# Performance Analysis of Water-Filling for Outdated CSI Multiple Hops MIMO Relay Systems

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*Abstract*—**Water-filling methods is one of techniques to improve a performance of multiple input multiple output (MIMO) system. However, the application of water-filling method to multiple hops MIMO relay system hasn't been discussed, especially in amplify and forward scheme. In this paper, the water-filling method for multiple hops MIMO relay system with amplify and forward scheme is proposed and evaluated in both perfect channel state information (CSI) and imperfect CSI. The effect of imperfect CSI on water-filling and on co-channel interference is taken into account. Compared to average transmit power, the end-to-end channel capacity of proposed water-filling method is higher in perfect CSI, however, it is subject to the effect of imperfect CSI. Additionally, a half duplex and a full duplex transmission method are also discussed in this paper. The end-to-end channel capacity of full duplex is higher and more robust than that of half duplex due to a double of allocation time and a half of delay time.**

*Index Terms*—**multiple hops MIMO relay system, water-filling method, perfect CSI and outdated CSI, half duplex and full duplex, end-to-end channel capacity.**

## I. Introduction

There are many researches on multiple-input multiple-output (MIMO) technique [1]-[4]. They indicated that a MIMO system can achieve the higher channel capacity than that of a single-input single-output (SISO). In order to further improve a channel capacity of MIMO system, a transmit power of every antenna is allocated depending on channel model meaning water-filling (WF) technique is applied to MIMO system [5]-[8].

Additionally, in order to reduce the transmit power and/or improve the performance of MIMO system, a multiple-hop MIMO relay system has been analyzed [9]-[11]. In multiple hops MIMO relay systems (MMRS), an end-to-end channel capacity is restricted by bottleneck. Therefore, in order to improve the end-to-end channel capacity, the performance of bottleneck should be improved. Distances between transceivers and transmit powers of every relay node can be optimized to obtain an upper bound of end-to-end channel capacity. We have proposed optimization method of distances and transmit powers for MMRS with amplify-and-forward (AF) scheme [12], [13] and decode-and-forward scheme (DF) [14]. However, in these researches, the transmit power of each relay was assumed to be divided equally to all antennas in the relay. The DF relay node decodes data before transferring to the next relay node, whereas the AF relay node amplifies the received data and transfers to the next relay node. The computational simplicity of AF scheme is greatly attractive and a strong candidate for the real-time application. Furthermore, in DF scheme MIMO relay system, since each hop is independent of the other hops, the water-filling of original MIMO system (the system hasn't a relay node) can be applied. Therefore, in this paper, we focus on the AF scheme MIMO relay system.

There are many literatures that focus on the AF scheme MIMO relay networks [9], [10]. The diversity multiplexing tradeoff for MMRSs also was investigated [15]. There are some works on beamforming design for special types of half-duplex AF scheme MMRSs [16]. The WF method for AF scheme MMRSs wasn't considered.

In this paper, some WF methods for AF scheme MMRS is proposed and compared to the conventional WF method. A half duplex and a full duplex transmission scheme are analyzed based on the end-to-end channel capacity. Moreover, the imperfect channel state information (CSI) is taken into account.

The rest of the paper is organized as follows. We introduce the system model of MMRS in Section II. The WF method for AF scheme MMRS is proposed in Section III and the numerical evaluation is described in Section IV. Finally, Section V concludes the paper.

## II. AF scheme MMRS

The detail of system model of AF scheme MMRS is described in [12], [13]. In this section, we introduce a brief review.

### A. System model of MMRSs

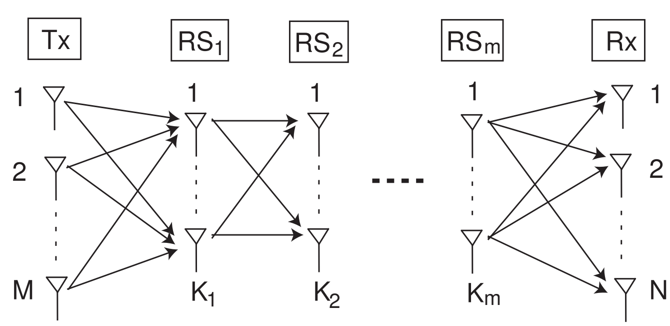


Fig. 1. System model of MMRSs.

As shown in Fig. 1, the MMRS with $m$ relay nodes intervened is considered. Here, let the $\mathrm{Tx}$, the $\mathrm{Rx}$ and

$RS_{\rm i}\ \ (i=1,\cdots,m)$ denote the transmitter, the receiver and $i^{\rm th}$ relay node, respectively. $M$, $N$ and $K_{\rm i}\ \ (i=1,\cdots,m)$ are the number of antenna elements corresponding to the Tx, the Rx and each relay. However, the number of antenna elements of all relays is assumed to be the same as the number of antenna elements of the transmitter and the final receiver, $M=N=K_{\rm i}\ \ (i=1,\cdots,m)$. Mathematical notations used in this paper are as follows. x and $X$ are scalar variables, $\mathbf{x}$ and $\mathbf{X}$ is vector variables or matrix variables, $\mathbf{X}^{\rm H}$ denotes a conjugate transpose matrix of $\mathbf{X}$.

### *B. Channel model*

In order to easily describe, the Tx, Rx are also be denoted as the $RS_0$ and $RS_{m+1}$, respectively. Since the path loss is taken into consideration, channel matrix is a composite matrix and we model as $\sqrt{l_{\rm ii+1}}\mathbf{H}_{\rm ii+1},\ \ {\rm i}=0,\cdots,{\rm m}$, of which $l_{\rm ii+1}$ and $\mathbf{H}_{\rm ii+1}$ represent the path loss and the $K_{\rm i+1}\times K_{\rm i}$ channel matrix between the $RS_{\rm i}$ and the $RS_{\rm i+1}$, respectively. $\mathbf{H}_{\rm ii+1}$ is a matrix with independent and identical distribution (i.i.d.), zero mean, unit variance, circularly symmetric complex Gaussian entries.

Let $\mathbf{H}_{\rm i}\ (i=1,\cdots,m+1)$ denote the channel matrix between the Tx and the $RS_i$ (as expressed in Fig. 2). The other parameters of system, i.e. transmit signal, received signal, noise vector, amplification factor, total transmit power, are summarized in Table I.

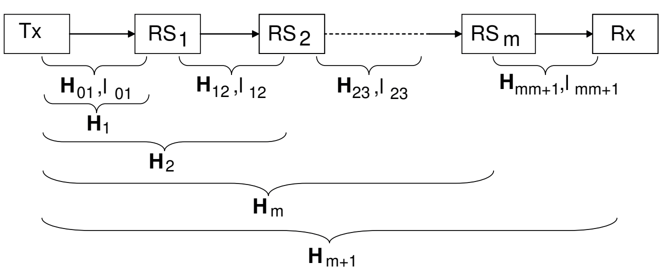


Fig. 2. Channel model of MMRS.

TABLE I
SYSTEM PARAMETERS OF EACH TRANSCEIVER

| - | Tx | $RS_1$ | $\cdots$ | $RS_{\rm m}$ | Rx |
|---|---|---|---|---|---|
| Transmit signal | $\mathbf{S}_0$ | $\mathbf{S}_1$ | $\cdots$ | $\mathbf{S}_{\rm m}$ | - |
| Received signal | - | $\mathbf{R}_1$ | $\cdots$ | $\mathbf{R}_{\rm m}$ | $\mathbf{R}_{\rm m+1}$ |
| Noise vector | - | $\mathbf{n}_1$ | $\cdots$ | $\mathbf{n}_{\rm m}$ | $\mathbf{n}_{\rm m+1}$ |
| Amplification factor | $\mathbf{\Gamma}_0$ | $\mathbf{\Gamma}_1$ | $\cdots$ | $\mathbf{\Gamma}_{\rm m}$ | - |
| Total transmit power | $E_0$ | $E_1$ | $\cdots$ | $E_{\rm m}$ | - |

Let's $\mathbf{P}_{\rm i}=diag(p_{\rm i1},p_{\rm i2},\cdots,p_{{\rm iK}_{\rm i}})$ denote a transmit power matrix and be assumed to be subject to a constraint $Tr(\mathbf{P}_{\rm i})=E_{\rm i}$. The $Tr(\cdot)$ and $p_{\rm ij}$ represent the trace and transmit power of $j^{\rm th}$ antenna element of the $RS_i$, respectively. Therefore, the received signal at the Rx can be expressed as

$$\mathbf{S}_{\rm m+1}=\sqrt{\frac{l_{\rm mm+1}\mathbf{P}_{\rm m}\cdots l_{01}\mathbf{P}_0}{|\mathbf{R}_{\rm m}|^2\cdots|\mathbf{R}_1|^2}}\mathbf{H}_{\rm mm+1}\cdots\mathbf{H}_{12}\mathbf{H}_{01}\mathbf{S}_0+\mathbf{n},\quad (1)$$

here the $\mathbf{n}$ denotes the noise vector of system. Therefore, the system channel matrix is written by

$$\mathbf{H}_{\rm m+1}=\mathbf{H}_{\rm mm+1}\cdots\mathbf{H}_{12}\mathbf{H}_{01}.\quad (2)$$

According to using the system channel matrix $\mathbf{H}_{\rm m+1}$, the multiple hops MIMO relay system can be analyzed as same as a conventional MIMO system.

### *C. End-to-end channel capacity*

Since AF scheme is applied to the channel model, the noise is also amplified and transmitted to the next relay while the signal is being amplified at every relay. The noise vector of system when $m$ relays intervened becomes as

$$\begin{aligned}\mathbf{n}\ &=\ \sqrt{\frac{l_{\rm mm+1}\mathbf{P}_{\rm m}\cdots l_{12}\mathbf{P}_1}{|\mathbf{R}_{\rm m}|^2\cdots|\mathbf{R}_1|^2}}\mathbf{H}_{\rm mm+1}\cdots\mathbf{H}_{12}\mathbf{n}_1 \quad (3)\\ &+\ \sqrt{\frac{l_{\rm mm+1}\mathbf{P}_{\rm m}\cdots l_{23}\mathbf{P}_2}{|\mathbf{R}_{\rm m}|^2\cdots|\mathbf{R}_1|^2}}\sqrt{|\mathbf{R}_1|^2}\mathbf{H}_{\rm mm+1}\cdots\mathbf{H}_{23}\mathbf{n}_2\\ &+\ \cdots\\ &+\ \sqrt{\frac{l_{\rm mm+1}\mathbf{P}_{\rm m}}{|\mathbf{R}_{\rm m}|^2\cdots|\mathbf{R}_1|^2}}\sqrt{|\mathbf{R}_{\rm m-1}|^2\cdots|\mathbf{R}_1|^2}\mathbf{H}_{\rm mm+1}\mathbf{n}_{\rm m}\\ &+\ \mathbf{n}_{\rm m+1}.\end{aligned}$$

However, at first, the channel state information (CSI) is assumed to be perfect, the imperfect CSI is considered in the following section. The channel matrix $\mathbf{H}_{01}$ can be expressed as follows in a singular-value decomposition (SVD) [17], [18].

$$\mathbf{H}_{01}=\mathbf{E}_{01_{\rm r}}\mathbf{D}_{01}\mathbf{E}_{01_{\rm t}}^{\rm H}=\sum_{\rm i=1}^{\rm M}\sqrt{\lambda_{01_{\rm i}}}\mathbf{e}_{01_{\rm r,i}}\mathbf{e}_{01_{\rm t,i}}^{\rm H},\quad (4)$$

here

$$\begin{aligned}\mathbf{D}_{01}\ &=\ {\rm diag}\left[\sqrt{\lambda_{01_1}}\ \sqrt{\lambda_{01_2}}\ \cdots\ \sqrt{\lambda_{01_{\rm M}}}\right],\quad (5)\\ \mathbf{E}_{01_{\rm r}}\ &=\ \left[\mathbf{e}_{01_{\rm r,1}}\ \mathbf{e}_{01_{\rm r,2}}\ \cdots\ \mathbf{e}_{01_{\rm r,M}}\right],\\ \mathbf{E}_{01_{\rm t}}\ &=\ \left[\mathbf{e}_{01_{\rm t,1}}\ \mathbf{e}_{01_{\rm t,2}}\ \cdots\ \mathbf{e}_{01_{\rm t,M}}\right],\end{aligned}$$

and $\lambda_{01_{\rm i}},\ i=1,2,\cdots,M$ is the $i^{th}$ eigenvalue of the correlation matrix $\mathbf{H}_{01}\mathbf{H}_{01}^{\rm H}$ (or $\mathbf{H}_{01}^{\rm H}\mathbf{H}_{01}$), $\mathbf{e}_{01_{\rm r,i}}$ is the eigenvector belonging to the eigenvalue $\lambda_{01_{\rm i}}$ of correlation matrix $\mathbf{H}_{01}\mathbf{H}_{01}^{\rm H}$ and $\mathbf{e}_{01_{\rm t,i}}$ is the eigenvector belonging to the eigenvalue $\lambda_{\rm i}$ of correlation matrix $\mathbf{H}_{01}^{\rm H}\mathbf{H}_{01}$. The SVD is applied to the others channel matrix similarly.

The channel matrix and the noise vector are assumed to be uncorrelated and the covariance of noise vector in all relays is denoted by $\sigma^2$. Therefore, the correlation of noise vector of system is represented as follows.

$$\mathbf{n}\mathbf{n}^{\rm H}=\frac{\sum_{i=0}^{m}\prod_{\rm j\neq i}^{\rm m}l_{\rm jj+1}\mathbf{P}_{\rm j}\mathbf{D}_{\rm jj+1}^2\sigma^2+\sigma^4 f(l_{\rm ii+1}\mathbf{P}_{\rm i}\mathbf{D}_{\rm ii+1}^2,\sigma^2)}{|\mathbf{R}_{\rm m}|^2\cdots|\mathbf{R}_1|^2},\quad (6)$$

here $f(l_{\rm ii+1}\mathbf{P}_{\rm i}\mathbf{D}_{\rm ii+1}^2,\sigma^2)$ denotes a polynomial of $l_{\rm ii+1}\mathbf{P}_{\rm i}\mathbf{D}_{\rm ii+1}^2$ and $\sigma^2$. Since $\sigma^2\ \le\ 1$ and the covariance of noise vector is smaller than the received power, the component containing $\sigma^4$ can be ignored. Thus (6) can be changed as

$$\mathbf{nn}^{\mathrm{H}} = \frac{\sum_{i=0}^{m} \prod_{\mathrm{j}\neq \mathrm{i}}^{\mathrm{m}} l_{\mathrm{jj+1}} \mathbf{P}_{\mathrm{j}} \mathbf{D}_{\mathrm{jj+1}}^2 \sigma^2}{|\mathbf{R}_{\mathrm{m}}|^2 \cdots |\mathbf{R}_1|^2}, \tag{7}$$

Consequently, the end-to-end channel capacity of AF scheme MMRS is expressed as

$$\begin{aligned} C &= log_2 \left( det \left( \frac{\mathbf{S}_{\mathrm{m+1}} \mathbf{S}_{\mathrm{m+1}}^{\mathrm{H}}}{\mathbf{n}_{\mathrm{m+1}} \mathbf{n}_{\mathrm{m+1}}^{\mathrm{H}}} \right) \right), \quad (8) \\ &= log_2 \left( det \left( \mathbf{I}_{\mathrm{M}} + \frac{\prod_{\mathrm{i}=0}^{\mathrm{m}} l_{\mathrm{jj+1}} \mathbf{P}_{\mathrm{j}} \mathbf{D}_{\mathrm{jj+1}}^2}{\sigma^2 \sum_{i=0}^{m} \prod_{\mathrm{j}\neq\mathrm{i}}^{\mathrm{m}} l_{\mathrm{jj+1}} \mathbf{P}_{\mathrm{j}} \mathbf{D}_{\mathrm{jj+1}}^2} \right) \right), \\ &= log_2 \left( det \left( \mathbf{I}_{\mathrm{M}} + \frac{1}{\sigma^2 \sum_{i=0}^{m} \frac{1}{l_{\mathrm{ii+1}} \mathbf{P}_{\mathrm{i}} \mathbf{D}_{\mathrm{ii+1}}^2}} \right) \right). \end{aligned}$$

Let

$$f(\lambda) = \sigma^2 \sum_{i=0}^{m} \frac{1}{l_{\mathrm{ii+1}} \mathbf{P}_{\mathrm{i}} \mathbf{D}_{\mathrm{ii+1}}^2}. \tag{9}$$

Thus the end-to-end channel capacity of system is inversely proportional to $f(\lambda)$. Therefore, $f(\lambda)$ can be considered instead of the end-to-end channel capacity. In order to maximize the end-to-end channel capacity, the $f(\lambda)$ has to be minimize. The $f(\lambda)$ is a function of transmit power, path loss meaning the distance between each transceiver, the variance of noise vector and the matrix of eigenvalue of channel model. The optimization of distance and transmit power were described in [12], [13], however, the transmit power of each antenna element in the same relay is assumed to be equal. In order to improve the performance of system, the WF method for AF scheme MMRS is proposed. The $f(\lambda)$ can be changed as

$$f(\lambda) = \sum_{i=1}^{m+1} \frac{1}{\mathbf{SNR}_{\mathrm{i-1i}} \mathbf{D}_{\mathrm{i-1i}}^2}, \tag{10}$$

here $\mathbf{SNR}_{\mathrm{i-1i}} = \frac{\mathrm{l}_{\mathrm{i-1i}} \mathbf{P}_{\mathrm{i-1}}}{\sigma^2} = \mathrm{diag}(\gamma_{\mathrm{i-1i_1}}, \gamma_{\mathrm{i-1i_2}}, \cdots, \gamma_{\mathrm{i-1i_M}})$ represents the signal to noise ratio (SNR) of hop between the $RS_{i-1}$ and the $RS_i$ regardless the other relays. Since, the distance between the transmitter and the final receiver as well as the total transmit powers of all transmitters are assumed to be fixed. Let's $Tr(\mathbf{SNR}_{\mathrm{i-1i}})$ is fixed as $\frac{l_{\mathrm{i-1i}} E_{\mathrm{i-1}}}{\sigma^2} = snr_i$ where $snr_i$ is the ratio of signal received power to noise power by one receive antenna element.

## III. Water-filling for AF scheme MMRS

The WF method can be applied to AF scheme MMRS by three schemes. Scheme 1: as conventional WF, the WF method is applied to each hop in MMRS independently of transmission profile of other hops. Scheme 2: the WF method is applied to each process. It means that the WF method for a hop depends on the transmission profile of all previous hops. Scheme 3: the same WF is applied to all hops of MMRS depending on the transmission profile from the $\mathrm{Tx}$ to the $\mathrm{Rx}$. The detail of each scheme is explained as follows.

### A. WF method for AF scheme MMRS with perfect CSI

**Scheme 1:** WF for each hop of MMRS: in this scheme, the channel model of each hop is used to optimize the transmit power of transmitter. The hop between the $RS_{i-1}$ and the $RS_i$ is considered and regardless of the other relay nodes. As conventional WF for MIMO system in which there are only one transmitter and one receiver, the channel capacity is described as follows.

$$\begin{aligned} C_{\mathrm{i-1i}} &= \log_2 \prod_{k=1}^{M} (1 + \lambda_{\mathrm{i-1i_k}} \gamma_{\mathrm{i-1i_k}}), \quad (11) \\ &= \sum_{k=1}^{M} \log_2 \lambda_{\mathrm{i-1i_k}} + \sum_{k=1}^{M} \log_2 \left( \frac{1}{\lambda_{\mathrm{i-1i_k}}} + \gamma_{\mathrm{i-1i_k}} \right). \end{aligned}$$

The channel capacity maximization problem is now changed to the choice of the maximal number of real, nonnegative values $\gamma_{\mathrm{i-1i_k}}$ subject to the power constraint $\sum_{k=1}^{M} \gamma_{\mathrm{i-1i_k}} = snr_i$. The well-known approach to solving constrained optimization problems is the method of Lagrange multipliers. Accordingly, we need to maximize

$$g(\gamma) = \sum_{k=1}^{M} \log_2 \left( \frac{1}{\lambda_{\mathrm{i-1i_k}}} + \gamma_{\mathrm{i-1i_k}} \right) + \alpha \left( \sum_{k=1}^{M} \gamma_{\mathrm{i-1i_k}} - snr_i \right), \tag{12}$$

where $\alpha$ is the Lagrange multiplier. The partial differentiation with respect to $\gamma_{\mathrm{i-1i_k}}$ leads to

$$\frac{\partial g(\gamma)}{\partial \gamma_{\mathrm{i-1i_k}}} = \frac{\lambda_{\mathrm{i-1i_k}}}{1 + \lambda_{\mathrm{i-1i_k}} \gamma_{\mathrm{i-1i_k}}} + \alpha. \tag{13}$$

The optimized $\gamma_{\mathrm{i-1i_k}}$ can be obtained when $\frac{\partial g(\gamma)}{\partial \gamma_{\mathrm{i-1i_k}}} = 0$. Hence

$$\gamma_{\mathrm{i-1i_k}} = -\frac{1}{\alpha} - \frac{1}{\lambda_{\mathrm{i-1i_k}}} \tag{14}$$

The Lagrange multiplier is relabeled as $\beta = -\frac{1}{\alpha}$ to simplify the notation in later formulas. This problem really has two constraints: the power constraint and being nonnegative $\gamma_{\mathrm{i-1i_k}}$. The latter constraint can be described as

$$\begin{aligned} \gamma_{\mathrm{i-1i_k}} &= \max\{0, \beta - \frac{1}{\lambda_{\mathrm{i-1i_k}}}\}, \quad (15) \\ &\equiv [\beta - \frac{1}{\lambda_{\mathrm{i-1i_k}}}]^+. \end{aligned}$$

The former constraint can be described as

$$\begin{aligned} snr_{\mathrm{i}} &= \sum_{k=1}^{M} [\beta - \frac{1}{\lambda_{\mathrm{i-1i_k}}}]^+, \quad (16) \\ &= \sum_{k=1}^{q} [\beta - \frac{1}{\lambda_{\mathrm{i-1i_k}}}]^+, \\ &= q\beta - \sum_{k=1}^{q} \frac{1}{\lambda_{\mathrm{i-1i_k}}}. \end{aligned}$$

Thus, the Lagrange multiplier and the optimal SNR meaning the optimal transmit power of each path responding to eigenvalues are obtained.

$$\beta = \frac{1}{q}\left(\sum_{k=1}^{q}\frac{1}{\lambda_{\mathrm{i-1i_k}}}+snr_i\right), \quad (17)$$

$$\gamma_{\mathrm{i-1i_k}} = \frac{1}{q}\left(\sum_{k=1}^{q}\frac{1}{\lambda_{\mathrm{i-1i_k}}}+snr_i\right)-\frac{1}{\lambda_{\mathrm{i-1i_k}}}.$$

A computational procedure for determining the optimal value of $q$ is to compute for $q = M, M-1, \cdots$ the value of $\gamma_{\mathrm{i-1i_k}}$ until this quantity is greater than zero for all $k$ from 1 to $q$. By substituting the optimal $\lambda_{\mathrm{i-1i_k}}$ to (10) and (8), the end-to-end channel capacity can be obtained.

Similar to scheme 1, in schemes 2 and 3, the transmit power of transmitters also can be optimized based on multiplication of channel models.

**Scheme 2:** WF for each process, the transmit power of transmitter is optimized based on multiplication of channel models until the receiver. It means that the WF of hop between the $RS_{i-1}$ and the $RS_i$ is analyzed based on the channel model $\mathbf{H}_{\mathrm{i}} = \mathbf{H}_{\mathrm{i-1i}}\cdots\mathbf{H}_{01}$ (refer to Fig. 2 ). Let's $\mathbf{D}_{\mathrm{i}}$ denote the eigenvalue matrix of $\mathbf{H}_{\mathrm{i}}$ in SVD, thus,

$$\mathbf{D}_{\mathrm{i}} = \mathbf{D}_{\mathrm{i-1i}}\cdots\mathbf{D}_{01}, \quad (18)$$

and the $\mathbf{SNR}_{\mathrm{i-1i}}$ is optimized based on $\mathbf{D}_{\mathrm{i}}$ as mentioned above.

**Scheme 3:** WF for all channel models, the WF for all hops is assumed to be the same and the transmit power is optimized based on channel matrix of system, $\mathbf{H}_{\mathrm{m+1}}$, in (2). The eigenvalue matrix of $\mathbf{H}_{\mathrm{m+1}}$ is denoted by $\mathbf{D}_{\mathrm{m+1}}$:

$$\mathbf{D}_{\mathrm{m+1}} = \mathbf{D}_{\mathrm{mm+1}}\cdots\mathbf{D}_{01}. \quad (19)$$

The WF of all hops is dependent on $\mathbf{D}_{\mathrm{m+1}}$.

Up to now, the perfect CSI at both transmitter and receiver is assumed and the analysis of WF for MMRS was proposed. However, in actuality, the perfect CSI assumption is not always practical due to channel estimation errors, feedback channel delay, and noise, especially in schemes 2 and 3, the CSI of another hops is also necessary. Compared to channel estimation errors, the CSI imperfection introduced by feedback channel delay is sometimes more significant and inevitable.

### *B. WF for AF scheme MMRS with imperfect CSI*

Similar to previous section, the channel model between the $RS_{i-1}$ and the $RS_i$ is considered. The channel time variation is described by the first-order Markov process as follows.

$$\mathbf{H}_{\mathrm{i-1i}}(\mathrm{t}) = \rho\mathbf{H}_{\mathrm{i-1i}}(\mathrm{t}-\tau) + \delta\bar{\mathbf{H}}_{\mathrm{i-1i}}(\mathrm{t}), \quad (20)$$

where $\tau$ denotes the total delay caused by signal processing, feedback, and other system delays, $\rho = J_0(2\pi f_D\tau)$ is the time correlation coefficient, $J_0(\cdot)$ is the zero-th order Bessel function of the first kind and $f_D$ denotes the maximum Doppler frequency shift. $\delta$ denotes as $\sqrt{1-\rho^2}$. The innovation term $\bar{\mathbf{H}}_{\mathrm{i-1i}}$ also has i.i.d entries, zero mean, unit variance, circularly symmetric complex Gaussian entries. For notation brevity, we drop off the time index

$$\mathbf{H}_{\mathrm{i-1i}} = \rho\hat{\mathbf{H}}_{\mathrm{i-1i}} + \delta\bar{\mathbf{H}}_{\mathrm{i-1i}}, \quad (21)$$

where $\hat{\mathbf{H}}_{\mathrm{i-1i}}$ denotes the outdated channel matrix while $\bar{\mathbf{H}}_{\mathrm{i-1i}}$ is the true one. Notice that there are two ways the outdated CSI affects the system. One is affecting to separate every transmission path between each transceiver, the other one is affecting to optimize the transmit power of all antenna elements based on WF. The former one just depends on the CSI between each transceiver regardless of other CSIs, whereas the latter one depends on the scheme of WF. The CSI of the latter one is explained as follows. Notice that the full-duplex is assumed, all relays transmit and receive in the same time. The delay of half-duplex is a double delay of full-duplex.

**Scheme 1:** For every pair of transceiver, we assume that the transmitter adds the CSI symbol in the data packet and transmits to the receiver, hence the receiver knows both $\bar{\mathbf{H}}_{\mathrm{i}}$ and $\hat{\mathbf{H}}_{\mathrm{i}}$. After receiving the CSI symbol, the receiver feedbacks the CSI to the transmitter immediately. However, the transmitter receives the CSI after a delay. Therefore the CSI that is used at transmitter is the outdated CSI. It means that the transmitter only knows the outdated channel model, $\hat{\mathbf{H}}_{\mathrm{i}}$, while the receiver knows both $\bar{\mathbf{H}}_{\mathrm{i}}$ and $\hat{\mathbf{H}}_{\mathrm{i}}$. The delay at each hop is assumed to be the same as $\tau$. Therefore, in scheme 1, the delay of CSI at each transmitter is the same as $\tau$.

**Scheme 2:** the CSI symbol that was added at the $\mathrm{Tx}$ is assumed to be forwarded at all relays, therefore all relays and the $\mathrm{Rx}$ can know the CSI from the $\mathrm{Tx}$ to itself. However, when a relay receives the CSI symbol, the CSI of previous hop has become the outdated. The delay of forward link (from the $\mathrm{Tx}$ to the $\mathrm{Rx}$) and backward link (from the $\mathrm{Rx}$ to the $\mathrm{Tx}$) is assumed to be the same as $\tau$. The $RS_{i-1}$ starts to optimize transmit power based on the WF after receiving the feedback CSI from the $RS_i$, therefore, the delay of CSI of $\mathbf{H}_{\mathrm{i-1i}}, \cdots, \mathbf{H}_{01}$ is $1\tau, \cdots, i\tau$, respectively.

**Scheme 3:** We assumed that the CSI information is added at the $\mathrm{Tx}$ and detected at the $\mathrm{Rx}$. After detecting the CSI information, the $\mathrm{Rx}$ transmits it to the $RS_{\mathrm{m}}$ and every relay forwards the feedback CSI to the previous relay until the TX receives. All transmitters change their WF every time they received the feedback of CSI. When the $RS_{\mathrm{i}}$ receives the feedback of CSI, the delay corresponded to $\mathbf{H}_{01}, \cdots, \mathbf{H}_{\mathrm{mm+1}}$ is $(m+i)\tau, (m-1+i)\tau, \cdots, i\tau$.

The influence of outdated CSI on separation of multiple paths of MIMO system is explained as follows. It is exact for all schemes 1, 2 and 3. In an SVD-based MIMO system, the relay $RS_{\mathrm{i-1}}$ steers the modulated signal vector $\mathbf{S}_{\mathrm{i-1}}$ within the eigenspace spanned by the right singular vectors contained in $\hat{\mathbf{E}}_{\mathrm{i-1i_t}}$:

$$\mathbf{S}_{\mathrm{i-1_t}} = \hat{\mathbf{E}}_{\mathrm{i-1i_t}}\mathbf{S}_{\mathrm{i-1}}. \quad (22)$$

The relay $RS_{\mathrm{i}}$ receives the signal with noise vector as the same in Sec. II:

$$\mathbf{R}_{\mathrm{i_r}} = \mathbf{H}_{\mathrm{i-1i}}\mathbf{S}_{\mathrm{i-1_t}} + \mathbf{n}_{\mathrm{i}}, \quad (23)$$

and preprocesses $\mathbf{R}_{\mathrm{i_r}}$ using $\hat{\mathbf{E}}_{\mathrm{i-1i_r}}$

$$\begin{aligned}\mathbf{R}_{\mathrm{i}} &= (\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\mathbf{R}_{\mathrm{i_r}}, \quad (24)\\ &= (\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\mathbf{H}_{\mathrm{i-1i}}\hat{\mathbf{E}}_{\mathrm{i-1i_t}}\mathbf{S}_{\mathrm{i-1}} + (\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\mathbf{n}_{\mathrm{i}},\\ &= \rho\hat{\mathbf{D}}_{\mathrm{i-1i}}\mathbf{S}_{\mathrm{i-1}} + \delta(\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\bar{\mathbf{H}}_{\mathrm{i-1i}}\hat{\mathbf{E}}_{\mathrm{i-1i_t}}\mathbf{S}_{\mathrm{i-1}} + (\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\mathbf{n}_{\mathrm{i+1}}.\end{aligned}$$

The $p^{\mathrm{th}}$ component of vector $\mathbf{x}$ and the $p^{\mathrm{th}}$ row of matrix $\mathbf{X}$ are denoted by $\mathbf{x}(\mathrm{p})$ and $\mathbf{X}(\mathrm{p})$, respectively. The component-wise form is described as follows.

$$\begin{aligned}\mathbf{R_i}(\mathrm{p}) = \Big(\rho\hat{\mathbf{D}}_{\mathrm{i-1i}}(\mathrm{p}) + \delta(\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\bar{\mathbf{H}}_{\mathrm{i-1i}}\hat{\mathbf{E}}_{\mathrm{i-1i_t}}(\mathrm{p})\Big)\mathbf{S}_{\mathrm{i-1}} \quad (25)\\ +\delta\sum_{\mathbf{t}\neq\mathbf{p}}(\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\bar{\mathbf{H}}_{\mathrm{i-1i}}\hat{\mathbf{E}}_{\mathrm{i-1i_t}}(\mathrm{t})\mathbf{S}_{\mathrm{i-1}}\\ +(\hat{\mathbf{E}}_{\mathrm{i-1i_r}})^{\mathrm{H}}\mathbf{n}_{\mathrm{i+1}}(\mathrm{p}),\end{aligned}$$

where the received signal consists of three components: the information carrying term, the interference term and the noise term. For each component-wise, there are $M-1$ interference components that is multiplied by $\delta$. The correlation matrix is approximate to the unit matrix. Therefore, (10) is changed as

$$\begin{aligned}&f(\lambda) \quad (26)\\ &= \frac{\sigma^2\sum_{i=0}^{m}\prod_{\mathrm{j}\neq\mathrm{i}}^{\mathrm{m}} l_{\mathrm{jj+1}}\mathbf{P}_{\mathrm{j}}\mathbf{D}_{\mathrm{jj+1}}^2 + \delta(M-1)\prod_{\mathrm{i=0}}^{\mathrm{m}} l_{\mathrm{ii+1}}\mathbf{P}_{\mathrm{i}}\mathbf{D}_{\mathrm{ii+1}}^2}{\prod_{\mathrm{i=0}}^{\mathrm{m}} l_{\mathrm{ii+1}}\mathbf{P}_{\mathrm{i}}\mathbf{D}_{\mathrm{ii+1}}^2},\\ &= \sum_{i=1}^{m+1}\frac{1}{\mathbf{SNR}_{\mathrm{i-1i}}\mathbf{D}_{\mathrm{i-1i}}^2} + \delta(M-1).\end{aligned}$$

Compared to the system has perfect CSI at both the transmitter and the receiver (10), in the end-to-end channel capacity of the system has outdated CSI at the transmitter (26), the co-channel interference by outdated CSI is added. However, the co-channel interference is independent of SNR at each receiver. This is explained as that the information term and the interference term are increased or decreased together and the ratio of them is fixed as $\frac{1}{(M-1)\delta}$.

## IV. Numerical Evaluation

### A. System with perfect CSI

In this work, we focus on proposing WF scheme for MMRS, the optimization of distance and transmit power is omitted. The received SNR of all receivers is assumed to be the same regardless of another hops. The number of relays is fixed as 4 and the number of antenna elements of all relays, the $\mathrm{Tx}$ and the $\mathrm{Rx}$ is set as 6. Since the performance of system should be discussed in both the low SNR ranger and the high SNR ranger, the $snr_i, i = 1, \cdots, m+1$ are changed from 0 dB to 30 dB. Fig. 3 shows the end-to-end channel capacity of proposed WF methods with perfect CSI. In order to evaluate the proposed WF methods, the average transmit power meaning the transmit power of all antenna elements in a relay is equal, also is represented. The end-to-end channel capacity of average transmit power and three WF methods is compared. The end-to-end channel capacity of scheme 3 is the highest and that of scheme 2 is higher than that of scheme 1. It can be explained that the transmit power of all transmitters in scheme 3 is the most suitable to system channel model and the

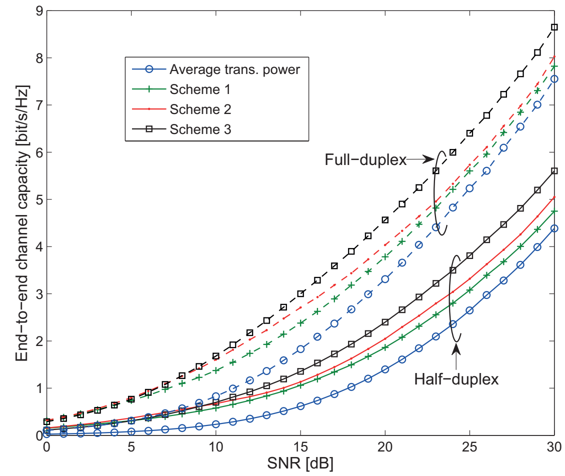


Fig. 3. The end-to-end channel capacity of half duplex and full duplex AF scheme MMRS with proposed WF methods.

transmit power of all transmitters in scheme 2 is more suitable to system channel model than that of scheme 1. Similarly, the system with proposed WF methods has more diversity than the system with average transmit power, therefore, the end-to-end channel capacity of proposed WF methods is higher than that of average transmit power. Although the number of antenna elements in the full duplex system is a half of number of antenna elements in half duplex system, the allocation time of full duplex system is double. Hence, compared to half duplex, the end-to-end channel capacity of full duplex is higher.

### B. System with outdated CSI

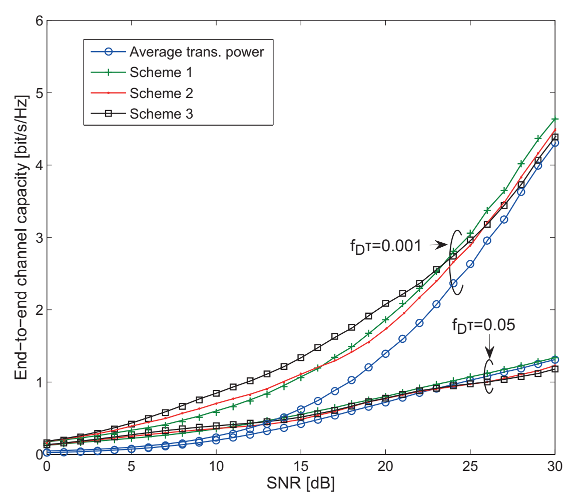


Fig. 4. The end-to-end channel capacity of half duplex system when $snr_i$ is changed, $f_D\tau$ is fixed as 0.001 and 0.05.

Figs. 4 and 5 that show the end-to-end channel capacity of half duplex and full duplex, respectively, when the term

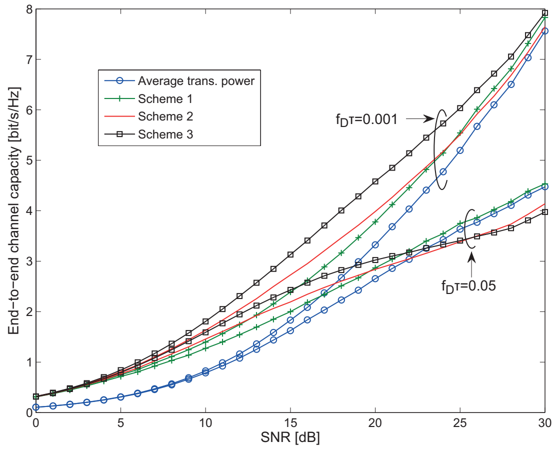


Fig. 5. The end-to-end channel capacity of full duplex system when $snr_i$ is changed, $f_D\tau$ is fixed as 0.001 and 0.05.

$f_D\tau$ is fixed as 0.001 and 0.05. When the term $f_D\tau$ is small (0.001), the influence of outdated CSI is weak, therefore, the end-to-end channel capacity increase when the $snr_i$ increases. However, when the term $f_D\tau$ is large (0.05), the influence of outdated CSI becomes stronger, therefore, the end-to-end channel capacity increase slowly, especially in half duplex system. The reason is the same as explanation in previous section.

## V. Conclusion

In this paper, we analyzed the end-to-end channel capacity of MMRS according to the proposed WF method. The WF method is divided into three schemes. Scheme 1: the WF method is applied to each hop; scheme 2: the WF is based on the multiplication of channel models from the $\mathrm{Tx}$ to each relay and scheme 3: The WF of all hops is the same and based on multiplication of channel models from the $\mathrm{Tx}$ to the $\mathrm{Rx}$. The end-to-end channel capacity of proposed WF methods is higher than that of average transmit power and the end-to-end channel capacity of scheme 3 is the highest in perfect CSI. However, when the imperfect CSI is taken into account, the outdated CSI not only affects the accuracy of WF method, but also affects the co-channel interference. In outdated CSI system, the end-to-end channel capacities decrease when the term $f_D\tau$ increases, especially in high SNR range due to the high power of co-channel interference. Additionally, the end-to-end channel capacities of WF methods decrease close to or drop below that of average transmit power. The average transmit power system is the most robust and the scheme 3 is the most sensitive in outdated CSI system. The half duplex and the full duplex also have been discussed in this paper, since the full duplex has double of allocation time and half of delay time, the full duplex system is more robust than the half duplex system does and the end-to-end channel capacity of full duplex system is higher than that of half duplex system.

Since the distance and the transmit power have been optimized in another literatures, in this paper, the same SNR at all receiver was assumed. However, the optimization of distance and transmit power based on each WF methods should be considered to increase performance of MMRS. Additionally, the transmission of all transmitters in both the full duplex and the half duplex was assumed to be controlled on MAC layer. However, the detail of control should be discussed. We leave them to the future works.

## References


[1] Wang, B., Zhang, J., host-Madsen, A.: On the capacity of mimo relay channel. IEEE Trans. Inf. Theory **Vol. 51**, 29–43 (2005)

[2] Shiu, D.S., Foschini, G.J., Gans, M.J., Kahn, J.M.: Fading correlation and its effect on the capacity of multi-element antenna systems. IEEE Trans. Commun. **Vol. 48**, 502–513 (2000)

[3] Gesbert, D., Bolcskei, H., Gore, D.A., Paulraj, A.J.: Mimo wireless channel: Capacity and performance prediction. In: Proceedings of GLOBECOM, vol. 2, pp. 1083–1088 (2000). IEEE communication society

[4] Liang, Y., Veeravalli, V.V.: Gaussian orthogonal relay channels: optimal resource allocation and capacity. IEEE Trans Inf. Theory **Vol. 51**, 3284–3289 (2005)

[5] Makki, B., Eriksson, T.: On the capacity of rayleigh fading correlated spectrum sharing networks. EURASIP Journal on Wireless Communications and Networking (2011)

[6] Kim, K., Ko, K., Lee, J.: Adaptive selection of antenna grouping and beamforming for mimo system. EURASIP Journal on Wireless Communications and Networking (2011)

[7] Shen, Z., Jr., R.W.H., Andrews, J.G., Evans, B.L.: Space time water filling for composite mimo fading channels. EURASIP Journal on Wireless Communications and Networking (2006)

[8] Lu, Y., Zhang, W.: Water-filling capacity analysis in large mimo system. In: Proceedings of Computing, Communications and IT Applications Conference (2013). ComComAp

[9] Lee, K.J., Kim, J.S., G., C., Lee, I.: Asymptotic ergodic capacity analysis for mimo amplify-and-forward relay network. IEEE Trans. Commun. **Vol. 9**, 2712–2717 (2010)

[10] Jin, S., McKay, M.R., Zhong, C., Wong, K.K.: Ergodic capacity analysis of amplify-and-forward mimo dual-hop systems. IEEE Trans. Inf. Theory **Vol. 56**, 2204–2224 (2010)

[11] Gastpar, M., Vetterli, M.: On the capacity of large gaussian relay networks. IEEE Trans. Inf. Theory **Vol. 51**, 765–779 (2005)

[12] Hiep, P.T., Kohno, R.: Optimizing position of repeaters in distributed mimo repeater system for large capacity. IEICE Trans. Commun. **Vol. E93-B**, 3616–3623 (2010)

[13] Hiep, P.T., Fumie, O., Ryuji, K.: Optimizing distance, transmit power and allocation time for reliable multi-hop relay system. EURASHIP Journal on Wireless Communications and Networking (2012)

[14] Hiep, P.T., Hoang, N.H., Chika, S., Ryuji, K.: End-to-end channel capacity of mac-phy cross-layer multiple-hop mimo relay system with outdated csi. EURASHIP Journal on Wireless Communications and Networking (2013)

[15] Yang, S., Belfiore, J.C.: Towards the optimal amplify-and-forward cooperative diversity scheme. IEEE Trans. Inform. Theory **Vol. 53**, 3114–3126 (2007)

[16] Khoshnevis, B., Yu, W., Adve, R.: Grassmannian beamforming for mimo amplify-and-forward relaying. IEEE J. Sel. Areas Commun **Vol. 26**, 1397–1407 (2008)

[17] Andersen, J.B.: Array gain and capacity for known random channels with multiple element arrays at both ends. IEEE J. Sel. Areas Commun **Vol. 18**, 2172–2178 (2000)

[18] Karasawa, Y.: Mimo propagation channel modeling. IEICE trans. commun. **Vol. E88-B**, 1829–1842 (2005)