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No-Zero-Entry Space-Time Block Codes Over Time-Selective Fading Channels for MIMO Systems

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Abstract In MIMO systems, space-time block code (STBC) is good solution for improving system performance. Among the STBCs, coordinate interleaved orthogonal designs (CIODs) combined with QR-decomposition-based decision-feedback decoding (QR-DDF) allow achieving good performance for time-selective fading channels. However, half of entries in codeword matrix of CIODs are zeros. These zero entries result in high peak-to-average power ratio (PAPR) and also impose a severe constraint on hardware implementation of the code when turning off some of the transmitting antennas whenever a zero is transmitted. In this paper, we propose a new design of space-time block codes without zero entry in codeword matrix (NZE-STBCs) for time-selective fading channels. The main advantage of the proposed NZE-STBCs is that its peak-to-average ratio (PAPR) is 3 dB lower than that of CIODs, and its hardware implementation is also easier due to eliminating on-off switchers without sacrificing performance. Moreover, similar as CIODs, the proposed NZE-STBCs can use low complexity QR-DDF decoder over time-selective fading channels to enhance performance and reduce decoding complexity. Simulation results show that the proposed NZE-STBCs outperform CIODs for three transmit antennas while performing the same for two and four transmit antennas.

Keywords Time-selective fading · Space-time block code · MIMO system

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1 Introduction

Space-time coding is an effective approach to achieve transmit diversity in multiple-input multiple-output (MIMO) systems [1]. Orthogonal space-time block codes (OSTBCs) [2,3] attain full diversity with single-symbol maximum likelihood (SML) decoder, but they suffer a rate loss when there are more than two transmit antennas. For example, the rate of OSTBCs is 1 (full rate) for two transmit antennas and 3/4 for three and four transmit antennas. By relaxing the orthogonal constraint to enable full rate, quasi-orthogonal STBCs (QOSTBCs) for four transmit antennas were proposed independently in researches [4–6]. These full rate QOSTBCs have two drawbacks: (1) they are not full diversity so their performance becomes worse than that of OSTBCs at high signal-to-noise regime; (2) they require pair-wise complex symbol maximum likelihood (PML) decoder. The first drawback can be eliminated by constellation rotation. For example, QOSTBCs with constellation rotation (QOSTBCs-CR) were proposed in researches [7,8]. While these QOSTBCs-CR outperform OSTBCs at all signal-to-noise values for four transmit antennas, they still require high complexity PML decoder.

To eliminate the drawback of QOSTBCs-CR, Khan et al. [9,10] proposed CIODs which achieve full diversity and full rate for two, three and four transmit antennas with single-symbol maximum likelihood (SML) decoding. In this paper, we call STBCs that requires only low complexity SML decoder are single-symbol decodable space-time block codes (SSDCs). However, codeword matrix of CIODs contains many zeros. Reducing the number of zeros in codeword matrix is important for many reasons, namely the improvement in PAPR and also the ease of practical implementation of these codes in wireless communication systems [11]. To forward this problem, QOSTBCs with minimum decoding complexity (MDC-QOSTBC) were proposed by Yuen et al. [12], Wang et al. [13] and Sinnokrot et al. [14].

When the quasi-static channel conditions are not satisfied (i.e., the channel is time-selective), performance of all above SSDCs (included CIODs and MDC-QOSTBCs) will be decreased, even appear *error floor* at high signal-to-noise ratios, if low complexity SML decoder is used. However, if we use conventional ML decoder then decoding complexity is extremely high (especially for high order modulations). To solve this problem, Hoo-Jin Lee [15] proposed QR-DDF decoder for CIODs. The QR-DDF decoder allows overcoming the detrimental effect of the time-selectivity of fading channels and improving decoding performance. Moreover, complexity of the QR-DDF decoder is comparable with that of low complexity SML decoder and much lower than that of conventional ML decoder [15]. Unfortunately, the QR-DDF decoder only can apply to CIODs [10] thanks to especial structure of CIODs, but can not apply to MDC-QOSTBCs [12–14]. This is reason that motivates us to find NZE-STBCs which work well with low complexity SML and QR-DDF decoders over the both quasi-static and time-selective fading channels.

In this paper, we propose a general structure to design new NZE-STBCs from CIODs. Then, we demonstrate that proposed NZE-STBCs have all desirable properties of CIODs such as: achieve full rate and full diversity with SML decoder over quasi-static fading channel, have good performance with QR-DDF decoder over time-selective fading channel. Moreover, by eliminating completely "zero-symbols" in codeword matrix, the proposed NZE-STBCs have lower PAPR of 3-dB and reduce difficult in hardware implementation in comparison to CIODs. We also provide simulation results in the both quasi-static and time-selective fading channels to demonstrate that advantage of the proposed NZE-STBCs over CIODs do not come with sacrificing the decoding performance. In addition, when number of transmit antenna is odd (i.e., for three transmit antennas) then performance of the proposed NZE-STBC is better 1.23-dB than that of CIODs.

We use the following notations throughout this letter. The superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$, respectively denote the conjugate, transpose and conjugate transpose operations. Bold-faced letters such as $\boldsymbol{s}$ and $\boldsymbol{S}$ represent vectors and matrices. $\det(\boldsymbol{A})$ is the determinant of matrix $\boldsymbol{A}$. $E[\cdot]$ is reserved for expectation with respect to all the random variables within the braces.

2 Backgrounds

2.1 Channel Model

We consider an uncorrelated multiple-input single-output (MIMO) system with N_{T} transmit antennas (Tx) and N_R receive antennas (Rx). A STBC encodes an input symbol vector of length K, $\boldsymbol{u} = [u_1 \quad u_2 \ldots u_K]^{\mathrm{T}}$, into an $L \times N_{\mathrm{T}}$ matrix $\boldsymbol{S}$, where L is the number of time slots. Symbol rate of the STBC $\boldsymbol{S}$ is K / L. if $K = L$ then $\boldsymbol{S}$ called full-rate code. The received signal at k-th receive antenna at t-th time slot $r_k(t)$ is given by

$$r_k(t) = \sum_{i=1}^{N_T} h_{ki}(t) s_{ti} + n_k(t) \tag{1}$$

where $h_{ki}(t)$ and $n_k(t)$ denote path gain from i-th transmit antenna to k-th receive antenna and noise at t-th time slot, respectively. s_{ti} denotes transmitted signal on i-th transmit antenna at t-th time slot.

In this letter, we make the following assumptions about the channel model (1): white Gaussian noise, so that n_t is a zero-mean circularly symmetric complex Gaussian random variable satisfying $E\left[n_k(t)n_k^*(t)\right] = N_0$; spatially symmetric Rayleigh fading, so that, $h_{ki}(t)$ is identically distributed, zero-mean unit-variance circularly symmetric complex jointly Gaussian random variables satisfying $E\left[|h_{ki}(t)|^2\right] = 1$; sufficient antenna spacing, so that $E\left[h_{ki}(\cdot)h_{kl}^*(\cdot)\right] = 0$ if $i \neq l$; relaxing this constraint would be possible, but it would complicate the analysis and it would detract from our main aim of studying the impact of time variations; temporally symmetric Rayleigh fading, so that the correlation $\rho(m)$ between $h_{ki}(t)$ and $h_{ki}(t+m)$ is the same for $\forall i = 1, .., N_T$, namely $E\left[h_{ki}(t)h_{ki}^*(t+m)\right] = E\left[h_{kl}(t)h_{kl}^*(t+m)\right] = \rho(m) \quad \forall i, l$; perfect knowledge of $h_{ki}(t)$ at the receiver.

According to Jakes'model [16], we have $\rho(m) = J_0(2\pi m f_d T_s)$ where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, f_d is the maximum Doppler shift and T_s is the period of each symbol. If $f_d T_s = 0$ we obtain quasi-static fading channel model, else we obtain time-selective fading channel model.

The PAPR for the m-th transmit antenna of a space-time code is [17]:

$$PAPR_m = \frac{\max_{t\in\{1,\ldots,L\}} |s_{tm}|^2}{L^{-1}\sum_{t\in\{1,\ldots,L\}} E\left(\left||s_{tm}|^2\right|\right)} \tag{2}$$

where the maximum and the expectation operators are taken over all possible codeword matrices. In all the space-time codes considered in this paper, the PAPR is the same for all the antennas, so that the subscript on PAPR in (2) may be dropped.

2.2 Structure of CIODs

From [9,10], the encoding procedure of CIODs is summarized as follows.

1. Information bits are mapped into complex information data symbols $x_i = x_{iI} + \mathrm{j}x_{iQ}$, $i = 1, \ldots, K$ from a constellation $\mathcal{A}$, where x_{iI} and x_{iQ} are real-part and image-part of complex signal x_i, respectively.
2. The mapped complex symbols x_i are rotated by θ to generate intermediate symbols u_i, such that $u_i \in \mathcal{A}e^{j\theta}$, which can be easily generated by $\begin{bmatrix} u_{iI} \\ u_{iQ} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_{iI} \\ x_{iQ} \end{bmatrix}$. For a square lattice constellation (e.g., square M-QAM), the optimum phase rotation is given as $\theta = 31.7175°$, which maximizes coding gain defined by [9,10].

The transmitted complex symbols s_i are generated by coordinate interleaving as $s_i = u_{iI} + \mathrm{j}u_{\langle i+K \rangle_K Q}$, $i = 1, \ldots, K$. Then transmitted complex symbols s_i are encoded yield CIODS for 2, 3 and 4 Tx (where $\boldsymbol{S}_3$ is obtained by deleting the last column of $\boldsymbol{S}_4$) as:

$$\boldsymbol{S}_2(s_1, s_2) = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}; \boldsymbol{S}_3(s_1, s_2, s_3, s_4) = \begin{bmatrix} s_1 & s_2 & 0 \\ -s_2^* & s_1^* & 0 \\ 0 & 0 & s_3 \\ 0 & 0 & -s_4^* \end{bmatrix};$$

$$\boldsymbol{S}_4(s_1, s_2, s_3, s_4) = \begin{bmatrix} s_1 & s_2 & 0 & 0 \\ -s_2^* & s_1^* & 0 & 0 \\ 0 & 0 & s_3 & s_4 \\ 0 & 0 & -s_4^* & s_3^* \end{bmatrix} \tag{3}$$

It is clear that a half of transmitted signals of CIODs in (3) are zeros. This results in high PAPR and increasing complexity in hardware implementation. In next section we propose new full rate STBCs where zeros are eliminated completely.

2.3 The Low Complexity SML Decoder and QR-DDF Decoder for CIODs

In this sub-section we summarize low complexity SML and DDF decoding methods for CIODs which presented in [15].

2.3.1 The SML Decoder

We consider to CIOD $\boldsymbol{S}_4$ with $N_T = 4$ given in (3). After some straightforward manipulations of channel model (1), the received signal of CIOD can be expressed in vector/matrix form as

$$\boldsymbol{r} = \boldsymbol{\mathcal{H}}\boldsymbol{s} + \boldsymbol{v} \tag{4}$$

where $\boldsymbol{r}$ is $4N_R \times 1$ received signal vector, $\boldsymbol{s} = [s_1 \quad s_2 \quad s_3 \quad s_4]^{\mathrm{T}}$ is transmitted signal vector, $\boldsymbol{v}$ is $4N_R \times 1$ AWGN noise vector and $\boldsymbol{\mathcal{H}} = \left[\boldsymbol{\mathcal{H}}_1^T, \boldsymbol{\mathcal{H}}_2^T, \ldots, \boldsymbol{\mathcal{H}}_{N_R}^T\right]^T$ is the effective channel matrix, for instance, whose sub-matrix $\boldsymbol{\mathcal{H}}_{\mathbf{n}}$, $n = 1, 2, \ldots, N_R$ for CIOD with $N_T = 4$ is given by

$$\boldsymbol{\mathcal{H}}_n = \begin{bmatrix} h_{n1}(1) & h_{n2}(1) & 0 & 0 \\ h_{n2}^*(2) & -h_{n1}^*(2) & 0 & 0 \\ 0 & 0 & h_{n3}(3) & h_{n4}(3) \\ 0 & 0 & h_{n4}^*(4) & -h_{n3}^*(4) \end{bmatrix} \tag{5}$$

where the index inside $(\cdot)$ is the discrete time index with respect to the symbol time duration in each codeword period. After channel-matched filtering operation by $\mathcal{H}^H$ to obtain sufficient statistics for detection at the receiver, we obtain

$$\underbrace{\begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix}}_{\boldsymbol{y}=\mathcal{H}^{\mathrm{H}}\boldsymbol{r}} = \underbrace{\begin{bmatrix} \alpha_1 & \varepsilon_1 & 0 & 0 \\ \varepsilon_1^* & \alpha_2 & 0 & 0 \\ 0 & 0 & \beta_1 & \varepsilon_2 \\ 0 & 0 & \varepsilon_2^* & \beta_2 \end{bmatrix}}_{\boldsymbol{G}=\mathcal{H}^{\mathrm{H}}\mathcal{H}} \underbrace{\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}}_{\boldsymbol{s}} + \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}}_{\bar{\boldsymbol{v}}=\mathcal{H}^{\mathrm{H}}\boldsymbol{v}} \tag{6}$$

where

$$\alpha_1 = \sum_{n=1}^{N_R} |h_{n1}(1)|^2 + |h_{n2}(2)|^2; \alpha_2 = \sum_{n=1}^{N_R} |h_{n1}(2)|^2 + |h_{n2}(1)|^2 \tag{7}$$

$$\beta_1 = \sum_{n=1}^{N_R} |h_{n3}(3)|^2 + |h_{n4}(4)|^2; \beta_2 = \sum_{n=1}^{N_R} |h_{n3}(4)|^2 + |h_{n4}(3)|^2 \tag{8}$$

$$\varepsilon_1 = \sum_{n=1}^{N_R} h_{n1}^*(1)\, h_{n2}(1) - h_{n1}^*(2)\, h_{n2}(2)\,; \varepsilon_2 = \sum_{n=1}^{N_R} h_{n3}^*(3)\, h_{n4}(3) - h_{n3}^*(4)\, h_{n4}(4) \tag{9}$$

When the channel is quasi-static, $\mathcal{H}$ is orthogonal (i.e., the Grammian matrix $\boldsymbol{G}$ is diagonal) with $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$ and $\varepsilon_1 = \varepsilon_2 = 0$. Then, by separating and rearranging the in-phase and quadrature-phase components of $y(t)$, we have the following ML decision metrics

$$\hat{u}_i = \underset{\hat{u}\in\mathcal{A}e^{j\theta}}{\arg\min} \left[\beta \left|y_I(i) - \alpha\hat{u}_I\right|^2 + \alpha \left|y_Q(i+2) - \beta\hat{u}_Q\right|^2\right], i = 1, 2; \tag{10}$$

$$\hat{u}_i = \underset{\hat{u}\in\mathcal{A}e^{j\theta}}{\arg\min} \left[\alpha \left|y_I(i) - \beta\hat{u}_I\right|^2 + \beta \left|y_Q(i-2) - \alpha\hat{u}_Q\right|^2\right], i = 3, 4; \tag{11}$$

The above two ML decision metrics reveal that the variables are completely decoupled, so that full-rate transmission and SML decoding are fulfilled at the receiver.

2.3.2 The QR-DDF Decoder

When the channel is time-selective, $\mathcal{H}$ is non-orthogonal (i.e., the Grammian matrix $\boldsymbol{G}$ is not diagonal) and the off-diagonal terms ε_1 and ε_2 are not zero, which introduces interference between consecutive symbols. Thus, the above SML decoder suffers from performance degradation. In this case, the low complexity QR-DDF decoder is applied [15]. The firstly, we perform a QR-decomposition of the channel matrix $\mathcal{H}$ in (4) as

$$\mathcal{H} = \boldsymbol{QR} \tag{12}$$

where the $4N_R \times 4$ matrix $\boldsymbol{Q}$ is unitary (i.e., $\boldsymbol{Q}^H\boldsymbol{Q} = \mathbf{I}_{4\times4}$) and the 4×4 matrix $\boldsymbol{R}$ is upper triangular with real diagonal elements. Then, by multiplying the received vector $\boldsymbol{r}$ by $\boldsymbol{Q}^H$, we obtain

$$\bar{\boldsymbol{y}} = \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{r} = \boldsymbol{Rs} + \bar{\boldsymbol{v}} \tag{13}$$

where $\bar{\boldsymbol{\nu}} = \boldsymbol{Q}^{\mathrm{H}}\boldsymbol{\nu}$ has the same statistical properties as $\boldsymbol{\nu}$ due to the property of the unitary matrix $\boldsymbol{Q}$, and the upper triangular matrix $\boldsymbol{R}$ is given by

$$\boldsymbol{R} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \tag{14}$$

From the above peculiar structure of $\boldsymbol{R}$, u_2 and u_4 (i.e., ultimately x_2 and x_4) can be directly decoded without considering the interference caused by the other transmitted symbols (i.e., x_1 and x_3) as

$$\hat{u}_2 = \underset{\hat{u}\in\mathcal{A}e^{j\theta}}{\arg\min}\left[\left|\bar{y}_I(2) - a_{22}\hat{u}_I\right|^2 + \left|\bar{y}_Q(4) - a_{44}\hat{u}_Q\right|^2\right] \tag{15}$$

$$\hat{u}_4 = \underset{\hat{s}\in\mathcal{A}e^{j\theta}}{\arg\min}\left[\left|\bar{y}_I(4) - a_{44}\hat{u}_I\right|^2 + \left|\bar{y}_Q(2) - a_{22}\hat{u}_Q\right|^2\right] \tag{16}$$

Accordingly, after re-interleaving the in-phase and quadrature components of $\hat{u}_2$ and $\hat{u}_4$, the decision statistics for s_1 and s_3 are obtained as

$$\tilde{y}(1) = \bar{y}(1) - a_{12}\left(\hat{s}_{2I} + j\hat{s}_{4Q}\right) \tag{17}$$

$$\tilde{y}(3) = \bar{y}(3) - a_{34}\left(\hat{s}_{4I} + j\hat{s}_{2Q}\right) \tag{18}$$

Therefore, the estimates of u_1 and u_3 can be obtained from the following decision metrics as

$$\hat{u}_1 = \underset{\hat{u}\in\mathcal{A}e^{j\theta}}{\arg\min}\left[\left|\tilde{y}_I(1) - a_{11}\hat{u}_I\right|^2 + \left|\tilde{y}_Q(3) - a_{33}\hat{u}_Q\right|^2\right] \tag{19}$$

$$\hat{u}_3 = \underset{\hat{u}\in\mathcal{A}e^{j\theta}}{\arg\min}\left[\left|\tilde{y}_I(3) - a_{33}\hat{u}_I\right|^2 + \left|\tilde{y}_Q(1) - a_{11}\hat{u}_Q\right|^2\right] \tag{20}$$

Although SML and QR-DDF decoders are presented for the CIOD with $N_T = 4(i.e., \boldsymbol{S}_4)$, but they can be directly applied for the CIOD with $N_T = 3$(i.e., $\boldsymbol{S}_3$) over quasi-static and time-selective fading channels. Moreover, the CIOD with $N = 2$(i.e., $\boldsymbol{S}_2$) achieves full-diversity even in time-selective fading channels, so the SML decoder can be employed regardless of the time-selectivity of fading channels.

Remark 1 As showed in [15], the decoding complexity of each decoder is linearly proportional to the constellation size M, and the decoding complexity of the QR-DDF decoder is comparable to that of the conventional SML decoder.

Remark 2 For other single-symbol ML decodable STBCs as MDC-QOSTBCs [12–14], their upper triangular matrix $\boldsymbol{R}$, which achieved from QR-decomposition of their equivalent channel matrix $\boldsymbol{\mathcal{H}}$, has form

$$\boldsymbol{R} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{4} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \tag{21}$$

Comparing (21) and (14) we can see that low complexity QR-DDF decoder can not use for MDC-QOSTBCs.

3 The Proposed NZE-STBCs and Their Properties

3.1 The Proposed NZE-STBCs

From (3), we can see that CIODs for 2 and 4 transmit antennas have the same structure $\begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B} \end{bmatrix}$, where $\boldsymbol{A}, \boldsymbol{B}$ is matrix of Alamouti code [2]. To eliminate completely zeros in codeword matrix of CIODs, we propose a new structure $\begin{bmatrix} \boldsymbol{A} & \boldsymbol{A} \\ -\boldsymbol{B} & \boldsymbol{B} \end{bmatrix}$ to design NZE-STBCs from CIODs. Based on the structure $\begin{bmatrix} \boldsymbol{A} & \boldsymbol{A} \\ -\boldsymbol{B} & \boldsymbol{B} \end{bmatrix}$ and CIODs (3) we propose NZE-STBCs for 2, 3 and 4 Tx as:

$$\mathbf{Z}_2(s_1, s_2) = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 & s_1 \\ -s_2 & s_2 \end{bmatrix} \tag{22a}$$

$$\mathbf{Z}_3(s_1, s_2, s_3, s_4) = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 & s_2 & s_1 \\ -s_2^* & s_1^* & -s_2^* \\ -s_3 & -s_4 & s_3 \\ s_4^* & -s_3^* & -s_4^* \end{bmatrix} \tag{22b}$$

$$\mathbf{Z}_4(s_1, s_2, s_3, s_4) = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 & s_2 & s_1 & s_2 \\ -s_2^* & s_1^* & -s_2^* & s_1^* \\ -s_3 & -s_4 & s_3 & s_4 \\ s_4^* & -s_3^* & -s_4^* & s_3^* \end{bmatrix} \tag{22c}$$

where $\boldsymbol{Z}_3$ is obtained by deleting the last column of $\boldsymbol{Z}_4$, transmitted signals s_i in NZE-STBCs (22a, 22b, 22c) are similar as transmitted signals s_i in CIODs (3). The factor $\frac{1}{\sqrt{2}}$ ensures total power on each transmit antenna of the both CIODs and our NZE-STBCs is the same, i.e., $E\left\{\|\boldsymbol{S}_i\|_F^2\right\} = E\left\{\|\boldsymbol{Z}_i\|_F^2\right\}, i = 2, 3, 4$.

Remark 3 It is a worth to notice that, the structure $\begin{bmatrix} \mathcal{A} & \mathcal{A} \\ -\mathcal{B} & \mathcal{B} \end{bmatrix}$ also can apply to reduce zero entries for all CIODs [10] for arbitrary number of transmit antennas. However, in this paper we only consider to full rate CIODs. Thus, other CIODs are out of scope of this chapter.

3.2 Properties of the Proposed NZE-STBCs

3.2.1 Full Rate and Full Diversity

Theorem 1 *The proposed NZE-STBCs for 3 and 4 transmit antennas are full-rate full-diversity codes in quasi-static fading channels while the NZE-STBC for 2 transmit antennas is full-rate full-diversity code in the both quasi-static and time-selective fading channels.*

Proof It is easily to see that the proposed NZE-STBCs are full-rate codes. In order to meet the full-diversity criterion, the codeword difference matrix $\mathbf{B} = \mathbf{C} - \mathbf{C}'$ should be of full-rank (i.e., det($\boldsymbol{B}^{\mathrm{H}}\boldsymbol{B}$) is non-zero) [1], where $\mathbf{C}$ and $\mathbf{C}'$ are two distinct codeword matrices obtained from a STBC $\mathbf{C}$. From (3) and (22a, 22b, 22c), we obtain

$$\det\left(\left(\mathbf{Z}_2-\mathbf{Z}_2'\right)^H\left(\mathbf{Z}_2-\mathbf{Z}_2'\right)\right)=\det\left(\left(\mathbf{S}_2-\mathbf{S}_2'\right)^H\left(\mathbf{Z}_2-\mathbf{S}_2'\right)\right)=\Delta_1^2\Delta_2^2 \tag{23}$$

$$\det\left(\left(\mathbf{Z}_4-\mathbf{Z}_4'\right)^H\left(\mathbf{Z}_4-\mathbf{Z}_4'\right)\right)=\det\left(\left(\mathbf{Z}_4-\mathbf{S}_4'\right)^H\left(\mathbf{Z}_4-\mathbf{S}_4'\right)\right)=\left(\Delta_1^2+\Delta_2^2\right)^2\left(\Delta_3^2+\Delta_4^2\right)^2 \tag{24}$$

where $\Delta_i^2=\left|s_i-s_i'\right|^2, i=1,2,3,4.$ □

Equations (23) and (24) demonstrate that NZE-STBCs $\mathbf{Z}_2, \mathbf{Z}_4$ and CIODs $\mathbf{S}_2, \mathbf{S}_4$ have the same coding gain under the same conditions. This means that NZE-STBCs $\mathbf{Z}_2, \mathbf{Z}_4$ are full diversity codes and have the same coding gain with CIODs $\mathbf{S}_2, \mathbf{S}_4$. For the NZE-STBC $\mathbf{Z}_3$ in (22a, 22b, 22c), we have

$$\begin{aligned}
&\det\left(\left(\mathbf{Z}_3-\mathbf{Z}_3'\right)^H\left(\mathbf{Z}_3-\mathbf{Z}_3'\right)\right)\\
&=\frac{1}{2}\left(\Delta_1^2+\Delta_2^2\right)\left(\Delta_3^2+\Delta_4^2\right)\left(\Delta_1^2+\Delta_2^2+\Delta_3^2+\Delta_4^2\right)\\
&=\frac{1}{2}\left\{\left(u_{1I}-u_{1I}'\right)^2+\left(u_{3Q}-u_{3Q}'\right)^2+\left(u_{2I}-u_{2I}'\right)^2+\left(u_{4Q}-u_{4Q}'\right)^2\right\}\\
&\quad\times\left\{\left(u_{3I}-u_{3I}'\right)^2+\left(u_{1Q}-u_{1Q}'\right)^2+\left(u_{4I}-u_{4I}'\right)^2+\left(u_{2Q}-u_{2Q}'\right)^2\right\}\\
&\quad\times\left\{\begin{array}{l}\left(u_{1I}-u_{1I}'\right)^2+\left(u_{3Q}-u_{3Q}'\right)^2+\left(u_{2I}-u_{2I}'\right)^2+\left(u_{4Q}-u_{4Q}'\right)^2+\cdots\\ \left(u_{3I}-u_{3I}'\right)^2+\left(u_{1Q}-u_{1Q}'\right)^2+\left(u_{4I}-u_{4I}'\right)^2+\left(u_{2Q}-u_{2Q}'\right)^2\end{array}\right\}
\end{aligned} \tag{25}$$

Clearly, the above determinant is minimum if and only if u_k differs from u_{k}' for only one k. Therefore assume, without loss of generality, that the codeword matrices $\mathbf{Z}_3$ and $\mathbf{Z}_3'$ are such that they differ by only one variable, say u_1 taking different values from the rotated signal set$\tilde{\mathcal{A}}=\mathcal{A}e^{j\theta}$. Then,

$$\det\left(\left(\mathbf{Z}_3-\mathbf{Z}_3'\right)^H\left(\mathbf{Z}_3-\mathbf{Z}_3'\right)\right)=\left(u_{1I}-u_{1I}'\right)^2\left(u_{1Q}-u_{1Q}'\right)^2\frac{\left(u_{1I}-u_{1I}'\right)^2+\left(u_{1Q}-u_{1Q}'\right)^2}{2} \tag{26}$$

From (26) we obtain

$$\begin{aligned}
&\min_{u_1\neq u_1'}\det\left(\left(\mathbf{Z}_3-\mathbf{Z}_3'\right)^H\left(\mathbf{Z}_3-\mathbf{Z}_3'\right)\right)\\
&\quad=\min_{u_1\neq u_1'}\left(\left(u_{1I}-u_{1I}'\right)^2\left(u_{1Q}-u_{1Q}'\right)^2\frac{\left(u_{1I}-u_{1I}'\right)^2+\left(u_{1Q}-u_{1Q}'\right)^2}{2}\right)\\
&\quad\geq\min_{u_1\neq u_1'}\left\{\left(\left|u_{1I}-u_{1I}'\right|\left|u_{1Q}-u_{1Q}'\right|\right)^3\right\}=\left(CPD\left(\tilde{\mathcal{A}}\right)\right)^3
\end{aligned} \tag{27}$$

Here, the metric $\min_{x_k\neq x_k'\in\mathcal{A}}\left|x_{kI}-x_{kI}'\right|\left|x_{kQ}-x_{kQ}'\right|$ is called the co-ordinate product distance (CPD) of $\mathcal{A}$ and denoted as $CPD(\mathcal{A})$ [10]. From the Eq. (27) it is clear that the NZE-STBC $\mathbf{Z}_3$ will give full-diversity if and only if the CPD of the signal set is nonzero. Research results in [10] showed that a rotated constellation$\tilde{\mathcal{A}}=\mathcal{A}e^{j\theta}$ always ensures nonzero $CPD\left(\tilde{\mathcal{A}}\right)$. Therefore, the NZE-STBC $\mathbf{Z}_3$ is full diversity code.

Finally, to complete proof of Theorem 1, we need demonstrate that the NZE-STBC $\boldsymbol{Z}_2$ achieves full diversity over time-selective fading channels. In [10], authors also show that a STBC achieve full-diversity in time-selective fading channels if its extended codeword difference matrix is full rank. For $\boldsymbol{Z}_2$, its extended codeword matrix (ExCM) is $\mathbb{Z}_2 = \begin{bmatrix} s_1 & s_1 & 0 & 0 \\ 0 & 0 & -s_2 & s_2 \end{bmatrix}$ and the difference of two distinct ExCM $\mathbb{Z}_2$ is $\mathbb{B}_2 = \mathbb{Z}_2 - \mathbb{Z}_2' = \begin{bmatrix} s_1 - s_1' & s_1 - s_1' & 0 & 0 \\ 0 & 0 & -s_2 + s_2' & s_2 - s_2' \end{bmatrix}$. It is easily to demonstrate that the optimum phase rotation, which makes codeword difference matrix $\boldsymbol{B}_2 = \boldsymbol{S}_2 - \boldsymbol{S}_{2'}$ full rank, makes $\mathbb{B}_2$ full rank, so $\boldsymbol{Z}_2$ is full diversity code in time-selective fading channels. The proof is completed.

3.2.2 Low Complexity Decoding

In this sub-section we will show that low complexity SML and QR-DDF decoders which are applied for CIODs, also can apply for the proposed NZE-STBCs. We consider to the NZE-STBC $\boldsymbol{Z}_4$ with $N_T = 4$ given in (22a, 22b, 22c). After some straightforward manipulations of (1), the received signal of the NZE-STBC $\boldsymbol{Z}_4$ can be expressed in vector/matrix form as

$$\boldsymbol{r} = \boldsymbol{\mathcal{H}}\boldsymbol{s} + \boldsymbol{v} \tag{28}$$

where $\boldsymbol{r}$ is $4N_R \times 1$ received signal vector, $\boldsymbol{s} = [s_1 s_2 s_3 s_4]^{\mathrm{T}}$ is transmitted signal vector, $\boldsymbol{v}$ is $4N_R \times 1$ AWGN noise vector and $\boldsymbol{\mathcal{H}} = \left[\boldsymbol{\mathcal{H}}_1^T, \boldsymbol{\mathcal{H}}_2^T, \ldots, \boldsymbol{\mathcal{H}}_{N_R}^T\right]^T$ is the effective channel matrix, for instance, whose sub-matrix $\boldsymbol{\mathcal{H}}_n$, $n = 1, 2, \ldots, N_R$ for the NZE-STBC with $N_T = 4$ is given by

$$\begin{aligned} \boldsymbol{\mathcal{H}}_n &= \frac{1}{\sqrt{2}} \begin{bmatrix} h_{n1}(1) + h_{n3}(1) & h_{n2}(1) + h_{n4}(1) & 0 & 0 \\ h_{n2}^*(2) + h_{n4}^*(2) & -h_{n1}^*(2) - h_{n3}^*(2) & 0 & 0 \\ 0 & 0 & h_{n3}(3) - h_{n1}(3) & h_{n4}(3) - h_{n2}(3) \\ 0 & 0 & h_{n4}^*(4) - h_{n2}^*(4) & -h_{n3}^*(4) + h_{n1}^*(4) \end{bmatrix} \\ &\triangleq \begin{bmatrix} c_{n1}(1) & c_{n2}(1) & 0 & 0 \\ c_{n2}^*(2) & -c_{n1}^*(2) & 0 & 0 \\ 0 & 0 & c_{n3}(3) & c_{n4}(3) \\ 0 & 0 & c_{n4}^*(4) & -c_{n3}^*(4) \end{bmatrix} \end{aligned} \tag{29}$$

From (5) and (29) we see that the effective channel matrixes of CIODs and NZE-STBCs have the same structure. Therefore, low complexity SML and QR-DDF decoders can apply to the proposed NZE-STBCs without sacrificing decoding performance

4 Performance Comparisons

As demonstrated by Hoo-Jin Lee in Ref. [15] (Section 5.2.3, pp. 98–104), the performance of existing QOSTBCs with pair-wise symbol maximum likelihood decoder is worse than that of CIODs with QR-DDF decoder. So, we only perform comparing performance between the proposed NZE-STBC with CIODs and we feel that comparing with existing QOSTBCs is not necessary.

It has been shown in [1] that the performance of a space-time code can be optimized by maximizing the minimum determinant of the codeword distance matrix (i.e., coding gain). For practical implementation, it has further been pointed out in [11] that the probability P_0 that an antenna transmits the "zero" symbol, should be kept as low as possible, to achieve a

Table 1 Comparisons between CIODs and the proposed NZE-STBCs with 4QAM modulation

	Optimum constellation angle (°)	Minimum determinant	P_0 (%)	PAPR (dB)
For 2 Tx				
CIOD	31.7175	3.20	50	5.79
Proposed NZE-STBC	31.7175	3.20	0	2.79
For 3 Tx				
CIOD	31.7175	3.53	50	5.79
Proposed NZE-STBC	31.7175	6.40	0	2.79
For 4 Tx				
CIOD	31.7175	10.24	50	5.79
Proposed NZE-STBC	31.7175	10.24	0	2.79

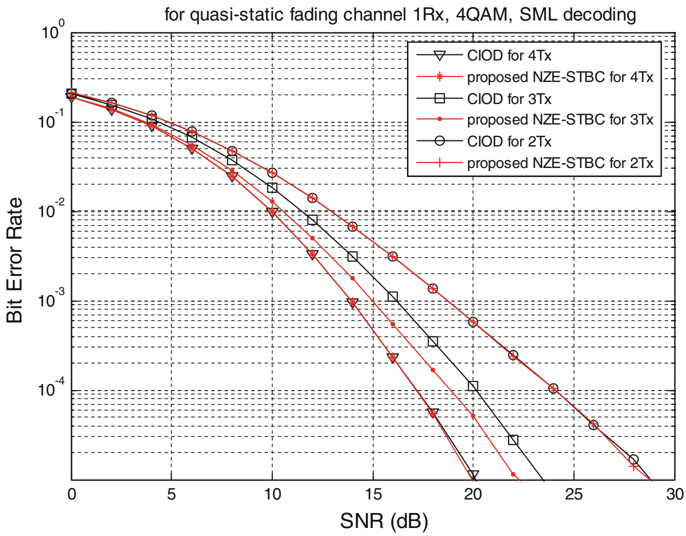

Fig. 1 Comparison of BER performance between the CIODs [10] and proposed NZE-STBCs under average power constraint over quasi-static flat Rayleigh fading channel with parameters: one receive antenna, rotated 4QAM constellation with rotation phase $\theta = 31.7175°$, and the SML decoding

low PAPR. The optimum constellation-rotation angle, minimum determinant (coding gain), and P_0 values of CIODs and our NZE-STBC with 4-QAM constellation are compared in Table 1, while their bit error rates (BER) under average power constrain are compared in Figs. 1 and 2.

From Table 1 and Figs. 1 and 2, we can observe that, for cases of 2 and 4 transmit antennas, although they have almost identical decoding performance, our proposed NZE-STBC does not require any transmit antenna to transmit zero (hence, achieving the ideal value of $P_0 = 0$), while CIODs requires half of the transmit antennas to transmit zero at any one time (hence $P_0 = 50\,\%$). Eliminating zero-entries in the proposed NZE-STBCs results in two advantages over CIOD codes:

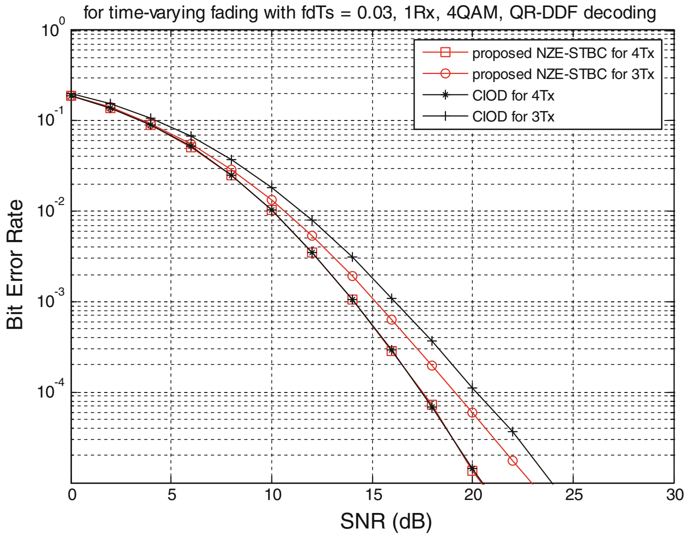

Fig. 2 Comparison of BER performance between the CIODs [10] and proposed NZE-STBCs under average power constraint over time-selective flat Rayleigh fading channel with parameters: normalized Doppler frequency shift $f_d T_s = 0.03$, one receive antenna, rotated 4QAM constellation with rotation phase $\theta = 31.7175°$, and the QR-DDF decoding

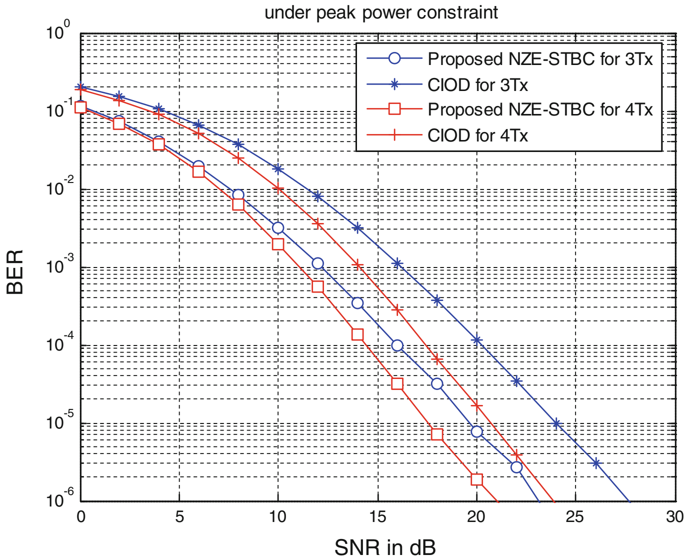

Fig. 3 Comparison of BER performance between the CIODs [10] and proposed NZE-STBCs under peak power constraint over time-selective flat Rayleigh fading channel with parameters: normalized Doppler frequency shift $f_d T_s = 0.03$, one receive antenna, rotated 4QAM constellation with rotation phase $\theta = 31.7175°$, and the QR-DDF decoding

✓ The first advantage is eliminating low-frequency interferences. Because, the regular transmission of "zeros" implies turning off the transmit antennas at regular intervals. This leads to undesirable low-frequency interference.

✓ The second advantage is achieving lower PAPR of 3 dB. The PAPR is an important property of a space-time block code. High PAPR requires a power amplifier with high power consumption and a large back off; such an amplifier is inefficient, bulky and expensive.

For case of three transmit antennas, these results show that our proposed NZE-STBC not only achieve lower PAPR of 3-dB, but also has a higher minimum determinant (hence lower BER) than CIODs. Therefore, our proposed NZE-STBCs are more effective than CIODs in practical implementation. Similar results can be found for different constellations with different number of receive antennas; details are omitted for brevity.

Figure 3 presents simulation results under peak power constraint. The simulation results show that the proposed code under peak power constraint has a significant performance gain of 3 dB compared with CIODs.

5 Conclusions

We have provided a general structure $\begin{bmatrix} A & A \\ -B & B \end{bmatrix}$ to construct NZE-STBCs for 2, 3 and 4 transmit antennas from CIODs. The proposed NZE-STBCs have all desirable properties of CIODs, such as full-rate and low decoding complexity over the both quasi-static and time-selective fading channels. Compared with CIODs, the proposed NZE-STBCs have a better power-distribution property as it does not require any transmit antenna to be turned off. Simulation results demonstrated that, the advantage of the proposed NZE-STBCs do not come with sacrificing decoding performance. In addition, NZE-STBC has better decoding performance than CIODs for three transmit antennas. Therefore, the proposed NZE-STBCs are better solutions than the CIODs in practical implementation. Moreover, the proposed structure $\begin{bmatrix} A & A \\ -B & B \end{bmatrix}$ also can be applied to generate SSDCs which have less zeros from CIODs when number of transmit antenna is greater than four.

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