Proceedings of the 2014 IEEE
International Conference on Robotics and Biomimetics
December 5-10, 2014, Bali, Indonesia

Synthetic Tracking Controller for Robot Manipulator with Flexible Joints, Dynamics of Executive Motors and Affect of Disturbance Based on Radial Basic Function (RBF) Neural Network

Tien Thanh Nguyen, Hai Van Nguyen, Tien Vu Hoa

***Abstract*— This paper presents synthetic checking controller for robot manipulator with flexible joints, dynamics of executive motors and affect of disturbance based on radial basic function (RBF) neural network. A control algorithm is synthesized basing on the combination of backsteppping and RBF neural network in order to approximate the unknown functions. Finally, simulation results of a manipulator robot with single flexible joints based on Matlab-Simulink are presented to demonstrate the effectiveness of the proposed control algorithms**

I. Introduction

Synthesizing controller for robot manipulator with the assumption that links beten joints are flexible has received considerably research attention in recent years [1,2,3,4,5,6]. In [2], the problem of designing robust tracking controllers for a class of robotic manipulators with flexible joints, only using position measurements is developed with assumption that the input signal of the system is the torque generated by the executive motor. In this paper, control algorithms are synthesized to ensure the tracking error converged to zero after a certain time.

Several approaches have been introduced to solve the control problem of flexible-joint robots without including the actuator dynamics.

In [1], the researchers synthesize the control algorithms for robot manipulators with flexible joints based on sliding mode control. In fact, the simulated results have shod higher quality of control parameters with torque input generated from the executive motor.

As can be seen, torque placed on the joints is always generated by motors such as DC motors, stepper motors, BLDC motors and others. Hover, the assumption that the input signal of the manipulator robot is torque placed on the joints will create higher difficulty in practical control algorithms. Moreover, manipulators include uncertain parameters, therefore synthesizing controller with assumption of certain parameters is not accurate in real performance. Thus, the above research basically has achieved certain results in terms of control theory. Hover, realizing the above algorithms is huge obstacle in fact.

In order to apply these control algorithms, the synthesized controller must have signal inputs that voltage is placed on executive motors and the torque is placed on these joints. The performance of motor will depend on the voltage as ll as suitable methodology which need to value uncertain parameters. One of most effective solution is based on the adaptive control law.

Hover, when considering the dynamics of motor, the order of system will increase considerably. Therefore, not only the synthesis of control algorithm will be more complicated, but also the system with torque placed the joints can be not stable. Furthermore, when uncertain parameters are included on the manipulator, it will be quite difficult to synthesize the control algorithms

In recent years, the analytical study of adaptive nonlinear control systems using RBF universal function approximation has received much attention, typically, these methods are mentioned in documents [6–11].

The RBF network adaptation can effectively improve the control performance against large uncertainty of the system. The adaptation law is derived from the Lyapunov method so that the stability of the entire system and the convergence of the ight adaptation are guaranteed.

By using RBF control, significant improvement has been achieved when the system is subjected to a sudden change with system uncertainty.

Past research of universal approximation theorems on RBF [12,13] is shown that any nonlinear function with arbitrary accuracy can be approximated by RBF neural network over a compact set.

In order to realize these control algorithms in practice, this paper focuses on synthesizing control algorithm for robot manipulator, taking into account flexibility of joints and dynamics of executive motors. The methodology of Backstepping control combining with and based on basic radial function (BRF) neural network is also applied on this algorithm. The result is proven by simulation on Matlab – Simulink environment.

This paper consists five sections. Section I demonstrates the state equations and setting tracking issues. Section II represents the backstepping controller design. Section III focuses on synthesizing adaptive Backstepping control with RBF for single-link flexible joint robot. The estimated simulation is made in Section IV. In Section V, the conclusions are given.

Manuscript received Juler 30, 2014. (Write the date on which you submitted your paper for review.)

T.T. Nguyen Le Quy Don University, Hanoi. He is now with the Department of Control Engineering (e-mail: ngttienktd@gmail.com)

H.V.Nguyen Phu Tho Vocational College of Technology and Agroforestry (e-mail: haind.nguyen@gmail.com)

T.H. Vu. Le Quy Don University, Hanoi (hoatien57@yahoo.com)

978-1-4799-7397-2/14/$31.00 © 2014 IEEE

II. MODEL DESCRIPTION AND PROBLEM STATEMENT

2.1. Establishing state model and tracking problem

The mathematical model of a flexible-joint manipulator with DC excited permanent magnet motor can be described by

$$\begin{aligned} &J_l(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) = k_e(q_2 - q_1) + d_1 \\ &J_r(q_2)\ddot{q}_2 + \mu(q_2,\dot{q}_2)\dot{q}_2 + k_e(q_2 - q_1) = K_i I_u + d_2 \\ &L_u \dot{I}_u + R_u I_u + K_b \dot{q}_2 = u \end{aligned} \qquad (1)$$

From the mathematical model, the dynamic structure of flexible-joint robot manipulator with dc excited permanent magnet motors is shown in figure 1

For the convenience of representations, the state variables are used as below:

$$x_1 = q_1;\ x_2 = \dot{q}_1;\ x_3 = q_2;\ x_4 = \dot{q}_2;\ x_5 = I_u \qquad (2)$$

Where $x_m = [x_{m1}, x_{m2} \ldots x_{mn}]^T$, $m = 1,2..5$.

Substituting Equation (2) into Equation (1) yields and through some simple transformations obtain;

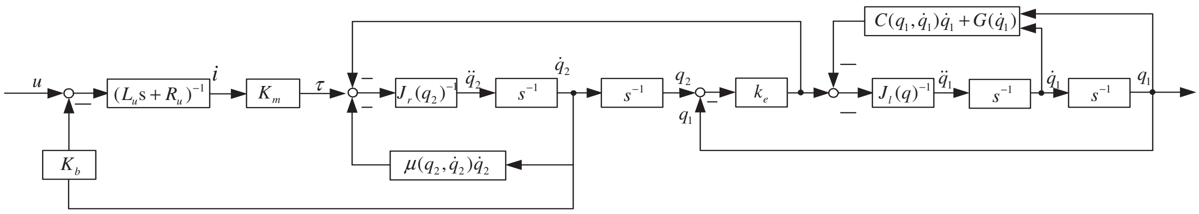

Fig. 1. Dynamic structure diagram of flexible joint robot manipulator with dc excited permanent magnets motor

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = x_3 + g(x) \\ &\dot{x}_3 = x_4 \\ &\dot{x}_4 = x_5 + h(x) \\ &\dot{x}_5 = f(x) + bu \end{aligned} \qquad (3)$$

Where

$$g(x) = -x_3 + J_l^{-1}\left[k_e(x_3 - x_1) - C(x_1,x_2) - G(x_1) + d_1\right]$$

$$h(x) = -x_5 + J_r^{-1}\left[K_i x_5 - \mu(x_3,x_4)x_4 - k_e(x_3 - x_1) + d_2\right]$$

$$f(x) = -\frac{R_u}{L_u}x_5 - \frac{K_B}{L_u}x_4;\ b = \frac{1}{L_u}$$

The purpose is to find the structure of input signal to meet the expected requirement that the output position converges to the desired reference trajectories ($x_1 \to x_{1d}$), with assumptions of existence of high order derivative of reference trajectories

Obviously, the output of the previous block is input of the next block in the cascade system [3]. With this cascade property, backstepping synthesis method can be applied easily.

2.2. Backstepping controller design

Regarding to the main aim of adaptive backstepping sliding controller design for single-link flexible joint robot manipulator proposed in [4], controller is designed in several steps as follows:

Step 1: $e_1 = x_1 - x_{1d}$ is error beten output value and desired trajectories in tracking problem

Differential expression of above equation is as below

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d} \qquad (4)$$

To realize $e_1 \to 0$, define a Lyapunov function as

$$V_1 = \frac{1}{2}e_1^2 \qquad (5)$$

Then $\dot{V}_1 = e_1\dot{e}_1 = e_1(x_2 - \dot{x}_{1d}) \qquad (6)$

To realize $\dot{V}_1 < 0$, if choose $x_2 = -k_1 e_1 + \dot{x}_{1d}, k_1 > 0$,

then $\dot{V} = -k_1 e_1^2$

Step 2: To realize $x_2 = -k_1 e_1 + \dot{x}_{1d}$, using virtual control as

$$x_{2d} = -k_1 e_1 + \dot{x}_{1d} \qquad (7)$$

To realize $x_2 \to x_{2d}$, getting a new error $e_2 = x_2 - x_{2d}$

So $\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} \qquad (8)$

Then $\dot{e}_2 = x_3 + g(x) - \dot{x}_{2d} \qquad (9)$

Then $\dot{V}_1 = e_1(x_{2d} + e_2 - \dot{x}_{1d}) = -k_1 e_1^2 + e_2 e_1 \qquad (10)$

To realize $e_2 \to 0$ and $e_1 \to 0$, using a Lyapunov function as:

$$V_2 = V_1 + \frac{1}{2}e_2^2 = \frac{1}{2}(e_1^2 + e_2^2)$$

Then $\dot{V}_2 = -k_1 e_1^2 + e_2 e_1 + e_2(x_3 + g(x) - \dot{x}_{2d}) \qquad (11)$

To realize $\dot{V}_2 < 0$, selecting

$$x_3 + g(x) - \dot{x}_{2d} = -e_1 - k_2 e_2 \qquad (12)$$

Then $\dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 \qquad (13)$

Step 3: To realize $x_3 + g(x) - \dot{x}_{2d} = -e_1 - k_2 e_2$, selecting virtual control as

$$x_{3d} = -e_1 - k_2 e_2 - g(x) + \dot{x}_{2d} \qquad (14)$$

To realize $x_3 \to x_{3d}$, defining the error

$$e_3 = x_3 - x_{3d} \qquad (15)$$

have $\dot{e}_3 = \dot{x}_3 - \dot{x}_{3d} = x_4 - \dot{x}_{3d} \qquad (16)$

To realize $e_3 \to 0$ and $e_2 \to 0$ and $e_1 \to 0$, choose a Lyapunov function as

$$V_3 = V_2 + \frac{1}{2}e_3^2 = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \qquad (17)$$

Substituting Equations (11) and (16) into (17) yields

$$\dot{V}_3 = -k_1e_1^2 - k_2e_2^2 + e_3(e_2 + x_4 - \dot{x}_{3d}) \quad (18)$$

To realize $\dot{V}_3 < 0$, choose

$$e_2 + x_4 - \dot{x}_{3d} = -k_3e_3,\ k_3 > 0 \quad (19)$$

Then $\dot{V}_3 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2$

Step 4: To realize (19), choose virtual control as

$$x_{4d} = \dot{x}_{3d} - k_3e_3 - e_2 \quad (20)$$

To realize $x_4 \to x_{4d}$, define a new error $e_4 = x_4 - x_{4d}$

have $\dot{e}_4 = \dot{x}_4 - \dot{x}_{4d} = x_5 + h(x) - \dot{x}_{4d}$ (21)

To realize $e_4 \to 0$ and $e_3 \to 0$ and $e_2 \to 0$ and $e_1 \to 0$, define a Lyapunov function as

$$V_4 = V_3 + \frac{1}{2}e_4^2 = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2)$$

Then

$$\dot{V}_4 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_4(e_3 + x_5 + h(x) - \dot{x}_{4d}) \quad (22)$$

To realize $\dot{V}_4 < 0$, choose

$$e_3 + x_5 + h(x) - \dot{x}_{4d} = -k_4e_4 \quad (23)$$

Then $\dot{V}_4 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2$

Step 5: To realize (23), choose virtual control as

$$x_{5d} = \dot{x}_{4d} - k_4e_4 - h(x) - e_3 \quad (24)$$

To realize $x_5 \to x_{5d}$, get a new error $e_5 = x_5 - x_{5d}$

have $\dot{e}_5 = \dot{x}_5 - \dot{x}_{5d} = f(x) + bu - \dot{x}_{5d}$ (25)

To realize $e_5 \to 0$ and $e_4 \to 0$ and $e_3 \to 0$ and $e_2 \to 0$ and $e_1 \to 0$, define a Lyapunov function as

$V_5 = V_4 + \frac{1}{2}e_5^2 = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2)$, Then

$$\dot{V}_5 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + e_5(e_4 + f(x) + bu - \dot{x}_{5d})$$

To realize $\dot{V}_5 < 0$, choose

$$e_4 + f(x) + bu - \dot{x}_{5d} = -k_5e_5,\ k_5 > 0 \quad (26)$$

Then $\dot{V}_5 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_5^2$

From (26) can get the control law:

$$u = \frac{1}{b}(-f(x) + \dot{x}_{5d} - k_5e_5 - e_4) \quad (27)$$

To realize (27), need to obtain information about $g(x), h(x), f(x)$. For manipulators, the above functions consist of uncertain parameters, therefore in the practice it is hard to get above information from objective model. In order to have the information on those functions, a method approximated them is used necessarily.

2.3. Adaptive backstepping control with RBF for single link flexible joint robot manipulation

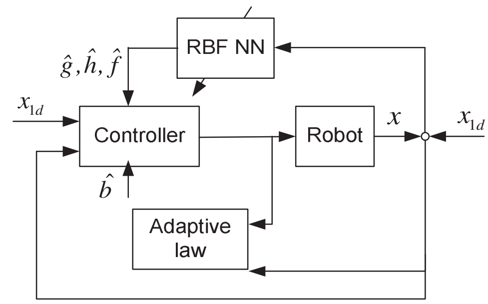

Fig. 2. Block diagram of the adaptive backstepping –RBF control

In (3), the functions $g(x), h(x), f(x)$ and b are unknown, the bound of b is known $b \geq \zeta(\zeta > 0)$. The unknown functions of $\hat{g}, \hat{h}, \hat{f}$ can be approximated by neural network and $\hat{b}$ can be estimated by adaptive law as in figure 2

2.3.1. Backstepping Controller Design estimating unknown Functions

Refer to the main idea of adaptive neural network backstepping sliding controller design for single-link flexible joint robot, several steps of designing controller are as follows:

Step 1: Similar as above,

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(x_2 - \dot{x}_{1d}) \quad (28)$$

To realize $\dot{V}_1 < 0$, if choose $x_2 = -k_1e_1 + \dot{x}_{1d}, k_1 > 0$

Step 2:

$$\dot{V}_2 = -k_1e_1^2 + e_2e_1 + e_2(x_3 + g(x) - \dot{x}_{2d}) \quad (29)$$

To realize $\dot{V} < 0$,

$$x_3 + g(x) - \dot{x}_{2d} = -e_1 - k_2e_2 \quad (30)$$

Step 3 Similar as above,

$$\dot{V}_3 = -k_1e_1^2 - k_2e_2^2 + e_2(g(x) - \hat{g}(x)) + + e_3(e_2 + x_4 - \dot{x}_{3d}) \quad (31)$$

To realize $\dot{V}_3 < 0$,

$$e_2 + x_4 - \dot{x}_{3d} = -k_3e_3,\ k_3 > 0 \quad (32)$$

Then $\dot{V}_3 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_2(g(x) - \hat{g}(x))$

Step 4: To realize $e_2 + x_4 - \dot{x}_{3d} = -k_3e_3$, choose virtual control as $x_{4d} = \dot{x}_{3d} - k_3e_3 - e_2$ (33)

Similar as above,

$$\dot{V}_4 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_2(g(x) - \hat{g}(x)) + + e_4(e_3 + x_5 + h(x) - \dot{x}_{4d}) \quad (34)$$

To realize $\dot{V}_4 < 0$,

$$e_3 + x_5 + h(x) - \dot{x}_{4d} = -k_4e_4 \quad (35)$$

Then $\dot{V}_4 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + e_2(g(x) - \hat{g}(x))$

Step 5: To realize $e_3 + x_5 + h(x) - \dot{x}_{4d} = -k_4e_4$

choose virtual control as

$$x_{5d} = \hat{\dot{x}}_{4d} - k_4e_4 - \hat{h}(x) - e_3 \quad (36)$$

Where $\dot{x}_{4d} = \dot{x}_{4d1} - d$, $\hat{\dot{x}}_{4d} = \dot{x}_{4d1} - \hat{d}$

$\dot{x}_{4d1}$ composes of known values, d consists of the unknown parts, where $\hat{d}$ is estimated value of d , with

$$\begin{aligned}\dot{x}_{4d1} = &-(k_2+k_3)x_4 + (k_2k_3-2)x_3 - k_3x_2 + \\ &+k_3\dot{x}_{1d} + \ddot{x}_{1d} + (k_2k_3+1)\dot{x}_{2d} + (k_2+k_3+1)\ddot{x}_{2d}\end{aligned} \tag{37}$$

and

$$d = (k_3k_2+2)g(x) + k_2\dot{g}(x) + k_3\dot{\hat{g}}(x) + \ddot{\hat{g}}(x) \tag{38}$$

To realize $x_5 \to x_{5d}$, define a new error $e_5 = x_5 - x_{5d}$ then

$$\dot{e}_5 = \dot{x}_5 - \dot{x}_{5d} = f(x) + bu - \dot{x}_{5d} \tag{39}$$

where

$$\dot{x}_{5d} = \hat{\dot{x}}_{4d} - k_4e_4 - \hat{h}(x) - e_3 = \dot{x}_{5d1} + \dot{x}_{5d2} \tag{40}$$

$\dot{x}_{5d1}$ composes of known values, $\dot{x}_{5d2}$ consists of the unknown parts. With

$$\begin{aligned}\dot{x}_{5d1} = &-k_4x_5 - (k_2+k_3-1)x_4 - (k_3+1)x_2 + \\ &+(k_2k_3-k_2-2)x_3 + (k_3+1)\dot{x}_{1d} + \ddot{x}_{1d} + \\ &+(k_2k_3+k_2+1)\dot{x}_{2d} + (k_2+k_3+2)\ddot{x}_{2d} + k_4\dot{x}_{4d}\end{aligned} \tag{41}$$

$$\dot{x}_{5d2} = -k_4h(x) - k_2g(x) - \dot{\hat{g}}(x) - \dot{\hat{d}} - \dot{\hat{h}}(x)$$

Define $\overline{f} = f - \dot{x}_{5d2}$.

$$\dot{e}_5 = \overline{f} - \dot{x}_{5d1} + (b-\hat{b})u + \hat{b}u \tag{42}$$

Where $\hat{b}$ is estimation value of b .

To realize $e_5 \to 0$ and $e_4 \to 0$ and $e_3 \to 0$ and $e_2 \to 0$ and $e_1 \to 0$. Similar as above,

$$\begin{aligned}\dot{V}_5 = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + e_2(g(x) - \hat{g}(x)) + \\ &+e_4\left(d-\hat{d}\right) + e_4\left(h(x)-\hat{h}(x)\right) + e_5(b-\hat{b})u + \\ &+e_5(e_4 + \overline{f} - \dot{x}_{5d1} + \hat{b}u)\end{aligned} \tag{43}$$

To realize $\dot{V}_5 < 0$, choose virtual control as

$$u = \frac{1}{\hat{b}}(-e_4 - \hat{\overline{f}} + \dot{x}_{5d1} - k_5e_5) \tag{44}$$

Then

$$\begin{aligned}\dot{V}_5 = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_5^2 + e_2(g-\hat{g}) + \\ &+e_4\left(d-\hat{d}\right) + e_4\left(h-\hat{h}\right) + e_5(\overline{f} - \hat{\overline{f}}) + e_5(b-\hat{b})u\end{aligned} \tag{45}$$

If $b = \hat{b}, g = \hat{g}, h = \hat{h}, f = \hat{f}, d = \hat{d}$, get $\dot{V}_5 < 0$

2.3.2.. Backstepping Controller Design with RBF Approximation

To approximate the unknown function $g(x), h(x), f(x)$, use RBF neural network as in below [5]:

$$\begin{cases} g = W_1^T h_1 + \varepsilon_1 \\ h = W_2^T h_2 + \varepsilon_2 \\ d = W_3^T h_3 + \varepsilon_3 \\ \overline{f} = W_4^T h_4 + \varepsilon_4 \end{cases} \tag{46}$$

Where W_i is the neural network weighted value, h_i is the Gaussian function,, ε_i is the approximation error and $\|\varepsilon\| = \left\|[\varepsilon_1\ \varepsilon_2\ \varepsilon_3\ \varepsilon_4]^T\right\| < \varepsilon_N$, $\|W\|_F \le W_m$. Define

$$\begin{cases} \hat{g} = \hat{W}_1^T h_1 \\ \hat{h} = \hat{W}_2^T h_2 \\ d = \hat{W}_3^T h_3 \\ \overline{f} = \hat{W}_4^T h_4 \end{cases} \tag{47}$$

Where $\hat{W}_i^T$ is weighted value estimation, Define

$$Z = \begin{bmatrix} 0 & & & & \\ & W_1 & & & \\ & & W_2 & & \\ & & & W_3 & \\ & & & & W_4 \end{bmatrix}, \|Z\|_F \le Z_M \tag{48}$$

$$\hat{Z} = \begin{bmatrix} 0 & & & & \\ & \hat{W}_1 & & & \\ & & \hat{W}_2 & & \\ & & & \hat{W}_3 & \\ & & & & \hat{W}_4 \end{bmatrix}, \tilde{Z} = Z - \hat{Z} \tag{49}$$

Define a Lyapunov function as

$$V = \frac{1}{2}\xi^T\xi + \frac{1}{2}tr(\tilde{Z}^T\Gamma^{-1}\tilde{Z}) + \frac{1}{2}\eta\tilde{b}^2 \tag{50}$$

Where $V_5 = \frac{1}{2}\xi^T\xi, \eta > 0, \Gamma$ is a positive-definite matrix with proper dimension

$$\Gamma = \begin{bmatrix} 0 & & & & \\ & \Gamma_2 & & & \\ & & \Gamma_3 & & \\ & & & \Gamma_4 & \\ & & & & \Gamma_5 \end{bmatrix},$$

and $\tilde{b} = b - \hat{b}$

Let adaptive law as

$$\dot{\hat{Z}} = \Gamma h\xi^T - n\Gamma\|\xi\|\hat{Z} \tag{51}$$

Where $h = [0\ \ h_1\ \ h_2\ \ h_3\ \ h_4]^T, n$ is a positive number, and $\hat{b}(0) \ge \underline{b}$. Then have

$$\begin{aligned}\dot{V} = \xi^T\dot{\xi} + tr(\tilde{Z}^T\Gamma^{-1}\dot{\tilde{Z}}) + \eta\tilde{b}\dot{\tilde{b}} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 \\ &-k_5e_5^2 + (\tilde{W}_1^T h_1 + \varepsilon_1)e_2 + (\tilde{W}_2^T h_2 + \varepsilon_2)e_4 + (\tilde{W}_3^T h_3 + \varepsilon_3)e_4 \\ &+e_5(\tilde{W}_4^T h_4 + \varepsilon_4) + tr(\tilde{Z}^T\Gamma^{-1}\dot{\tilde{Z}}) + \tilde{b}e_5u + \eta\tilde{b}\dot{\tilde{b}}\end{aligned}$$

Where $\tilde{W}_i^T = W_i^T - \hat{W}_i^T, i = 1,2,3,4$,then

$$\dot{V} = -\xi^T K_e\xi + \xi^T\varepsilon + tr(\tilde{Z}^T\Gamma^{-1}\dot{\tilde{Z}} + \tilde{Z}^T h\xi^T) + \tilde{b}e_5u + \eta\tilde{b}\dot{\tilde{b}}$$

Where $K_e = [k_1\ \ k_2\ \ k_3\ \ k_4\ \ k_5]$ and $\varepsilon = [0\ \ \varepsilon_1\ \ \varepsilon_2\ \ \varepsilon_3\ \ \varepsilon_4]^T$

Then $\dot{\tilde{Z}} = -\dot{\hat{Z}}$, and $\dot{\tilde{b}} = \dot{\hat{b}}$, from (51), have

$$\begin{aligned}\dot{V} = &-\xi^T K_e\xi + \xi^T\varepsilon + n\|\xi\|tr(\tilde{Z}^T(Z-\tilde{Z}))) \\ &+\tilde{b}(e_5u - \eta\dot{\hat{b}})\end{aligned} \tag{52}$$

To guarantee $\tilde{b}(e_5u+\eta\dot{\hat{b}})\le 0$, at the same time to avoid singularity in (44) and guarantee $\hat{b}>\underline{b}$, using an adaptive law for $\hat{b}$ given in [6] is as follows:

$$\dot{\hat{b}}=\begin{cases}\eta^{-1}e_5u,\ if\ \ e_5u>0\\ \eta^{-1}e_5u,\ if\ \ e_5u\le 0\ and\ \hat{b}>\underline{b}\\ \eta^{-1},\quad\ if\ \ e_5u\le 0\ and\ \hat{b}\le\underline{b}\end{cases}\tag{53}$$

The adaptive law (53) can be analyzed as:

1. If $e_5u>0$, get $\tilde{b}(e_5u+\eta\dot{\hat{b}})=0$ and $\dot{\hat{b}}>0$; thus $\hat{b}>\underline{b}$
2. If $e_5u\le 0$ and $\hat{b}>\underline{b}$, get $\tilde{b}(e_5u+\eta\dot{\hat{b}})=0$
3. if $e_5u\le 0$ và $\hat{b}\le\underline{b}$ have $\tilde{b}=b-\hat{b}\ge b-\underline{b}>0$; thus $\tilde{b}(e_5u-\eta\dot{\hat{b}})=\tilde{b}e_5u-\tilde{b}\le 0$ and $\hat{b}$ will increase gradually,

and then $\hat{b}>\underline{b}$ s will be guaranteed with $\dot{\hat{b}}>0$

According to Schwarz inequality, have $tr(\tilde{Z}^T(Z-\tilde{Z}))\le\|\tilde{Z}\|_F\|Z\|_F-\|\tilde{Z}\|_F^2$, then $K_{\min}\|\xi\|^2\le\xi^TK\xi$, $K_{\min}$ is the minimum eigenvalue of *K*, then (54) becomes

$$\dot{V}\le -K_{\min}\|\xi\|^2+\varepsilon_N\|\xi\|+n\|\xi\|\left(\|\tilde{Z}\|_F\|Z\|_F-\|\tilde{Z}\|_F^2\right)+M$$
$$\le -\|\xi\|\left(K_{\min}\|\xi\|-\varepsilon_N+n\|\tilde{Z}\|_F\left(\|\tilde{Z}\|_F\right)-Z_M\right)$$

Since

$$K_{\min}\|\xi\|-\varepsilon_N+n\left(\|\tilde{Z}\|_F^2-\|\tilde{Z}\|_F Z_M\right)=K_{\min}\|\xi\|-$$
$$-\varepsilon_N+n\left(\|\tilde{Z}\|-\frac{1}{2}Z_M\right)^2-\frac{n}{4}Z_M^2$$

this implies that $\dot{V}<0$ as long as

$$\|\xi\|>\frac{\varepsilon_N+\frac{n}{4}Z_M^2}{K_{\min}}\ \text{or}\ \|\tilde{Z}\|_F>\frac{1}{2}Z_M+\sqrt{\frac{Z_M^2}{4}+\frac{\varepsilon_N}{n}}$$

2.4. Simulation and Result remarks

2.4.1. Description of the Single-link robot manipulator with flexible joint

To evaluate the dynamic behaviour of the proposed control algorithm, simulation results for single-link flexible-joint robots (figure 3) are presented to estimate the effectiveness of the proposed control algorithms.

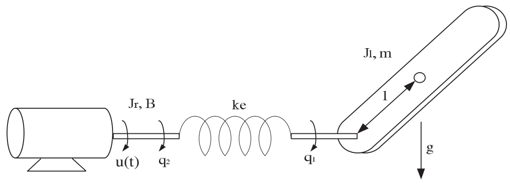

Fig. 3. Single-link robot manipulator with flexible joint

Considering the dynamics of the single-link manipulator with rotational joints as described by

$$J_l(q_1)\ddot{q}_1+mgl\sin q_1=k_e(q_2-q_1)$$
$$J_r\ddot{q}_2+\mu\dot{q}_2+k_e(q_2-q_1)=K_ii_u\tag{54}$$
$$L_u\dot{I}_u+R_uI_u+K_B\dot{q}_2=u$$

TABLE I

PHYSICAL PARAMETERS OF THE ROBOT MANIPULATOR

Symbol	Value	Unit
J_l	1,33x10^{-3}	[Kg m^2]
m	0,2	Kg
l	0,02	m
ke	5	Nm/rad
J_r	2,16x10^{-1}	[Kg m^2]
μ	0,1	[Kg m^2 /sec]
R_u	5	Ω
L_u	0.2	mH
K_b	0.1	V/rad/sec
K_i	0.1	Nm/A

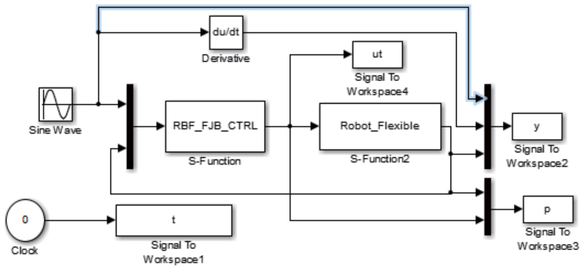

Fig. 4. The structure diagram of proposed controller

The parameters of the proposed controller

$k_1=k_2=k_3=k_4=k_5=3,5$; $\Gamma_2=\Gamma_3=\Gamma_4=\Gamma_5=250$

$c_j=[-1\ \ -0.5\ \ 0\ \ 0.5\ \ 1], b_j=1.5, \eta=150$

The simulation results are shown in figs. 5-11, respectively

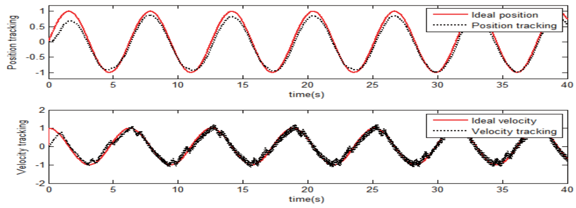

Fig. 5. The joint angular position and velocity tracking of joint

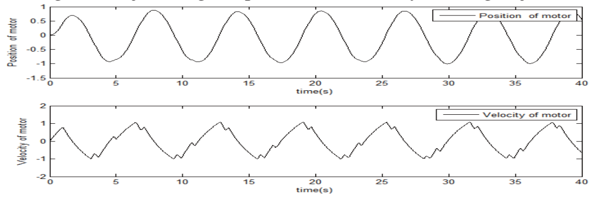

Fig. 6. The joint angular position and velocity tracking of motor

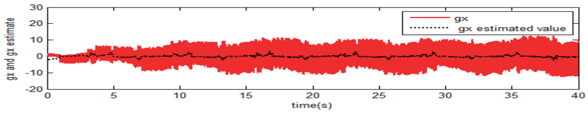

Fig. 7. g(x) and its estimated value

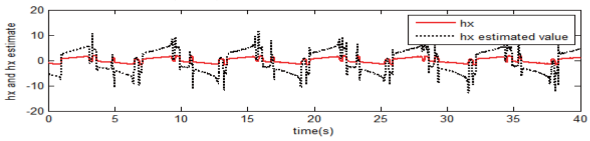

Fig. 8. h(x) and its estimated value

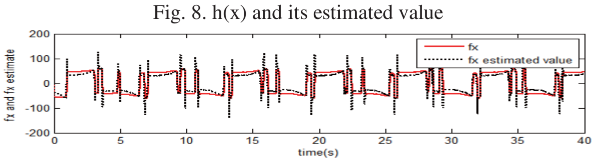

Fig. 9. f(x) and its estimated value

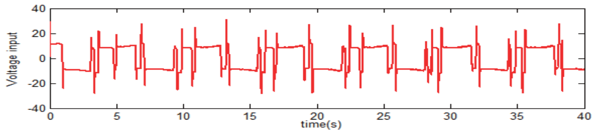

Fig. 10. Voltage value of motor

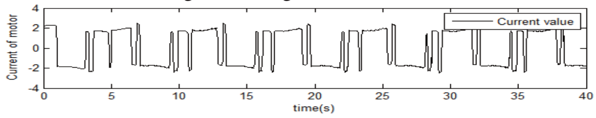

Fig. 11. Current value of DC motor

2.4.2. Result remarks

The simulation results for single-link flexible-joint robots based on proposed control algorithms are presented in figures 5-11. Several remarks can be seen as below:

- The angular position of joints was tracked to the desired reference trajectories with the error values converging towards 0 in a certain period of time.

- The values of unknown functions $g(x)$, $h(x)$, $f(x)$ are approximated basically by RBF neural network, therefore this permit to build control algorithm with higher quality.

- From the achieved results, it is concluded that the proposed controller design brings to the expected results.

III. Conclusion

The study successfully establishes the synthesized controller for robot manipulator with flexible joints, based on radial basic function (RBF) neural network. This controller permit to control the joint positions of robot manipulator including dynamics of electric motors in order to achieving high-precision position control. Although the order of systems is higher, but the angular position of joints was tracked to the desired reference trajectories with the error values converge towards zero in a certain period of time.

From above results, this study can be extended to handle a broader class of complicated flexible-joint robots

References

[1] Nguyen Van Hai, Vu Hoa Tien, Nguyen Thanh Tien. *The Synthesizing Controller for Flexible Joints Robot Manipulator.* The Journal of Science and Technology, No.154(4-2013)

[2] Y.-C. Chang, H.-M. Yen. *Design of a Robust Position Feedback Tracking Controller for Flexible - joint Robots.* IET Control Theory Appl., Vol. 5, Iss. 2, 2011, pp. 351-363.

[3] Dong Hwan Kim and Won Ho Oh. *Robust control Design for flexible Joint Manipulator: Theory and Experimental Verifivation.* International Journal of Control, Automation, and Systems, Vol. 4, No. 4, August 2006, pp. 495-505.

[4] Huang AC, Chen YC (2004) *Adaptive sliding control for single – link flexible joint robot with mismatched uncertainties.* IEEE trans Control Syst Technol 12(5):770-775

[5] Jinkun Liu (2013) *Radial Basis Function (RBF) Neural Network Control for Mechanical Systems.* Tsinghua University Press, Beijing and Springer-Verlag Berlin Heidelberg

[6] Sundararajan N, Saratchandran P, Li Y (2002) *Fully tuned radial basis function neural networks for flight control.* Klur, Boston

[7] Huang SN, Tan KK, Lee TH (2002) *Adaptive motion control using neural network approximations.* Automatica 38(2):227–233

[8] Ge SS, Wang C (2002) *Direct adaptive NN control of a class of nonlinear systems.* IEEE Trans Neural Netw 13(1):214–221

[9] . Li Y, Qiang S, Zhuang X, Kaynak O (2004) *Robust and adaptive backstepping control for nonlinear systems using RBF neural networks.* IEEE Trans Neural Netw 15(3):693–701

[10] Huang S, Tan KK, Lee TH, Putra AS (2007) *Adaptive control of mechanical systems using neural networks.* IEEE Trans Syst Man, Cybern Part C 37(5):897–903

[11] Zhu Q, Fei S, Zhang T, Li T (2008) *Adaptive RBF neural-networks control for a class of timedelay nonlinear systems.* Neurocomputing 71(16–18):3617–3624

[12] Hartman EJ, Keeler JD, Kowalski JM (1990) *Layered neural networks with Gaussian hidden units as universal approximations.* Neural Comput 2(2):210–215

[13] Park J, Sandberg LW (1991) *Universal approximation using radial-basis-function networks.* Neural Comput 3(2):246–257