Interval Type-2 Fuzzy C-means Clustering using Intuitionistic Fuzzy Sets

Dzung Dinh Nguyen, Long Thanh Ngo and Long The Pham
Department of Information Systems, Faculty of Information Technology
Le Quy Don Technical University, 236, Hoang Quoc Viet, Hanoi, Vietnam
Email: dinhdung1082@gmail.com, ngotlong@gmail.com

***Abstract*—In this paper, intuitionistic interval type-2 fuzzy c-means clustering (InIT2FCM) method is proposed for the clustering problems. Intuitionistic fuzzy sets (IFS) and intuitionistic type-2 fuzzy sets (InIT2FS) were introduced with the aim to better handle the uncertainty. Utilizing the advantages of the IFS and InT2FS, we have combined them with fuzzy clustering algorithms to overcome some drawbacks of the "conventional" FCM in handling uncertainty. The experiments were completed for different types of images which show the advantages of the proposed algorithms, especially with noisy images.**

***Index Terms*—Intuitionistic fuzzy sets, intuitionistic type-2 fuzzy sets, Intuitionistic fuzzy c-means clustering, type-2 fuzzy c-means clustering.**

I. Introduction

Clustering technique is applied in many fields such as data mining, pattern recognition, image processing etc. It is used to detect any structures or patterns in the data set, in which objects within the cluster level data show certain similarities. Clustering algorithms have different shapes from simple clustering as k-means and its variants [2], [3], [4], [5], development of family of fuzzy c-mean clustering (FCM) [14]. With the framework of fuzzy theory, fuzzy techniques are suitable for the development of new clustering algorithms because they are able to remove vagueness/imprecision in the data [16]. The most popular fuzzy clustering algorithms and applications were introduced in [17], [18], [28].

Recently, type-2 fuzzy sets are extensions of original fuzzy sets, have the advantage of handling uncertainty, which have been developed and applied to many different problems [6], [7], [8], [9] including data clustering problems. In addition, interval type-2 fuzzy c-means clustering (IT2FCM) [1] has developed a new way in the fuzzy clustering method in which FOU (footprint of uncertainty) is formed by using two fuzzifiers for handling uncertainty and making clustering more efficiently.

Besides, the intutionistic fuzzy set (IFS) was introduced [19], [20] and used for representing the hesitance of an expert on determining the membership functions and the non-membership functions. This capability has created a different research direction to handle the uncertainty based on IFS [22], [25], [26]. IFSs also have been recently used for the clustering problem [23], [24], [27].

In light of this brief review, we found the outstanding developments of the type-2 fuzzy sets and the intuitionistic fuzzy sets. They are applied to handle the uncertainty. However, their uncertainty processing is not the same. When the uncertainty treatment of type-2 fuzzy sets based on the uncertain selection of membership functions, the intuitionistic fuzzy sets handle uncertainty based on the identification of the membership function and the non-membership function with the hesitance assessment function. We can see that the difference here is the uncertainty and the hesitance. Many people mistakenly believe the uncertainty and the hesitance are the same, we can see that there is a little difference between them, sometime in the uncertainty, there is still hesitance and vice versa.

Therefore, in this paper, we introduced intuitionistic type-2 fuzzy sets (InT2FS) on the basis of the extension of intuition fuzzy set. It has a ability to handle both the hesitance and the uncertainty. Next, intuitionistic type-2 fuzzy set is applied in fuzzy clustering algorithm for image segmentation. The experimental results show that the proposed algorithm gives better results than the traditional clustering method, especially with noisy images.

Remain of the paper is organized as follows: Section II briefly introduces about some backgrounds about intuitionistic fuzzy sets and fuzzy clustering; Section III proposes the intuitionistic fuzzy C-means clustering algorithm, Section III describes intuitionistic type-2 fuzzy C-means; Section IV offers some experimental results and section V concludes the paper.

II. Background

A. Intuitionistic Fuzzy sets

Let X be an ordinary finite non-empty set. An IFS in X is an expression $\tilde{A}$ given by:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \to [0,1]$ $v_{\tilde{A}} : X \to [0,1]$ satisfy the condition $\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$. for all $x \in X$. The numbers $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in set $\tilde{A}$.

Considering IFSs(X) as the set of all the intuitionistic fuzzy sets in X. For each IFS $\tilde{A}$ in X, the values $\pi_{\tilde{A}(x)} = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x)$ is called the degree of uncertainty of x to $\tilde{A}$, or the degree of hesitancy of x to $\tilde{A}$.

Note that for an IFS $\tilde{A}$, if $\mu_{\tilde{A}}(x) = 0$, then $v_{\tilde{A}}(x) + \pi_{\tilde{A}}(x) = 1$, and if $\mu_{\tilde{A}}(x) = 1$ then $v_{\tilde{A}}(x) = 0$ and $\pi_{\tilde{A}}(x) = 0$.

978-1-4799-3230-6/13/$31.00 ©2013 IEEE

1) Entropy on Intuition fuzzy sets: Most of the fuzzy algorithms select the best threshold t using the concept of fuzzy entropy. In this paper, we will focus on the definition and characterization of the intuitionistic fuzzy entropy. The entropy on IFSs is defined as a magnitude that measures the degree of IFS that a set is with respect to the fuzziness of this set which satisfy the following conditions:

1. The entropy will be null when the set is a FSs(X),
2. The entropy will be maximum if the set is an A-IFS; that is $\mu(x) = v(x) = 0$ for all $x \in X$.
3. As in fuzzy sets, the entropy of an IFS will be equal to its respective complement.
4. If the degree of membership and the degree of non-membership of each element increase, the sum will as well, and therefore, this set becomes more fuzzy, and therefore the entropy should decrease. One of the simplest expressions that satisfy the conditions previously mentioned in [?]

$$IE\left(\tilde{A}\right) = \frac{1}{n}\sum_{k=1}^{n} \pi_{\tilde{A}}\left(x_k\right) \tag{1}$$

Equation (1) is a base for segmentation algorithm.

B. Fuzzy C-means Clustering

Fuzzy c-means (FCM) was first introduced by Dunn in [13] and was improved by Bezdek in [14]. It is a method of clustering which allows a data point can belong to more than one cluster with different membership grades. It assumes that number of clusters c is known in priori and minimizes the objective function (J_m) as:

$$J_m(U,v) = \sum_{k=1}^{n}\sum_{i=1}^{c} (u_{ik})^m (d_{ik})^2 \tag{2}$$

where

$$d_{ik} = d(x_k - v_i) = \|x_k - v_i\| = \left[\sum_{j=1}^{d} (x_{kj} - v_{ij})^2\right]^{1/2}$$

and m is a constant, known as the fuzzifier, which controls the fuzziness of the resulting partition and can be any real number greater than 1 but generally it can be taken as 2 .

Predefined parameters to the problem: the number of clusters $c\,(1 < c < n)$, fuzifier $m(1 < m < +\infty)$ and error ε. This algorithm can be briefly described as follows:

Algorithm 1 Fuzzy c-means algorithm

Step 1: Initialize centroid matrix

$$V = [v_{ij}]\,, V^{(0)} \in R^{M\times c}, j = 0$$

, by choosing random from dataset $X = \{x_i, x_i \in R^M\}, i = 1..n$ and the membership matrix U^0 by using the equation:

$$u_{ik} = \frac{1}{\sum\limits_{j=1}^{c}\left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m-1}}}, 1 \le i \le c, 1 \le k \le n \tag{3}$$

Where $d_{ij} = d(x_j - v_i) = \|x_j - v_i\|$

Step 2:

repeat :

Update the centroid matrix $V^{(j)} = \left[v_1^{(j)}, v_2^{(j)}, ..., v_c^{(j)}\right]$ by

$$v_i = \frac{\sum\limits_{k=1}^{n} (u_{ik})^m x_k}{\sum\limits_{k=1}^{n} (u_{ik})^m}, 1 \le i \le c$$

Update the membership matrix $U^{(j)}$ by using(3)

Assign data x_j to cluster c_i if data $(u_i\,(x_j) > u_k\,(x_j))$, $k = 1, .., c$ and $j \ne k$.

until :

$$Max\left(||U^{(j+1)} - U^{(j)}||\right) \le \varepsilon$$

Step 3: **Return** U and V.

C. Interval Type-2 fuzzy C-means Clustering

IT2FCM is an extension of the FCM clustering in which we use two fuzzification coefficients m_1, m_2 to form FOU, corresponding to the upper and lower values of membership; refer also to [1]. The use of fuzzifiers gives rise to different objective functions to be minimized:

$$\begin{cases} J_{m_1}(U,v) = \sum_{k=1}^{N}\sum_{i=1}^{C} (u_{ik})^{m_1} d_{ik}^2 \\ J_{m_2}(U,v) = \sum_{k=1}^{N}\sum_{i=1}^{C} (u_{ik})^{m_2} d_{ik}^2 \end{cases} \tag{4}$$

in which $d_{ik} =\| x_k - v_i \|$ is the Euclidean distance between the pattern x_k and the centroid of the cluster v_i, C is number of clusters and N is number of patterns. Upper/lower degrees of membership $\overline{u}_{ik}$ and $\underline{u}_{ik}$ were similar to FCM algorithm except they were formed by two fuzzifiers m_1, m_2 (m_1 ¡ m_2) and determined as follows:

$$\overline{u}_{ik} = \begin{cases} \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})^{2/(m_1-1)}} & if\ \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})} < \dfrac{1}{C} \\ \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})^{2/(m_2-1)}} & if\ \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})} \ge \dfrac{1}{C} \end{cases} \tag{5}$$

$$\underline{u}_{ik} = \begin{cases} \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})^{2/(m_1-1)}} & if\ \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})} \ge \dfrac{1}{C} \\ \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})^{2/(m_2-1)}} & if\ \dfrac{1}{\sum_{j=1}^{C} (d_{ik}/d_{jk})} < \dfrac{1}{C} \end{cases} \tag{6}$$

in which $i = \overline{1,C}$, $k = \overline{1,N}$.

Because each pattern comes with the membership interval as the bounds $\overline{u}$ and $\underline{u}$, each centroid of the cluster is represented by the interval between v^L and v^R. Cluster centroids is computed in the same way in the case of FCM:

$$v_i = \frac{\sum_{k=1}^{N} (u_{ik})^m x_k}{\sum_{k=1}^{N} (u_{ik})^m} \tag{7}$$

in which $i = \overline{1,C}$.

Algorithm 2 Finding Centroids

Step 1: Find $\overline{u}_{ij}, \underline{u}_{ij}$, by the equations (5)-(6).

Step 2: Set m = arbitrary and $m \ge 1$;

Compute $v_j' = (v_{j1}', ..., v_{jM}')$ by (7) with $u_{ij} = \frac{(\overline{u}_{ij} + \underline{u}_{ij})}{2}$.

Sort N patterns on each of M features in ascending order.

Step 3: Find index k such that: $x_{kl} \leq v'_{jl} \leq x_{(k+1)l}$ with $k = 1, .., N$ and $l = 1, .., M$.
▷ *Update* u_{ij}*:*
if $i \leq k$ **then do**
    $u_{ij} = \underline{u}_{ij}$.
else
    $u_{ij} = \overline{u}_{ij}$.
▷ *Define* v_L *or* v_R
Step 4: Define v_L or v_R
    Compute v''_j by (**??**).
    Compare v'_{jl} with v''_{jl}
if $v'_{jl} = v''_{jl}$ **then do**
    $v_R = v'_j$.
else
    Set $v'_{jl} = v''_{jl}$.
    Back to Step 3.
▷ *In Case, to define* v_L:
In step 3 we modify
Update u_{ij}:
if $i \leq k$ **then do**
    $u_{ij} = \overline{u}_{ij}$.
else
    $u_{ij} = \underline{u}_{ij}$.
and in Step 4 replace V_R with v_L.

After obtaining v_i^R, v_i^L, type-reduction is applied to forming centroid of clusters as follows:

$$v_i = (v_i^R + v_i^L)/2 \tag{8}$$

For membership grades:

$$u_i(x_k) = (u_i^R(x_k) + u_i^L(x_k))/2, j = 1, ..., C \tag{9}$$

in which

$$u_i^L = \sum_{l=1}^{M} u_{il}/M, u_{il} = \begin{cases} \overline{u}_i(x_k) & \text{if } x_{il} \text{ uses } \overline{u}_i(x_k) \text{ for } v_i^L \\ \underline{u}_i(x_k) & otherwise \end{cases} \tag{10}$$

$$u_i^R = \sum_{l=1}^{M} u_{il}/M, u_{il} = \begin{cases} \overline{u}_i(x_k) & \text{if } x_{il} \text{ uses } \overline{u}_i(x_k) \text{ for } v_i^R \\ \underline{u}_i(x_k) & otherwise \end{cases} \tag{11}$$

Next, defuzzification done for IT2FCM follows the rule if $u_i(x_k) > u_j(x_k)$ for $j = 1, ..., C$ and $i \neq j$ then x_k is assigned to cluster i.

III. Intuitionistic Fuzzy C-means Clustering

As an enhancement of classical FCM, the intuitionistic Fuzzy C-means Clustering (IFCM) use the intuitionistic fuzzy sets with the aim to better handle the uncertainty.

In order to incorporate intuitionistic fuzzy sets in conventional fuzzy C-means clustering algorithm, we first redefine the membership function for IFCM. Several studies have proposed the membership function as in [19], [22] as follows:

$$u_{ij}^* = u_{ij} + \pi_{ij} \tag{12}$$

where u_{ij} is the membership function in FCM (Eq. (3)) and π_{ij} is the hesitance degree of the j^{th} data in i^{th} cluster.

However, Eq.12 seems ineffective. we take a simple example as follows : Predefined a data x , a set A. To assess a membership function of x in set A based on intuitionistic fuzzy theory. Let u is the membership function, v is the non-membership function and the hesitance degree π (u+v+π=1)

Case 1:

Assuming that: u = 0.6 , v = 0.1 and π = 0.3 we have $u^* = 0.9$ according to Eq.12

Case 2:

Assuming that: u = 0.8 , v=0.1 and π = 0.1. we also have $u^* = 0.9$ according to Eq.12

One can easily realize that: case 2 is better than case 1. However, with the above Eq.12, the aggregate membership functions u^* are 0.9 in both two cases and we can not determine the better one.

Remark: The fact that hesitance degree π usually is not affect performance results. we are only interested in two factors that affect the decision are the membership function u and the non-membership function v.

Therefore, we propose a new way to define the aggregate membership functions u^* satisfying properties: the maximum membership function and the minimum non-membership function.

$$u_{ij}^* = u_{ij} - v_{ij} \tag{13}$$

where u_{ij} is the membership function Eq. (3) and v_{ij} is the the non-membership function of the j^{th} data in i^{th} cluster.

Substituting $v_{ij} = 1 - u_{ij} - \pi_{ij}$ in Eq. (13), we have:

$$u_{ij}^* = u_{ij} - v_{ij} \tag{14}$$
$$= u_{ij} - (1 - u_{ij} - \pi_{ij}) \tag{15}$$
$$= 2u_{ij} + \pi_{ij} \tag{16}$$

From Eq. (16), obviously see that $-1 \leq u_{ij}^* \leq 1$, we normalize u_{ij}^* so that $0 \leq u_{ij}^* \leq 1$.

we have

$$0 \leq \frac{u_{ij}^* + 1}{2} \leq 1$$

and

$$0 \leq u_{ij} + \frac{\pi_{ij}}{2} \leq 1$$

.

We define the aggregate membership functions for intuitionistic fuzzy sets:

$$\tilde{u}_{ij}^* = u_{ij} + \frac{\pi_{ij}}{2}. \tag{17}$$

Next, substituting Eq. (17) in FCM, the modified cluster center $\tilde{v}_i^*$ is calculated as:

$$v_i^* = \frac{\sum_{j=1}^{n} \tilde{u}_{ij}^* x_j}{\sum_{j=1}^{n} \tilde{u}_{ij}^*} \tag{18}$$

Thus, the objective function in FCM is modified when using IFSs:

$$J_1 = \sum_{i=1}^{c} \sum_{j=1}^{n} \tilde{u}_{ij}^{*m} d\big(x_j, v_j^*\big)^2 \tag{19}$$

In order to build the intuitionistic fuzzy sets, we know that the choice of the membership functions is conditioned by the hesitance degree. In this approach, the hesitance degree is represented by means of intuitionistic fuzzy index (π) referred in [25], [26]. We define the hesitance degree π_j of the j^th data with the clusters as follows:

$$\pi_j = \wedge\left(1 - u_{1j}, 1 - u_{2j}, ..., 1 - u_{cj}\right) \tag{20}$$

where c is the number of clusters and $u_{1j}, ..., u_{cj}$ is the membership functions of the j^{th} data in the corresponding clusters.

A second objective function is added, which is the intuitionistic fuzzy entropy (IFE) was above described (1):

$$J_2 = \frac{1}{n}\sum_{j=1}^{n}\pi_j \tag{21}$$

From (19) and (21), the final objective function that contains two terms is minimized and is as follows:

$$\tilde{J} = J_1 + J_2 \tag{22}$$

$$= \sum_{i=1}^{c}\sum_{j=1}^{n}\tilde{u}_{ij}^{*m} d\left(x_j, v_j^*\right)^2 + \frac{1}{n}\sum_{j=1}^{n}\pi_j \tag{23}$$

Predefined parameters to the problem: the number of clusters $c\,(1 < c < n)$, fuzzifier $m(1 < m < +\infty)$ and error ε. This algorithm can be briefly described as follows:

Algorithm 3 Intuitionistic Fuzzy c-means algorithm

Step 1: Initialize centroid matrix

$$V = [v_{ij}], V^{(0)} \in R^{M \times c}, j = 0$$

, by choosing random from dataset $X = \{x_i, x_i \in R^M\}, i = 1..n$ and the membership matrix U^0 by using the Eq.16 and Eq.20

Step 2:

repeat :

Update the centroid matrix by Eq.17

Update the membership matrix $U^{(j)}$ by using Eq.16 and Eq.20

Assign data x_j to cluster c_i if data $(\tilde{u}_i^*(x_j) > \tilde{u}_k^*(x_j))$, $k = 1, .., c$ and $j \neq k$.

until :

$$Max\left(||U^{(j+1)} - U^{(j)}||\right) \leq \varepsilon$$

Step 3: **Return** U and V.

IV. Intuitionistic Type-2 Fuzzy C-means Clustering

A. Basic concepts

We give some basic concepts which are used for thresholding algorithm using InT2FSs for image segmentation problems. These basic concepts are extended by combining of type-2 fuzzy sets and intuitionistic fuzzy sets.

1) Intuitionistic type-2 fuzzy sets: A intuitionistic type-2 fuzzy set in X is denoted $\tilde{A}^*$, and its membership grade of $x \in X$ is $\mu_{\tilde{A}^*}(x, u_1)$ with $u_1 \in J_x^1 \subseteq [0, 1]$, its non-membership grade of $x \in X$ is $v_{\tilde{A}^*}(x, u_2)$ with $u_2 \in J_x^2 \subseteq [0, 1]$. The elements of domain of $(x, u_1), (x, u_2)$ are called primary membership and primary non-membership of x in $\tilde{A}^*$, respectively, memberships of primary memberships $\mu_{\tilde{A}^*}(x, u_1)$ and non-memberships of primary memberships $v_{\tilde{A}^*}(x, u_2)$ are called secondary memberships and secondary non-memberships, respectively, of x in $\tilde{A}^*$, with $u_1 \in J_x^1 \subseteq [0, 1]$, $u_2 \in J_x^2 \subseteq [0, 1]$, which are intuitionistic fuzzy sets.

Type-2 intuitionistic fuzzy sets are called an Interval type-2 intuitionistic fuzzy sets if the secondary membership function $\mu_{\tilde{A}}(x, u_1) = 1$ and $\mu'_{\tilde{A}^*}(x, u_2) = 1$ $\forall u_1, u_2 \in J_x)$ i.e. an Interval type-2 intuitinistic fuzzy set are defined as follows:

Definition 4.1: A type-2 intuitionistic fuzzy set (InT2FS), denoted $\tilde{A}^*$, is characterized by two type-2 intuitionistic membership functions: $\mu_{\tilde{A}}(x, u_1)$, $\mu'_{\tilde{A}}(x, u_2)$ and two type-2 intuitionistic non-membership function $v_{\tilde{A}^*}(x, u_1)$, $v'_{\tilde{A}^*}(x, u_2)$ where $x \in X$ and $u_1 \in J_x^1 \subseteq [0, 1]$, $u_2 \in J_x^2 \subseteq [0, 1]$, i.e.,

$$\tilde{A}^* = \{((x, u_1), \mu_{\tilde{A}^*}(x, u_1), v_{\tilde{A}^*}(x, u_1)), ((x, u_2), \mu'_{\tilde{A}^*}(x, u_2), v'_{\tilde{A}^*}(x, u_2)) | \forall x \in X, \forall u_1 \in J_x^1 \subseteq [0, 1], \forall u_2 \in J_x^2 \subseteq [0, 1]\}$$

in which

$0 \leq \mu_{\tilde{A}^*}(x, u_1)), \mu'_{\tilde{A}^*}(x, u_2)), v_{\tilde{A}^*}(x, u_1)), v'_{\tilde{A}^*}(x, u_2)) \leq 1$

and $0 \leq v_{\tilde{A}^*}(x, u_1)) + \mu_{\tilde{A}^*}(x, u_1)) \leq 1$, $0 \leq v'_{\tilde{A}^*}(x, u_1)) + \mu'_{\tilde{A}^*}(x, u_1)) \leq 1$.

Intuitionistic type-2 fuzzy sets are called an interval InT2Fs if the secondary membership function $\mu_{\tilde{A}}(x, u_1) = 1$ and $\mu'_{\tilde{A}^*}(x, u_2) = 1$ $\forall u_1 \in J_x^1, u_2 \in J_x^2$ i.e. an interval type-2 intuitinistic fuzzy set is defined as follows:

Definition 4.2: An interval InT2FS $\tilde{A}^*$ is characterized by membership bounding functions $\overline{\mu}_{\tilde{A}^*}(x)$, $\underline{\mu}_{\tilde{A}^*}(x)$ and non-membership bounding functions $\overline{v}_{\tilde{A}^*}(x)$, $\underline{v}_{\tilde{A}^*}(x)$ where $x \in X$ in which

$$0 \leq \overline{\mu}_{\tilde{A}}(x) + \underline{v}_{\tilde{A}}(x) \leq 1 \tag{24}$$

$$0 \leq \underline{\mu}_{\tilde{A}}(x) + \overline{v}_{\tilde{A}}(x) \leq 1 \tag{25}$$

Thus, an Interval type 2 intuitionistic fuzzy set can be described through FOUs as follow:

$$\tilde{A} = \{x, \overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x), \overline{v}_{\tilde{A}}(x), \underline{v}_{\tilde{A}}(x) | \forall x \in X, \tag{26}$$

$$\forall \overline{\mu}_{\tilde{A}}(x), \underline{\mu}_{\tilde{A}}(x), \overline{v}_{\tilde{A}}(x), \underline{v}_{\tilde{A}}(x) \in [0, 1]\} \tag{27}$$

B. Intuitionistic type-2 fuzzy C-means Clustering

Note that: the developments of type-2 fuzzy sets and intuition fuzzy sets are applied to handle the uncertainty. However, their uncertainty processing is not the same. When the uncertainty treatment of type-2 fuzzy sets based on the uncertain selection of membership functions, the intuitionistic fuzzy sets handle uncertainty based on the identification of the membership function and the non-membership function with the hesitance assessment function. On the basis of the IFCM and IT2FCM algorithms were presented above, we proposed a intuitionistic type-2 fuzzy c-means clustering algorithm (IT2IFCM) as follows:

As described in the IFCM, We define the aggregate membership functions for intuitionistic interval type-2 fuzzy sets (InIT2FSs):

$$\overline{\tilde{u}}_{ij}^{*} = \overline{u}_{ij} + \frac{\underline{\pi}_{ij}}{2}. \quad (28)$$

$$\underline{\tilde{u}}_{ij}^{*} = \underline{u}_{ij} + \frac{\overline{\pi}_{ij}}{2}. \quad (29)$$

Where $\overline{u}_{ij}$ and $\underline{u}_{ij}$ was calculated from Eq. (5) and (6),respectively.

Under the same conditions in IFCM, to build the InIT2FSs, we know that the choice of the interval membership functions $[\underline{\tilde{u}}_{ij}^{*}, \overline{\tilde{u}}_{ij}^{*}]$ is conditioned by the interval hesitance degree $[\underline{\pi}_j, \overline{\pi}_j]$ of the j^th data with the clusters as follows:

$$\underline{\pi}_j = \wedge\left(1 - \overline{\tilde{u}}_{1j}^{*}, 1 - \overline{\tilde{u}}_{2j}^{*}, ..., 1 - \overline{\tilde{u}}_{cj}^{*}\right) \quad (30)$$

$$\overline{\pi}_j = \wedge\left(1 - \underline{\tilde{u}}_{1j}^{*}, 1 - \underline{\tilde{u}}_{2j}^{*}, ..., 1 - \underline{\tilde{u}}_{cj}^{*}\right) \quad (31)$$

where c is the number of clusters and $[\underline{\tilde{u}}_{1j}, \overline{\tilde{u}}_{1j}], ..., [\underline{\tilde{u}}_{cj}^{*}, \overline{\tilde{u}}_{cj}^{*}]$ is the membership functions of the j^{th} data in the corresponding clusters.

Because each pattern j^{th} has membership interval as the upper $\overline{\tilde{u}}_{ij}^{*}$ and the lower $\underline{\tilde{u}}_{ij}^{*}$, in the i^{th} cluster. Each centroid of cluster v_i is represented by the interval between v_i^L and v_i^R. Cluster centroids are computed in the same way in the case of FCM:

$$v_i = \frac{\sum_{j=1}^{n} (\tilde{u}_{ij}^{*})^m x_k}{\sum_{k=1}^{n} (\tilde{u}_{ij}^{*})^m} \quad (32)$$

in which $i = \overline{1,c}$ and $\tilde{u}_{ij}^{*} = \frac{\overline{\tilde{u}}_{ij}^{*} + \underline{\tilde{u}}_{ij}^{*}}{2}$

Finding cluster centroid v_i^L and v_i^R follow the algorithm 2 by replacing Eq. (5), Eq.(5) and Eq. (7) with Eq. (28), Eq. (29) and Eq. (32), respectively.

After obtaining v_i^R, v_i^L, type-reduction is applied to forming centroid of clusters as follows:

$$v_i = (v_i^R + v_i^L)/2 \quad (33)$$

For membership grades:

$$u_i(x_j) = (u_i^R(x_j) + u_i^L(x_j))/2, j = 1, ..., C \quad (34)$$

in which

$$u_i^L = \sum_{l=1}^{M} u_{il}/M, u_{il} = \begin{cases} \overline{\tilde{u}}_i^{*}(x_j) & \text{if } x_{il} \text{ uses } \overline{\tilde{u}}_i^{*}(x_j) \text{ for } v_i^L \\ \underline{\tilde{u}}_i^{*}(x_j) & otherwise \end{cases} \quad (35)$$

$$u_i^R = \sum_{l=1}^{M} u_{il}/M, u_{il} = \begin{cases} \overline{\tilde{u}}_i^{*}(x_j) & \text{if } x_{il} \text{ uses } \overline{\tilde{u}}_i^{*}(x_j) \text{ for } v_i^R \\ \underline{\tilde{u}}_i^{*}(x_j) & otherwise \end{cases} \quad (36)$$

Next, defuzzification for IT2IFCM is made as if $u_i(x_j) > u_k(x_j)$ for $k = 1, ..., c$ and $i \neq j$ then x_j is assigned to cluster i.

V. EXPERIMENTS

The experiments were completed for the well-known images with the predefined the number of clusters images in Table I. The results were measured on the basis of several validity indexes to assess the performance of the algorithms on the experimental images.

TABLE I
THE NUMBER OF CLUSTERS

Image	Number of cluster
Rose	3
Wolf	3
Mountain	4

We have done experiments on these test images with the FCM, IFCM, IT2FCM and IT2IFCM algorithms with predefined parameters: the number of clusters $c(1 < c < n)$, fuzifiers $m = 2$ for FCM and IFCM, fuzifier $m_1 = 1.5, m_2 = 5$ for IT2FCM and IT2IFCM, and error $\varepsilon = 0.00001$.

We performed the different validity indexes such as the Bezdeks partition coefficient (PC-I), the Dunns separation index (Dunn-I), the Davies-Bouldins index (DB-I), and the Separation index (S-I), Xie and Beni's index (XB-I), Classification Entropy index (CE-I) [11]. The various validity indexes are shown in the Table II.

Note that: the validity indexes are proposed to evaluate the quality of clustering. The better algorithm has smaller T-I, DB-I, XB-I, S-I, CE-I and larger PC-I and the best results are marked bold. The results in Table II show that the IT2IFCM (the proposed algorithm) has a better performance or higher quality clustering than the other typical algorithm such as FCM and IT2FCM.

VI. CONCLUSIONS

This paper presented a fuzzy clustering algorithm based on intuitionistic fuzzy sets and intuitionistic type-2 fuzzy sets which improved the clustering results and overcome the drawbacks of the conventional clustering algorithms in handling the uncertainty. The proposed approach have solved the problem of combining between IFSs and fuzzy clustering to improve the quality of clustering results, especially with noisy data.The experiments were completed for image segmentation with the statistics show that the proposed algorithm generates better results than other existing methods.

The next goal is some researches related to use the general type-2 intuitionistic fuzzy sets to better improvement of quality.

REFERENCES

[1] C. Hwang, F. C. H. Rhee, Uncertain Fuzzy clustering: Interval Type-2 Fuzzy Approach to C-Means, IEEE Tran. on Fuzzy Systems, Vol. 15, 107-120, 2007.

[2] T. Kanungo, D. M. Mount, N. S. Netanyahu, C. D. Piatko, R. Silverman, A. Y. Wu, An Efficient k-Means Clustering Algorithm: Analysis and Implementation, IEEE Trans. On Pattern Analysis and Machine Intelligence, Vol 24(7), 881-893, 2002.

[3] M.C. Hung, J. Wu, J.H. Chang, D.L. Yang, An Efficient k-Means Clustering Algorithm Using Simple Partitioning, J. of Info. Science and Engineering, Vol.21, 1157-1177, 2005.

Fig. 1. Test Images: a) Rose image; b) Wolf image; c) Mountain image

TABLE II
THE VARIOUS VALIDITY INDEXES ON THE EXPERIMENTAL IMAGES.

Validity Index	Rose image				Wolf image				Mountain image			
	FCM	IFCM	IT2FCM	IT2IFCM	FCM	IFCM	IT2FCM	IT2IFCM	FCM	IFCM	IT2FCM	IT2IFCM
DB-I	1.4357	1.2132	1.3522	**1.0122**	2.3122	1.3822	1.6531	**1.3293**	1.5352	1.2532	1.2312	**1.1892**
XB-I	0.5281	0.1936	0.1141	**0.0993**	0.7214	0.1811	0.2651	**0.1539**	0.9854	0.8653	0.6414	**0.6411**
S-I	0.9641	0.8382	0.3178	**0.3111**	0.9221	0.1654	0.1931	**0.1562**	0.7158	0.3862	0.3261	**0.3101**
CE-I	0.8432	0.6287	0.5531	**0.5131**	0.8131	0.6028	0.7152	**0.4642**	0.8643	0.5341	**0.2861**	**0.2861**
PC-I	0.8890	0.8912	0.8919	**0.9291**	0.8998	0.9413	0.9490	**0.9502**	0.9229	0.9291	0.9321	**0.9753**

[4] K.R. Zalik, An efcient k-means clustering algorithm, Pattern Recognition Letters Vol. 29, 1385 - 1391, 2008.

[5] K. A. Abdul Nazeer, M. P. Sebastian, Improving the Accuracy and Efficiency of the k-means Clustering Algorithm, Proceedings of the World Congress on Engineering, 2009.

[6] J.M. Mendel and R.I. John, Type-2 fuzzy set made simple, IEEE Trans.Fuzzy Syst., vol. 10(2), 117 - 127, 2002.

[7] N. Karnik, J.M. Mendel, Operations on Type-2 Fuzzy Sets, Fuzzy Sets and Systems, 122, 327-348, 2001.

[8] Q. Liang , J.M. Mendel, Interval Type-2 Fuzzy Logic Systems: Theory and Design, IEEE Trans. on Fuzzy Systems, 8(5), 535-550, 2000.

[9] J.M. Mendel, R.I. John, F. Liu, Interval Type-2 Fuzzy Logic Systems Made Simple, IEEE Trans. on Fuzzy Systems, 14(6), 808-821, 2006.

[10] J. M. Mendel, F. Liu, D. Zhai, z-Plane Representation for Type-2 Fuzzy Sets: Theory and Applications, IEEE Trans. on Fuzzy Systems, 17(5), 1189-1207, 2009.

[11] W. Wang,Y. Zhang, "On fuzzy cluster validity indices", Fuzzy Sets and Systems 158,2095-2117, 2007.

[12] F. Hoppner, F. Klawonn, R. Kruse, and T. Runkler,Fuzzy Cluster Analysis.New York: Wiley, 1999.

[13] J. C. Dunn, "A fuzzy relative of the ISODATA process and its use in detecting compacy well-separated clusters," J. Cybern., vol. 3, pp. 3257, 1973.

[14] J. C. Bezdek,Pattern Recognition With Fuzzy Objective Function Algorithms. New York: Academic, 1981.

[15] N. Pal and J. Bezdek, "On cluster validity for the fuzzy c-means model,"IEEE Trans. Fuzzy Syst., vol. 3, no. 3, pp. 370379, Aug. 1995.

[16] J. Yu and M.-S. Yang, "Optimality test for generalized FCM and its application to parameter selection,"IEEE Trans. Fuzzy Syst., vol. 13, no. 1, pp. 164176, Feb. 2005.

[17] H. Frigui and C. Hwang, "Fuzzy clustering and aggregation of relational data with instance-level constraints," IEEE Trans. Fuzzy Syst., vol. 16,no. 6, pp. 15651581, Dec. 2008.

[18] Y. A. Tolias., and S. M. Panas., "Image Segmentation by a Fuzzy Clustering Algorithm using Adaptive Spatially Constrained Membership Functions," IEEE Trans. Syst., Man, Cybern. A,Syst., Humans, vol. 28, no. 3, pp. 359369, May 1998. 263

[19] Atanassov, K. (1986a). Intuitionistic fuzzy sets.Fuzzy Sets and Systems, 20, 87-96.

[20] Atanassov, K. (1986b). Intuitionistic fuzzy sets. VII ITKRs Session, Deposed in Central Sci. Techn. Library of Bulg. Acd. of Sci., Sofia, pp. 1684-1697.

[21] Bezdek, J., Keller, J., Krisnapuram, R. and Pal, N. (1999).Fuzzy Models and algorithms for pattern recognition and image processing. The Handbooks of Fuzzy Sets Series.Boston/London/Dordrecht: Kluwer Academic Publishers.

[22] Burillo, P. and Bustince, H. (1996a). Construction theorems for intuitionistic fuzzy sets.Fuzzy Sets and Systems, 84, 271281.

[23] Carvalho, B., Gau, J., Herman, G. and Kong, T. (1999). Algorithms for fuzzy segmentation.Pattern Analysis and Applications, 2, 7381.

[24] Couto, P., 2006. Image segmentation using atanassov intuitionistic fuzzy sets. Ph.D. thesis, Trs-os-Montes e Alto Douro University, Vila Real, Portugal.

[25] Bustince,H.,Barrenechea,E., Pagola, M. and Orduna, R. (2007).Image Thresholding Computation Using Atanassovs Intuitionistic Fuzzy Sets, Journal of Advanced Comput. Int. and Int. Informatics, Vol.11 No.2, 187-194.

[26] Pinto,P. M.,Couto, P., Bustince, H., Barrenechea, E., Pagola, M. and Fernandez, J. (2013).Image segmentation using Atanassovs intuitionistic fuzzy sets, Expert Systems with Applications 40, 1526.

[27] Haralick, R. and Shapiro, L. (1985). Image segmentation techniques.Computer Vision,Graphics and Image Processing, 29, 100132.

[28] W. L. Cai., S. C. Chen., and D. Q. Zhang., "Fast and Robust Fuzzy C-Means Clustering Algorithms Incorporating Local Information for image Segmentation," Pattern Recognition, vol. 40, no. 3, pp. 825838, Mar. 2007.