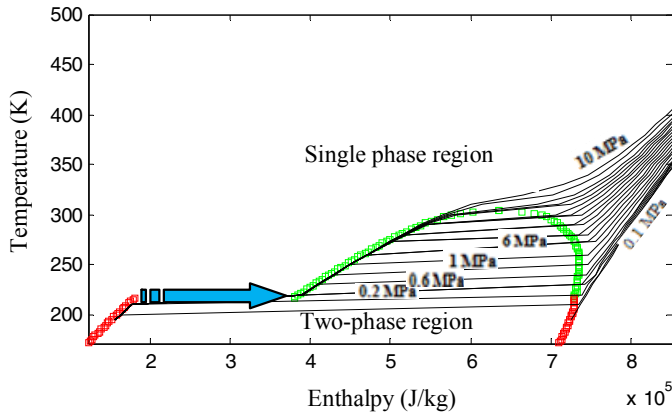


Figure 2. Temperature-specific enthalpy chart of carbon dioxide at different pressures

### III. DISCHARGE MATHEMATICAL MODEL

#### A. Discription and Bases of the Discharge Model

As it is seen in Fig. 3, the considered thermodynamic system consists of the supply tank and the receiver tank within

the control volume. Depending upon the initial temperature and pressure inside the supply tank,  $\text{CO}_2$  can exist in the form of single- or two-phase. In the case of single phase  $\text{CO}_2$  can exist in the form of vapor, if it is superheated, or in liquid form, if the supply tank is supercharged.

In this paper, we assume that in the supply tank  $\text{CO}_2$  exists in liquid-vapor equilibrium, in which the vapor phase occupies the upper part of the tank and the bottom tank portion is filled by the liquid phase. There exists an interface between the two phases.

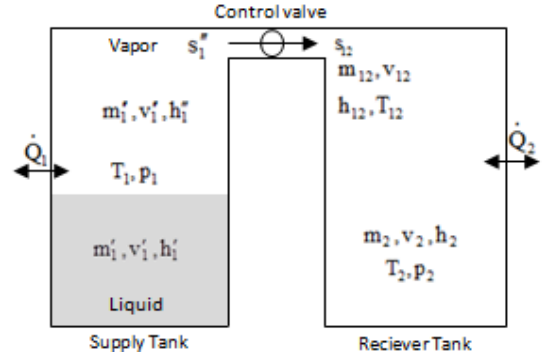


Figure 3. Schematic of the considered thermodynamic system

The zero-dimensional equilibrium model based on the mass and energy conservation [4] is developed for comprehensive simulation of the thermodynamic phenomena taking place in the supply and the receiver tank.

The exchange of mass and energy between the pressure tanks takes place in an open thermodynamic system (Fig. 4)

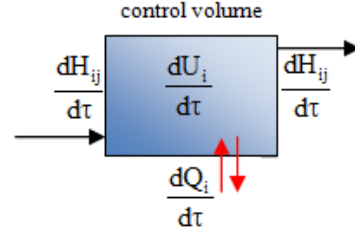


Figure 4. Schematic of the  $i$ -th thermodynamic system:  $dH_{ij}/d\tau$  – enthalpy rates crossing the boundary,  $dU_i/d\tau$  – internal energy time change,  $dQ_i/d\tau$  – heat transfer rate between the control volume and environment

During the unsteady-state processes, when the state quantities of a system vary in time and also the state and the amount of incoming and outgoing fluid can vary in time, the first law of thermodynamics acquires the common differential rate form with respect to a time-lag  $d\tau$

$$\frac{dQ}{d\tau} + \sum_{ij} \frac{dH_{ij}}{d\tau} = \frac{dU_i}{d\tau} + \frac{dV_i}{d\tau} \quad (1)$$

Terms appearing in (1) are as follows:

$\frac{dQ}{d\tau} = \alpha A (T - T_w)$  is the rate of convective heat transfer between the tank internal surfaces area  $A$  ( $\text{m}^2$ ) of temperature  $T_w$  and the gas within the tank of temperature  $T$  (K) by assuming that the temperature flows entering and leaving

across the system boundary, which can be expressed by the air mass flow rates  $\dot{m}_{ij}$  and the specific enthalpies  $h_{ij}$ .

$\frac{dU_i}{d\tau} = \frac{d(m_i u_i)}{d\tau} = \frac{dm_i}{d\tau} u_i + m_i \frac{du_i}{d\tau}$  is the internal energy time change, which is given by the change of mass  $m_i$  and by the change of specific internal energy  $u_i$  inside the chamber.

During the discharge process the whole tanks volume does not change in time. Hence, the term  $dV_i / d\tau$  appearing in (1) is equal to zero.

For a two-phase mixture, the quality, i.e. the ratio of the mass of vapor  $m''$  to the total mass of the mixture  $m_{\text{total}}$ , represented as  $x$  is given by

$$x = \frac{m''}{m_{\text{total}}} = \frac{m''}{m' + m''} \quad (2)$$

where  $m'$  (kg) is the mass of liquid or solid phase. Then the specific volume  $v$  ( $\text{m}^3/\text{kg}$ ), specific enthalpy  $h$  (J/kg), the density  $\rho$  ( $\text{kg}/\text{m}^3$ ) and the specific entropy  $s$  (J/kg/K) of a two-phase mixture can be determined as follows

$$v = xv'' + (1-x)v' \quad (3)$$

$$h = xh'' + (1-x)h' \quad (4)$$

$$\rho = x\rho'' + (1-x)\rho' \quad (5)$$

$$s = xs'' + (1-x)s' \quad (6)$$

where the upper subscript '' denotes vapor phase and ' denotes the liquid or solid phase quantities that are determined by the spline interpolation from the saturation tables.

### B. Assumptions

The mathematical model is built on the basis of the following assumptions:

- Open thermodynamic system.
- Single component ( $\text{CO}_2$  in various phases).
- One-dimensional flow of heterogeneous fluid.
- Two-phase or single phase fluid in the thermal equilibrium.
- In two-phase thermal equilibrium: specific enthalpy, specific volume, density and specific capacities of each phase are functions of temperature or pressure. In case of single-phase: specific enthalpy and specific capacities are functions of temperature and pressure.
- Isentropic expanding flow of  $\text{CO}_2$  through the discharge orifice.
- Heat transfer between the fluid within the tank and its walls.
- Constant temperature of the tank walls.

### C. Equations of Mathematical Model

First of all, we formulate the mathematical model simulating the thermodynamic processes taking place in the supply and receiver tank. In Fig. 3 it is seen that, when the control valve is open, the vapor immediately exits the supply tank and flows through the valve system into the receiver due to the pressure difference inside the tanks. Immediately, the phase equilibrium in the supply tank is broken and this causes an amount of liquid phase to evaporate in order to maintain the thermal phase equilibrium. Depending on the percent liquid fill and the valve shape and size, there are two different situations that may take place next:

- The amount of liquid phase is much enough to remain, until the vapor is totally discharged. In this case the thermodynamic state within the supply tank always remains in the two-phase thermal equilibrium.
- The liquid phase totally evaporates before the vapor is discharged. After the disappearance of liquid, there is only the single vapor phase inside the supply tank.

At first, we consider the case (i).

The change of the mass in the supply tank and the receiver tank is given by the mass balance of the system control volume generally expressed in the rate form as simulation of all the inlets and exits

$$\frac{dm_i}{d\tau} = \sum_{ij} \dot{m}_{ij} \quad (7)$$

Applying (7) for the supply tank (subscript 1) and the receiver tank (subscript 1) we obtain the time change in mass of  $\text{CO}_2$  within them as

$$\frac{dm_1}{d\tau} = -\dot{m}_{12} \quad (8)$$

$$\frac{dm_2}{d\tau} = \dot{m}_{12} \quad (9)$$

The mass gas flow rate  $\dot{m}_{12}$  from the supply tank through the control valve of the cross-sectional area  $A_{12}$  into the receiver tank is given by

$$\dot{m}_{12} = C_{12} \frac{v_{12} A_{12}}{v_{12}} \quad (10)$$

where  $v_{12}$  (m/s) is the discharge flow velocity,  $v_{12}$  ( $\text{m}^3/\text{kg}$ ) is the specific volume of fluid flowing from the discharge orifice,  $C_{12}$  is the discharge coefficient, it is usually assumed that the discharge coefficient is between 0.45-0.61.

The thermodynamic quantities of the two-phase fluid outgoing from the control valve orifice are calculated using expressions (3), (4), (5) and (6) as follows

$$v_{12} = x_{12} v_{12}'' + (1-x_{12}) v_{12}' \quad (11)$$

$$h_{12} = x_{12} h_{12}'' + (1-x_{12}) h_{12}' \quad (12)$$

$$s_{12} = x_{12}s_{12}'' + (1-x_{12})s_{12}' \quad (13)$$

$$\rho_{12} = x_{12}\rho_{12}'' + (1-x_{12})\rho_{12}' \quad (14)$$

Where quantities:  $v_{12}' = v_{12}'(p_2)$ ,  $v_{12}'' = v_{12}''(p_2)$ ,  $h_{12}' = h_{12}'(p_2)$ ,  $h_{12}'' = h_{12}''(p_2)$ ,  $s_{12}' = s_{12}'(p_2)$ ,  $s_{12}'' = s_{12}''(p_2)$  are functions of pressure  $p_2$ .

The  $x_{12}$  is the quality of the outgoing two-phase fluid that can be calculated by assuming that CO<sub>2</sub> expands from the supply tank through the valve orifice to the receiver tank isentropically. This assumption means that  $s_1'' = s_{12}$ , thus, we can determine  $x_{12}$  from (13) as follows

$$x_{12} = \frac{s_1'' - s_{12}'}{s_{12}'' - s_{12}'} \quad (15)$$

where  $s_1'' = s_1''(T_1)$  is a function of the supply tank's fluid temperature  $T_1$  (K).

The discharge flow velocity  $v_{12}$  is defined by

$$v_{12} = \min \left\{ \sqrt{2(h_1'' - h_{12})}, a_{\text{sonic}} \right\} \quad (16)$$

where  $a_{\text{sonic}}$  (m/s) is the speed of sound in the CO<sub>2</sub> two-phase fluid that is generally defined from the fluid  $p-v-T$  behavior using the following formula [5]

$$a_{\text{sonic}} = \left( \frac{\partial p}{\partial \rho} \right)_{\text{isentropic}}^{-\frac{1}{2}} \quad (17)$$

There are several mathematical models developed in previous researches to calculate the speed of sound in a two-phase fluid. The most notable models are given in [5-9]. In this study, we apply the pressure relaxed model provided in [9] due to its simplicity and accuracy for computing the speed of sound in the two-phase mixture of carbon dioxide. This model is also called Wood's model, in which the sound velocity is given by the following formula

$$a_{\text{sonic}} = \left[ \rho_{12} \left( \frac{\Theta_{12}}{\rho_{12}'' a_G^2} + \frac{1-\Theta_{12}}{\rho_{12}' a_{\text{SL}}^2} \right) \right]^{-\frac{1}{2}} \quad (18)$$

Here:  $\Theta_{12} = x_{12}v_{12}'' / v_{12}$  accounts for the void fraction,  $a_G$  is the speed of sound in gaseous phase and  $a_{\text{SL}}$  denotes the speed of sound in solid or liquid phase.

Values of speeds of sound  $a_G$  and  $a_{\text{SL}}$  for CO<sub>2</sub> are available in [3].

The plot of the speed of sound for two-phase CO<sub>2</sub> computed by (18) for temperatures 270 K and 285 K is shown in Fig. 5, where red lines indicate dependences for the two-phase fluid mass fractions  $x_{12}$  and blue lines are for the two-phase fluid void fractions  $\Theta_{12}$ .

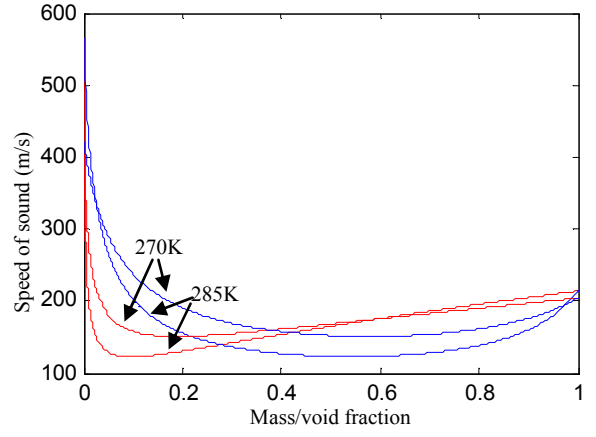


Figure 5. Speed of sound in liquid-vapor mixture of CO<sub>2</sub> versus mass fraction (red lines) and void fraction (blue lines)

The time change in the specific volume is determined by

$$\frac{dv_i}{d\tau} = \frac{1}{m_i} \frac{dV_i}{d\tau} - \frac{V_i}{m_i^2} \frac{dm_i}{d\tau}, \text{ for } \forall i = \{1, 2\} \quad (19)$$

The fact that while the mass of each phase changes in time, the sum of volumes of liquid and vapor phases in the supply tank is always equal to the volume of the supply tank  $V_s$  (m<sup>3</sup>). This statement can be mathematically expressed for the corresponding masses and specific volumes as

$$m_1'v_1' + m_1''v_1'' = V_s \quad (20)$$

If we replace  $m_1' = m_1 - m_1''$  in (19) and then take the derivative of its both sides with respect to time  $\tau$ , we can obtain the time change in mass of the gaseous phase in form

$$\frac{dm_1''}{d\tau} = -\frac{v_1'}{\Delta v_1} \frac{dm_1}{d\tau} - \phi(T_1) \frac{dT_1}{d\tau} \quad (21)$$

where  $\Delta v_1 = v_1'' - v_1'$  and the substitution function  $\phi(T_1)$  is defined as

$$\phi(T_1) = \frac{m_1''}{\Delta v_1} \left( \frac{dv_1''}{dT_1} + x_1 \frac{dv_1'}{dT_1} \right) \quad (22)$$

The derivatives appearing in (22) are determined by taking linear differences from the tables of saturated CO<sub>2</sub> properties provided in [1] along the saturation lines.

By differentiating  $x_1 = m_1''/m_1$  with respect to time  $\tau$  and by rearranging, we receive the time change in the quality of the two-phase mixture inside the supply tank in form

$$\frac{dx_1}{d\tau} = -\frac{1}{m_1} \left( \frac{v_1}{\Delta v_1} \frac{dm_1}{d\tau} - \phi(T_1) \frac{dT_1}{d\tau} \right) \quad (23)$$

The specific internal energy  $u_1$  of fluid inside the supply tank is given by the formula  $u_1 = h_1 - p_1v_1$ , in which  $h_1$  is determined using (4). The specific latent heat of vaporization at temperature  $T_1$  is given by  $L(T_1) = h_1'' - h_1'$  (J/kg).

The specific heat capacities of saturated phases are defined by  $c''_{p1} = dh''_1/dT_1$  and  $c'_{p1} = dh'_1/dT_1$  (J/kg/K).

Next, if we define  $\tilde{c}_{p1}(T_1) = x_1 c''_{p1} + (1-x_1) c'_{p1}$ , then, we can rewrite the governing equation (1) as bellows

$$\frac{dQ_1}{d\tau} + \frac{dm_1}{d\tau} (h''_1 - h_1) = m_1 \left[ \frac{dT_1}{d\tau} \left( \tilde{c}_{p1}(T_1) - v_1 \frac{dp_1}{dT_1} \right) + L(T_1) \frac{dx_1}{d\tau} \right] \quad (24)$$

The pressure change in temperature appearing in (24) can be expressed by using the Clausius-Clapeyron equation [4]:

$$\frac{dp_1}{dT_1} = \frac{1}{T_1} \frac{L(T_1)}{\Delta v_1} \quad (25)$$

By introducing (8), (23), and (25) into (24) and by rearranging them, we obtain the time change in temperature inside the supply tank in the following form

$$\frac{dT_1}{d\tau} = \frac{\frac{dQ_1}{d\tau} - \dot{m}_{12} v_1'' \frac{L(T_1)}{\Delta v_1}}{m_1 \tilde{c}_{p1}(T_1) - \frac{V_s}{T_1} \frac{L(T_1)}{\Delta v_1} - L(T_1) \phi(T_1)} \quad (26)$$

The thermodynamic quantities in the output of the valve depend on the pressure  $p_2$  in the receiver tank. We consider the phenomena occurring at the beginning of the process, as the first amount of gaseous phase flows into the receiver tank. Assuming that the initial pressure  $p_2$  equals to the pressure of the surrounding environment, thus, the mixture appearing at the output of the valve includes dry ice in the form of small solid particles and vapor of CO<sub>2</sub>. At the valve output, the mixture temperature  $T_{12}$  is a function of the downstream pressure  $p_2$  and it is interpolated from the saturation curves (Fig. 1). After entering to the receiver tank, this two-phase fluid expands to whole tank volume, where its equilibrium state is broken due to the change in temperature of  $T_2$  within the receiver tank. The initial temperature of  $T_2$  is assumed to be the surrounding temperature. It causes that the most of dry ice particles sublimates. Hence, the mixture in the receiver tank is in metastable state until the temperature and pressure conditions of two-phase mixture are met. Therefore, the presence of residual dry ice particles within the receiver tank can be neglected. Then, it is possible to describe the thermodynamic state of the gas inside the receiver tank by using the van der Waals equation of state [4] that can be written in form

$$p_2 = \frac{rT_2}{v_2 - b} - \frac{a}{v_2^2} \quad (27)$$

where  $r$  is the CO<sub>2</sub> specific gas constant, constant  $a$  provides a correction for the intermolecular forces and the constant  $b$  represents a correction for finite molecular size. The values of constants  $a$ ,  $b$  may be obtained by the theorem of corresponding states [4] using the CO<sub>2</sub> critical point conditions, which are listed in Tab. 1.

The time change in the specific internal energy  $u_2(T_2, v_2)$  of a real gas within the receiver tank is generally given by [4]

$$\frac{du_2}{d\tau} = c_{v2} \frac{dT_2}{d\tau} - \left[ T_2 \left( \frac{\partial p_2}{\partial T_2} \right)_v - p_2 \right] \frac{dv_2}{d\tau} \quad (28)$$

The specific heat capacity at constant volume  $c_{v2}$  for gaseous CO<sub>2</sub> appearing in the above equation is given by the difference of the specific heat capacity at constant pressure  $c_{p2}$  and the specific gas constant  $r$ . The values of  $c_{p2}$  are available in [10].

From (27), we can obtain the partial derivative appearing in (28) as

$$\left( \frac{\partial p_2}{\partial T_2} \right)_v = \frac{r}{v_2 - b} \quad (29)$$

By introducing (19) and (29) into (28) we can express the change in specific internal energy as a function of the time change in temperature and the specific volume:

$$\frac{du_2}{d\tau} = c_{v2} \frac{dT_2}{d\tau} - \frac{a}{v_2^2} \frac{dv_2}{d\tau} \quad (30)$$

The time change in temperature of gas within the receiver tank is given by rearranging the first law of thermodynamics (1) and the mass conservation (9) and by using (30), it yields

$$\frac{dT_2}{d\tau} = \frac{1}{m_2 c_{v2}} \left[ \frac{dQ_2}{d\tau} + \dot{m}_{12} \left( h_{12} - h_2 + p_2 v_2 - \frac{a}{v_2} \right) \right] \quad (31)$$

where  $h_{12} = h_2(p_2, T_2)$  is determined by using the spline interpolation of the thermodynamic table properties of CO<sub>2</sub> provided in [10].

Finally, for comprehensive simulation of the problem, we consider the case (ii), in which only the vapor phase remains within the supply tank. When the liquid is totally evaporated, the vapor, as a real gas, is governed by the van der Waals equation of state. Here, it refers to the single phase of vapor equilibrium. Therefore, we may consider the thermodynamic properties of the single vapor to equal to the saturated ones. Similarly to deriving (31), the time change in temperature within the supply tank  $T_1$  is given in form

$$\frac{dT_1}{d\tau} = \frac{1}{m_1 c_{v1}} \left[ \frac{dQ_1}{d\tau} - \dot{m}_{12} \left( h_{12} - h_1 + p_1 v_1 - \frac{a}{v_1} \right) \right] \quad (32)$$

In this case, the mixture quality  $x_2$  within the supply tank equals to 1.

#### D. Variations of Discharge Model

##### 1) Two-tanks with discharged liquid fluid

The above system of equations is valid only for the case, when vapor phase discharges out of the supply tank. If the liquid phase discharges from the supply tank instead of vapor (see Fig. 6), the phenomena appearing in the control valve output is similar to the previous case.

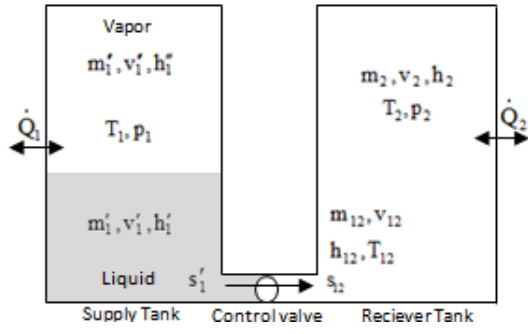


Figure 6. Schematic of two-tanks system with discharged liquid

The mathematical model is nearly same as for the previous case. Thermodynamic variables of the receiver tank are the same. But for evaluation of the supply tank variables, there are some changes in several equations, as seen in Tab. 2.

TABLE II. CHANGES IN EQUATIONS FOR SUPPLY TANK

Equation	Original quantity	Replaced by
(15)	$s_1''$	$s_1'$
(16)	$h_1''$	$h_1'$
(26)	$v_1''$	$v_1'$

## 2) Discharge from supply tank to surroundings

Here, it is necessary to take into consideration two different cases: vapor and liquid discharge. In fact, these cases are only modifications of the above described thermodynamic systems. The equations for changes in the variables within the supply tank are fully valid for these cases. Equations for determination of the time changes in variables for the receiver tank are not used here. Temperature  $T_{12}$  of the two-phase fluid in the output is only a function of the atmospheric pressure.

## IV. RESULTS OF SOLUTION

The above described problems were solved by numerical integration with MATLAB using the explicit fourth-order Runge-Kutta method. Mathematical model considers a range of input data parameters and boundary conditions (i.e. supply tank and receiver tank volumes, initial temperatures, the total mass of CO<sub>2</sub> and the percentage of liquid fill within the supply tank, the discharge coefficient, the heat transfer coefficient, etc.).

For the purpose of presenting results of solution, we chose the following input data: the total initial mass of carbon dioxide filled into the supply tank  $m_{1(0)} = 20$  g, the supply tank volume  $V_S = 76 \cdot \text{cm}^3$ , the convective heat transfer coefficient  $\alpha_1 = 12$  W/(m<sup>2</sup>K<sup>1</sup>), the total wall surface for the heat transfer is 0.011 m<sup>2</sup>, the control valve flow diameter is 8-mm, the initial temperature inside the supply tank  $T_{1(0)} = 288.15$  K corresponding the initial pressure  $p_{1(0)} = 5.07$  MPa. The initial mass of the CO<sub>2</sub> vapor phase in the supply tank is given by

$$m_{1(0)}'' = \left( \frac{V_S}{v_1'(T_{1(0)})} - m_{1(0)} \right) \frac{v_1''(T_{1(0)})}{v_1''(T_{1(0)}) - v_1'(T_{1(0)})} \quad (33)$$

The initial mass fraction of the CO<sub>2</sub> vapor phase within the supply tank is determined as

$$x_{1(0)} = \frac{m_{1(0)}''}{m_{1(0)}} \quad (34)$$

Firstly, we examine the case of the carbon dioxide discharge out from the supply tank through the control valve to the surrounding atmosphere. Results of the solution for the given example are clearly shown in Fig. 8, where we can see time courses of temperature, pressure and the mass of both CO<sub>2</sub> phases within the supply tank for the discharge of vapor (Fig. 8a) and liquid (Fig. 8b) phases.

Next, we examine the case of the pressurizing the receiver tank by the CO<sub>2</sub> flowing through the control valve from the supply tank. The design parameters of both tanks are the same. Results of the solution for this example are given in Fig. 9, where are plotted the time courses of temperature, pressure and the mass of CO<sub>2</sub> phases within the both tanks.

## V. VERIFICATION OF MATHEMATICAL MODEL

An experimental setup for studying properties of power gases for gas guns has been developed for the purpose of verification the proposed mathematical model.

The experimental device contains two high-pressure transparent tanks of volume 91 cm<sup>3</sup> connected by a connecting pipe. Access between tanks is provided through the electrically driven 2/2-way pilot operated solenoid valve GSR Type 46, rated for maximum pressure 10 MPa.

The time courses of pressure within the both chambers are measured by the piezoresistive pressure sensors DMP 333 with measure range from 0 to 25 MPa. Temperatures of both chamber walls are measured by resistive temperature sensors Jumo T90 2050 with measure range from -50 to 300 °C.

Measured data processing is performed by the software LabVIEW from National Instruments.

The view of the experimental setup is shown in Fig. 7.



Figure 7. View of experimental setup

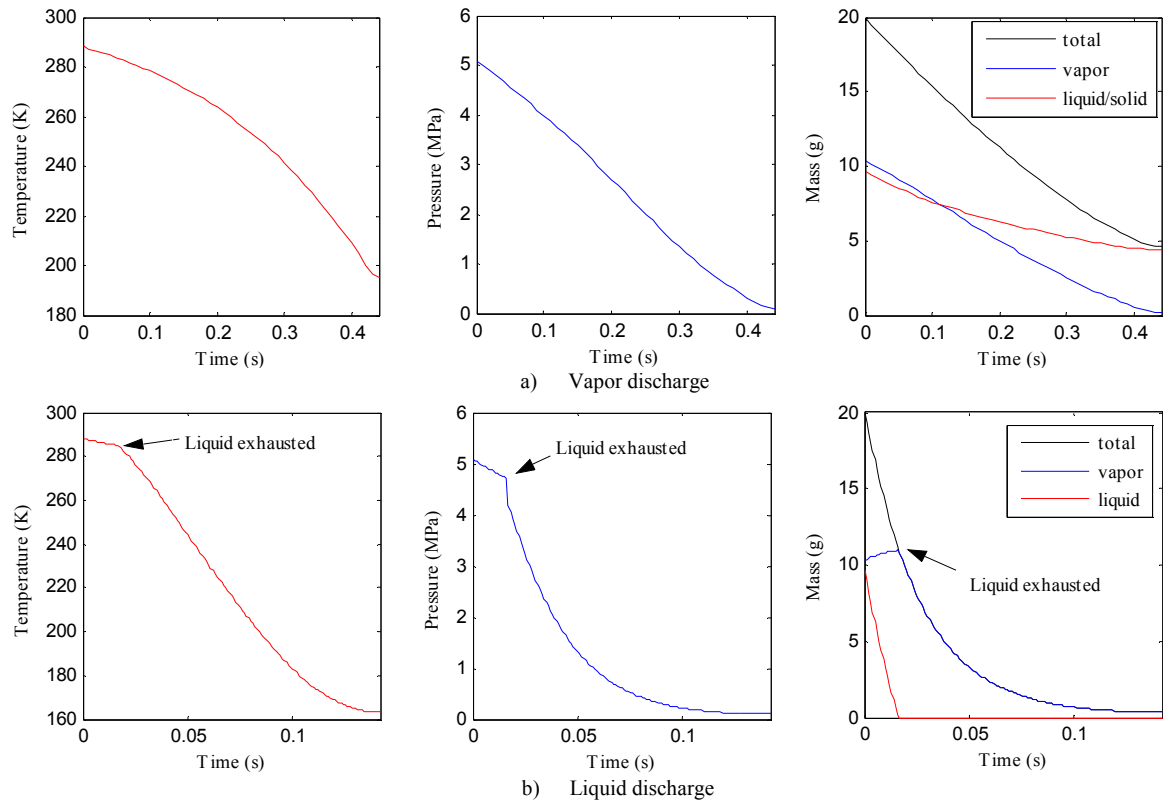


Figure 8. Discharge of CO<sub>2</sub> from supply tank to surroundings: temperature, pressure and mass as functions of time within the supply tank

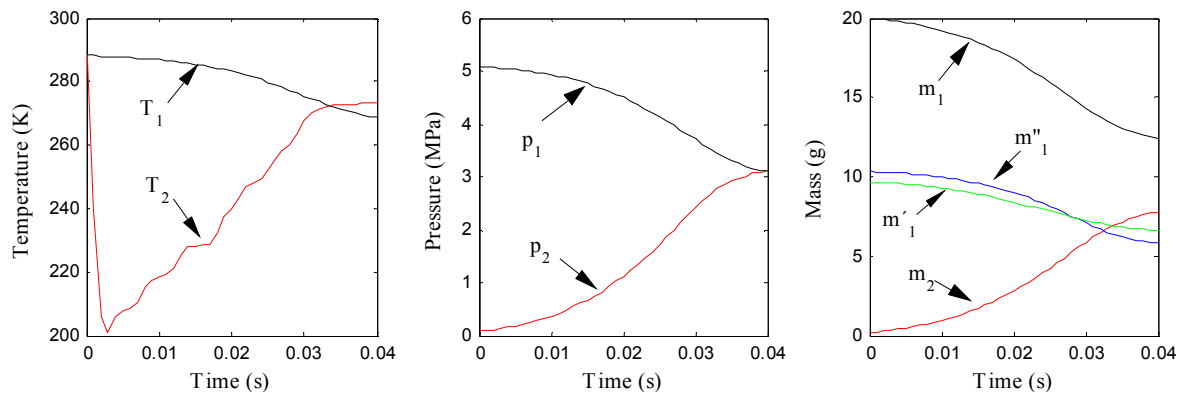


Figure 9. Pressurization of receiver tank by gaseous CO<sub>2</sub>: temperature, pressure and mass as functions of time within the both tanks.

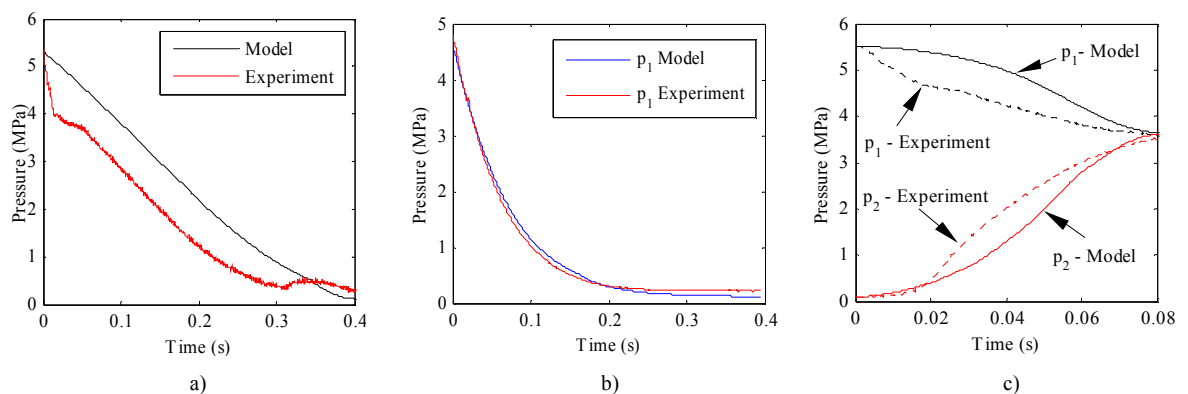


Figure 10. Comparison of calculated and measured time courses of pressure for: a) discharge of two-phase CO<sub>2</sub> to surroundings, b) discharge of gaseous CO<sub>2</sub> to surroundings, c) pressurization of receiver tank by two-phase CO<sub>2</sub> discharged from supply tank.

Three basic cases of carbon dioxide discharge were experimentally observed. The results of testing are given in Fig. 10 in comparison with results of theoretical solution by the proposed mathematical model.

The atmospheric conditions are the same for all three examined cases:  $T_a = 298.15$  K,  $p_a = 100.39$  kPa.

The first case (Fig. 10a) represents the discharge of a two-phase fluid into the surrounding atmosphere for the initial total CO<sub>2</sub> mass filling  $m_{1(0)} = 46.5$  g, which corresponds to the initial mass fraction of vapor phase  $x_{1(0)} = 0.1533$ .

The second case (Fig. 10b) is the discharge of gaseous CO<sub>2</sub> to the surroundings for the initial filling  $m_{1(0)} = 13.6$  g and the initial mass vapor fraction is naturally  $x_{1(0)} = 1$ .

The last case (Fig. 10c) examines the pressurization of the receiver tank by the two-phase fluid discharged from the supply tank. The initial total CO<sub>2</sub> mass filling of the supply tank is  $m_{1(0)} = 37.41$  g, which corresponds to the initial mass fraction of vapor phase  $x_{1(0)} = 0.2754$ .

Comparing the time courses of pressure for the above cases in Fig. 10, we can conclude fairly good agreement between the results of the equilibrium discharge mathematical model and experimentally obtained values.

Partial differences can be seen in Fig. 10a for the discharge of two-phase fluid. Beginning of the process after the valve opening represents discharge of gaseous CO<sub>2</sub> above the liquid level. This stage of depressurization corresponds to the pressure course in Fig. 10b. Then between 20 and 60 ms, we can observe thermal non-equilibrium effects of two-phase fluid flow which are present due to rapid depressurization. If the pressure and temperature drops under the CO<sub>2</sub> triple point values, the dry ice is created. This process of formation and following sublimation of the dry ice can be observed after the discharge time 0.3 s.

The best fit of experimental data is obtained in the case of discharge of gaseous CO<sub>2</sub> (Fig. 10b), where thermal non-equilibrium effects are not observed.

In the case of the pressurization of the receiver tank by the two-phase fluid discharged from the supply tank, we can see fairly good agreement in Fig. 10c. When the solenoid control valve is open, the two-phase fluid starts to discharge from the supply tank through the valve and the connecting pipe into the receiver tank (see Fig. 7). Here, we can also observe the thermal non-equilibrium effects of two-phase fluid flow. In the time about 80 ms, the process of the receiver tank pressurization is completed. The final value of pressure in the connected chambers is 3.5 MPa. It could be noted that, these values are almost identical with results of computation.

## VI. CONCLUSION

In this paper, the two-phase behavior of carbon dioxide is comprehensively examined. The discharge mathematical model for one- or two-pressure tanks is formulated. The problem is solved in MATLAB environment. The model was verified by comparison with measured data. Model provides very good results in describing equilibrium processes.

This discharge model can be used as a powerful tool not only in the gas guns technology but also in other fields of military technology. It is supposed applying e.g. for:

- The study of stability of systems using the liquid propellant during periodic discharge processes.
- Development of a device for the gun recoil simulation, whereas increased power gas consumption is supposed.
- Development of a simulation device for the sound effect of the RPG-7 anti-tank grenade launcher [11].
- The study of using of a liquid gas as the emergency power gas generator for pneumatic systems.

As the future work, it is supposed further development by the study and mathematical description of thermal non-equilibrium effects of two-phase fluid flow during the rapid depressurization. Such problems are observed e.g. in hybrid rocket feed systems [12].

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